

Machine Learning - Homework 2

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Problem 1

(a)
$$\mu_{y,j} = \sum_{i=1}^n \frac{\mathbb{1}\{y_i = y\}x_{i,j}}{\mathbb{1}\{y_i = y\}}$$

(b) Source code: hw2_p1b.m
Training Error Rate: 21.6257%
Test Error Rate: 37.6016%
Running Time: 30 secs

(c) Source Code: hw2_p1c.m
Training Error Rate: 5.7794%
Test Error Rate: 13.1383%
Running Time: 1.15 secs

(d) Source Code: hw2_p1d.m
 $\alpha_0 = -20.7849$
20 most positive words (corresponding to 20 largest values of α_j)
1: 'firearms'
2: 'occupied'
3: 'israelis'
4: 'serdar'
5: 'argic'
6: 'ohanus'
7: 'appressian'
8: 'sahak'
9: 'melkonian'
10: 'villages'
11: 'cramer'
12: 'armenia'
13: 'cpr'
14: 'sdpa'
15: 'handgun'
16: 'optilink'
17: 'palestine'

- 18: 'firearm'
- 19: 'budget'
- 20: 'arabs'

20 most negative words (corresponding to 20 smallest values of

α_j)

- 1: 'athos'
- 2: 'atheism'
- 3: 'atheists'
- 4: 'clh'
- 5: 'teachings'
- 6: 'revelation'
- 7: 'testament'
- 8: 'livesey'
- 9: 'atheist'
- 10: 'wpd'
- 11: 'solntze'
- 12: 'scriptures'
- 13: 'theology'
- 14: 'believers'
- 15: 'ksand'
- 16: 'alink'
- 17: 'benedikt'
- 18: 'jesus'
- 19: 'prophet'
- 20: 'mozumder'

Problem 2

- (a) The classifier f^* predicts 1 when $c^2_3 N(0, 1) \leq \frac{1}{3} N(2, \frac{1}{4})$

Thus, the range in which f^* predicts 1 can be given by the roots of:

$$c^2_3 N(0, 1) = \frac{1}{3} N(2, \frac{1}{4})$$

Thus, we get the quadratic equation

$$3x^2 - 16x + 16 + 2\log(c) = 0$$

which give the roots:

$$x = \frac{16 \pm \sqrt{256 - 12(16 + 2\log(c))}}{6}$$

$$x = \frac{16 \pm \sqrt{64 - 24\log(c)}}{6}$$

For $1 \leq c \leq 14$,

f^* predicts 1 when x is in the interval $[\frac{16 - \sqrt{64 - 24\log(c)}}{6}, \frac{16 + \sqrt{64 - 24\log(c)}}{6}]$

- (b) For $c \geq 15$, the roots are imaginary i.e. they don't exist. Thus, the classifier will always predict 0.

Problem 3

- (a) Covariance matrix can be written as:
 $\Sigma = U\Lambda U^T$ where U is orthonormal
 Thus, $\Sigma + \sigma^2 I = U\Lambda U^T + \sigma^2 U U^T$
 $\Sigma + \sigma^2 I = U(\Lambda + \sigma^2 I)U^T$
 Thus, the eigenvalues of $\Sigma + \sigma^2 I$ are

$$\lambda_1 + \sigma^2, \lambda_2 + \sigma^2, \dots, \lambda_d + \sigma^2$$

- (b) Similarly, we get the eigenvalues of $(\Sigma + \sigma^2 I)^{-2}$ as

$$(\lambda_1 + \sigma^2)^{-2}, (\lambda_2 + \sigma^2)^{-2}, \dots, (\lambda_d + \sigma^2)^{-2}$$

References

- [1] <https://www.cs.ubc.ca/~murphyk/Teaching/Stat406-Spring08/Lectures/linalg1.pdf>
- [2] <http://cs229.stanford.edu/section/gaussians.pdf>
- [3] https://ocw.mit.edu/courses/mathematics/18-06sc-linear-algebra-fall-2011/least-squares-determinants-and-eigenvalues/diagonalization-and-powers-of-a/MIT18_06SCF11_Ses2.9sum.pdf