# STAT 8010 - FINAL EXAM

1. In this question, the parameter of interest is the difference of means.

Since we are performing a hypothesis test on difference of means with an extraneous variable, i.e, same patient, we need to choose dependent or paired experimental design.

| Patient | Before | After | Difference ($d_i$) (Before - After) |
|---|---|---|---|
| 1 | 22.86 | 16.11 | 6.75 |
| 2 | 7.74 | -4.02 | 11.76 |
| 3 | 15.49 | 8.04 | 7.45 |
| 4 | 9.97 | 3.29 | 6.68 |
| 5 | 1.44 | -0.77 | 2.21 |

Let $\mu_B$ be the average of the readings before the treatment and $\mu_A$ be the average of the readings after the treatment.

$$\mu_D = \mu_B - \mu_A$$

Hypotheses:-

$H_0 : \mu_D = 0$

$H_a : \mu_D > 0$

Test statistic :-

$$t_{obs} = \frac{\bar{d} - D_0}{S_d/\sqrt{n}}$$

$$\bar{d} = \frac{d_1 + d_2 + \dots d_5}{5} = \frac{6.75 + 11.76 + 7.45 + 6.68 + 2.21}{5}$$

$$= 6.97$$

$$D_0 = 0$$

$$S_d = \sqrt{\frac{\Sigma (d_i - \bar{d})^2}{n-1}}$$

$$= \sqrt{\frac{(6.75-6.97)^2 + (11.76-6.97)^2 + (7.45-6.97)^2 + (6.68-6.97)^2 + (2.21-6.97)^2}{5-1}}$$

$$= 3.38$$

$$n = 5$$

$$t_{obs} = \frac{6.97 - 0}{3.38/\sqrt{5}} = 4.61$$

Rejection Region :-

$$t_{obs} > t_{n-1,\alpha} = t_{5-1,0.05} = t_{4,0.05} = 2.132$$

$$t_{obs} > 2.132$$

**Decision:-**

$t_{obs} > 2.132$

$\Rightarrow 4.61 > 2.132$ ✓

Therefore, we reject $H_0$

**Conclusion:-**

There is sufficient evidence to conclude $H_a$.

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2. a) This is a special case as the sample size of all brands is equal.

| Brand | Sample Size | Sample mean | Sample s.d |
|---|---|---|---|
| Low Tar | 100 | 8 | 0.3 |
| A | 100 | 10 | 0.4 |
| B | 100 | 10 | 0.4 |
| C | 100 | 11 | 0.5 |

**Hypotheses:-**

$H_0: \mu_{LT} = \mu_A = \mu_B = \mu_c$

$H_a$: Not all the means are equal

**Test statistics:-**

$$F_{obs} = S_B^2 / S_W^2$$

$S_B^2 = n \times (\text{variance of sample means})$

Average of sample means, $\bar{y} = \dfrac{8+10+10+11}{4} = 9.75$

Variance of sample means $= \dfrac{(9.75-8)^2+(9.75-10)^2+(9.75-10)^2+(9.75-11)^2}{4-1}$

$= 1.5833$

$S_B^2 = 100 \times 1.5833 = 158.33$

$S_W^2 = \dfrac{S_{LT}^2+S_A^2+S_B^2+S_C^2}{t} = \dfrac{0.3^2+0.4^2+0.4^2+0.5^2}{4} = 0.165$

$F_{obs} = \dfrac{158.33}{0.165} = 959.57$

Rejection Region :-

$F_{obs} > F_{t-1,\, nt-t,\, \alpha} = F_{3,\,396,\,0.05} = 2.60$

Decision :-

$959.57 > 2.60$

Reject $H_0$

Conclusion :-

There is sufficient evidence to conclude $H_a$.

b) Assumption 1 :- The samples are independent random samples.

The above assumption is satisfied for this problem.

Assumption 2 :- Each sample is selected from a normal population.

The above assumption is also satisfied.

Assumption 3 :-

Hypothesis

$H_0: \sigma^2_{LT} = \sigma^2_A = \sigma^2_B = \sigma^2_C$

$H_a$: Not all variances are equal

Test statistic

$$F_{max} = \frac{s^2_{max}}{s^2_{min}} = \frac{0.5^2}{0.3^2} \approx 2.77$$

Rejection Region

$$F_{max} > F_{max_{t,n-1,\alpha}} = F_{max_{4,99,0.05}} = 1.96$$

Decision

$2.77 > 1.96$ ✓ $\Rightarrow$ Reject $H_0$

Conclusion

There is sufficient evidence to conclude $H_a$. Therefore there is evidence of violation.

3. a) This is a randomized block design.

b) $TSS = 897.18$

$$SST = \sum_{i=1}^{t} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{bt}$$

$$= \frac{90.2^2 + 84.3^2 + 89.6^2 + 89.8^2}{4} - \frac{353.9^2}{4\times4}$$

$$= 5.85$$

$$SSB = \sum_{j=1}^{b} \frac{y_{.j}^2}{t} - \frac{y_{..}^2}{bt}$$

$$= \frac{55.8^2 + 125.4^2 + 69.3^2 + 103.4^2}{4} - \frac{353.9^2}{4\times4}$$

$$= 755.38$$

$$SSE = TSS - SST - SSB$$

$$= 897.18 - 5.85 - 755.38$$

$$= 135.95$$

$t = 4$

$b = 4$

$$MST = SST/(t-1) = \frac{5.85}{4-1} = 1.95$$

$MSB = SSB/(b-1) = \frac{755.38}{4-1} = 251.79$

$MSE = SSE/(b-1)(t-1) = \frac{135.95}{(4-1)(4-1)} = 15.1$

$F_{obs}(\text{Treatments}) = MST/MSE = 1.95/15.1 = 0.12$

$F_{obs}(\text{Blocks}) = MSB/MSE = 251.79/15.1 = 16.67$

ANOVA Table:-

| Source Due to | Sum of Squares (SS) | df | Mean Square (MS) | F |
|---|---|---|---|---|
| Treatments | 5.85 | 3 | 1.95 | 0.12 |
| Blocks | 755.38 | 3 | 251.79 | 16.67 |
| Error | 135.95 | 9 | 15.1 | |
| Totals | 897.18 | 15 | | |

c) Hypotheses:-

$H_0 : \alpha_A = \alpha_B = \alpha_C = \alpha_D = 0$

$H_a$ : Not all of the above equal to zero

Test statistic:-

$F_{Obs}$ (treatments) $= 0.12$ (from ANOVA table)

P-value:-

$$\text{p-value} = P\left(F_{t-1,(b-1)(t-1),\alpha} > F_{obs}\right)$$

$$= P\left(F_{3,9,0.05} > 0.12\right)$$

$> 0.25$ (from F table)

Decision:-

$\alpha = 0.05$

P-value $> \alpha$, therefore we fail to reject $H_0$

Conclusion:-

There is no sufficient evidence to claim $H_a$.

d) The extraneous variable is the car model.

Hypotheses:-

$H_0 : \beta_A = \beta_B = \beta_C = \beta_D = 0$

$H_a$ : Not all of the above are equal to zero

Test Statistic:-

$F_{obs}$ (blocks) $= 16.67$ (from ANOVA table)

P-value:-

$$p\text{-value} = P(F_{b-1,\,(b-1)(t-1),\,\alpha} > F_{obs})$$

$$= P(F_{3,9,0.05} > 16.67)$$

$$< 0.001$$

Decision:-

$\alpha = 0.05$

p-value $< \alpha$, therefore we reject $H_0$

Conclusion:-

There is enough evidence to claim $H_a$.

e) Relative efficiency $= \dfrac{MSE_{CR}}{MSE_{RB}}$

$$= \frac{(b-1)MSB + b(t-1)MSE_{RB}}{(bt-1)MSE_{RB}}$$

$$= \frac{(4-1)251.79 + 4(4-1)15.1}{((4\times4)-1)15.1}$$

$$= \underline{\underline{4.13}}$$

4. a) This is completely randomized design.

b) $TSS = 6.93$

$$SSB = \sum_{i=1}^{t} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{n_T}$$

$n_T = 12$

$$SSB = \frac{10.6^2 + 15.9^2 + 17.3^2}{4} - \frac{43.8^2}{12}$$

$$= 6.24$$

$SSW = TSS - SSB = 6.93 - 6.24 = 0.69$

$t = 3$

$s_B^2 = SSB/t-1 = 6.24/3-1 = 3.12$

$$S_W^2 = SSW/(n_T - t) = 0.69/(12-3) = 0.07$$

~~ANOVA Table:~~

$$F_{obs} = S_B^2 / S_W^2 = 3.12/0.07 = 44.57$$

ANOVA Table :-

| Source Due to | Sum of Squares(SS) | df. | Mean Square(MS) | F |
|---|---|---|---|---|
| Between samples | ~~SSB~~ 6.24 | 2 | 3.12 | 44.57 |
| Within samples | ~~SSW~~ 0.69 | 9 | 0.07 | |
| Totals | 6.93 | 11 | | |

c) Hypotheses :-

$H_0 : \mu_A = \mu_B = \mu_C$

$H_a$ : Not all the means are equal

Test statistic :-

$F_{obs} = 44.57$

p-value :-

$$\text{p-value} = P(F_{t-1, n_T - t} > F_{obs})$$

$$= P(F_{2,9} > 44.57)$$

$$< 0.01$$

Decision:-

$\alpha = 0.05$

P-value $< \alpha$, therefore we reject $H_0$

Conclusion:-

There is sufficient evidence to conclude $H_a$.

5. a) Dependent variable $\Rightarrow$ Quantity sold

Independent variable $\Rightarrow$ Price

b) Hypotheses:-

$H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

Test statistic:-

$$t_{obs} = \frac{\hat{\beta}_1 - \beta_{1_0}}{\frac{S_\epsilon}{\sqrt{S_{xx}}}}$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-29.5}{24} = -1.22$$

$$t_{obs} = \frac{-1.22 - 0}{1.8/\sqrt{24}} = -3.32$$

Rejection Region :-

$$|t_{obs}| > t_{n-2,\alpha/2} = t_{4,0.025} = 2.776$$

$$|t_{obs}| > 2.776$$

Decision :-

$$|-3.32| > 2.776 \Rightarrow 3.32 > 2.776 \checkmark$$

Therefore, we reject $H_0$

Conclusion :-

There is enough evidence to claim $H_a$.

6.a) To choose between pooled t-test and separate t-test, we need to perform F-test.

Hypotheses :-

$H_0 : \sigma_M^2 = \sigma_C^2$

$H_a : \sigma_M^2 \neq \sigma_C^2$

Test statistic :-

$$F_{obs} = \frac{S_i^2}{S_j^2}, \text{ where } S_i^2 > S_j^2$$

$$= \frac{S_c^2}{S_M^2} = \frac{480^2}{350^2} = 1.88$$

Rejection Region:-

$$F_{obs} > F_{n_c-1,\, n_M-1,\, 0.025} = F_{8,9,0.025} = 4.10$$

(or)

$$< \frac{1}{F_{n_M-1,\, n_c-1,\, 0.025}} = \frac{1}{F_{9,8,0.025}} = \frac{1}{4.36} = 0.22$$

$F_{obs} > 4.10$ (or) $< 0.22$

Decision:-

$F_{obs} = 1.88$

It is not greater than 4.10 or less than 0.22, we fail to reject $H_0$.

Conclusion:-

There is no sufficient evidence to conclude $H_a$.

Therefore we conduct pooled-t test.

b) Hypotheses:-

$H_0: \mu_c - \mu_M = 0$

$H_a: \mu_c - \mu_M > 0$

Test statistic:-

$$t_{obs} = \frac{(\bar{y}_c - \bar{y}_M) - D_0}{s_p \sqrt{\frac{1}{n_c} + \frac{1}{n_M}}}$$

$$S_p^2 = \frac{(n_c-1)S_c^2 + (n_M-1)S_M^2}{n_c+n_M-2} = \frac{8 \times 480^2 + 9 \times 350^2}{19-2}$$

$$= 173276.47$$

$$S_p = 416.26$$

$$t_{obs} = \frac{(8540-7358)-0}{416.26\sqrt{\frac{1}{9}+\frac{1}{10}}} \approx 6.18$$

Reject Region:-

$$t_{obs} > t_{n_c+n_M-2,\alpha} = t_{17,0.05} = 1.740$$

$$t_{obs} > 1.740$$

Decision:-

Reject $H_0$ since $6.18 > 1.740$

Conclusion:-

There is sufficient evidence to conclude $H_a$.

7.

| School/rating | Outstanding | average | Poor | |
|---|---|---|---|---|
| Undesirable | 10 | 4 | 2 | $R_1 = 16$ |
| Adequate | 12 | 8 | 2 | $R_2 = 22$ |
| | $C_1 = 22$ | $C_2 = 12$ | $C_3 = 4$ | 38 |

X = School type (independent variable)

Y = rating (dependent variable)

Hypotheses :-

$H_0$ : Variables X and Y are independent

$H_a$ : Variables X and Y are dependent

$$E_{11} = \frac{16 \times 22}{38} = 9.26 \qquad E_{21} = 12.73$$

$$E_{12} = \frac{16 \times 12}{38} = 5.05 \qquad E_{22} = 6.94$$

$$E_{13} = \frac{16 \times 4}{38} = 1.68 \qquad E_{23} = 2.31$$

Test statistic :-

$$\chi^2_{obs} = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(n_{ij} - E_{ij})^2}{E_{ij}}$$

$$= \frac{(10 - 9.26)^2}{9.26} + \cdots \frac{(2 - 2.31)^2}{2.31}$$

$$= 1.77$$

$$S_p^2 = \frac{(n_c-1)S_c^2 + (n_M-1)S_M^2}{n_c+n_M-2} = \frac{8 \times 480^2 + 9 \times 350^2}{19-2}$$

$$= 173276.47$$

$$S_p = 416.26$$

$$t_{obs} = \frac{(8540-7358)-0}{416.26\sqrt{\frac{1}{9}+\frac{1}{10}}} = 6.18$$

Reject Region:-

$$t_{obs} > t_{n_c+n_m-2,\alpha} = t_{17,0.05} = 1.740$$

$$t_{obs} > 1.740$$

Decision:-

Reject $H_0$ since $6.18 > 1.740$

Conclusion:-

There is sufficient evidence to conclude $H_a$.

Rejection Region :-

$$\chi^2_{obs} > \chi^2_{(r-1)(c-1),\alpha} = \chi^2_{2,0.05} = 5.991$$

$$\chi^2_{obs} > 5.991$$

Decision :-

Test statistic does not fall in rejection region

We fail to reject $H_0$

Conclusion :-

There is no sufficient evidence to conclude $H_a$

$\therefore$ Rating is not contingent on school type.