1.) STAT 8010 - Final Exam :-

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In this question, the Parameter of interest is the difference of means.

Since we are performing a hypothesis test on difference of means with an extraneous Variable i.e Same patient, we need to choose dependent (or) paired experimental design.

Patient	Before	After	Difference (d_i) (Before - After)
1	22.86	16.11	6.75
2	7.74	-4.02	11.76
3	15.49	8.04	7.45
4	9.97	3.29	6.68
5	1.44	-0.77	2.21

Let μ_B be the average of the readings before the treatment and μ_A be the average of the readings after the treatment.

$\mu_D = \mu_B - \mu_A$

Hypothesis:-

$H_0 : \mu_D = 0$

$H_a : \mu_D > 0$

Test Statistics:-

$$t_{obs} = \frac{\bar{d} - D_0}{S_d / \sqrt{n}}$$

$$\bar{d} = \frac{d_1 + d_2 + \cdots d_5}{5} = \frac{6.75 + 11.76 + 7.45 + 6.68 + 2.21}{5} = 6.97$$

$D_0 = 0$

$$S_d = \sqrt{\frac{\Sigma (d_i - \bar{d})^2}{n-1}}$$

$$= \sqrt{\frac{(6.75-6.97)^2 + (11.76-6.97)^2 + (7.45-6.97)^2 + (6.68-6.97)^2 + (2.21-6.97)^2}{5-1}}$$

$= 3.38.$

$n = 5$

$$t_{obs} = \frac{6.97 - 0}{3.38/\sqrt{5}} = 4.61$$

Rejection Region :-

$t_{obs} > t_{n-1, \alpha} = t_{5-1, 0.05} = t_{4, 0.05} = 2.132$

$t_{obs} > 2.132$

Decision :

$t_{obs} > 2.132$

$\Rightarrow 4.61 > 2.132$ ✓

Therefore we reject H_0.

Conclusion :-

There is sufficient evidence to conclude H_a.

2.)

Brand	Sample size	Sample mean	Sample S.D
Low tar	100	8	0.3
A	100	10	0.5
B	100	10	0.4

a) This is a special case as the sample size of all brands are equal.

Hypothesis:

$H_0: \mu_{low\,tar} = \mu_A = \mu_B$

H_a: Not all the means are equal.

Test Statistics:-

$$F_{obs} = \frac{s^2_B}{s^2_w}$$

$s^2_B = n \times$ (Variance of Sample mean)

Average of Sample mean, $\bar{y} = \frac{8+10+10}{3} = 9.33$

$$\text{Variance of Sample mean} = \frac{(9.33-8)^2 + (9.33-10)^2 + (9.33-10)^2}{3-1}$$

$$= \frac{(1.33)^2 + (-0.67)^2 + (-0.67)^2}{2} = 1.33$$

$s^2_B = 100 \times 1.33 = \underline{\underline{133}}$

$$s^2_w = s^2_{Lowtar} + s^2_A + s^2_B = \frac{0.3^2 + 0.5^2 + 0.4^2}{3} = 0.16$$

$$F_{obs} = \frac{133}{0.16} = 831.25$$

Rejection Region:-

$F_{obs} > F_{t-1,\, nt-t,\, \alpha} = F_{2,\,297,\,0.05} = 3.03$

Decision:-

$831.2 > 3.03$ ✓ (True)

$\therefore$ we Reject Ho.

Conclusion:- There is sufficient evidence to conclude Ha.

b.) Assumption - 1:- The samples are independent random samples.

The above assumption is satisfied for this problem.

Assumption - 2:- Each sample is selected from a normal population

The above assumption is also satisfied.

Assumption - 3:-

Hypothesis: $H_0: \sigma^2_{H} = \sigma^2_{A} = \sigma^2_{B}$

H_a: Not all variances are equal.

Test Statistic:-

$$F_{max} = \frac{s^2_{max}}{s^2_{min}} = \frac{0.5^2}{0.3^2} = 2.77$$

Rejection Region:-

$$F_{max} > F_{max\, t, n-1, \alpha} = F_{max\, 3, 99, 0.05}$$

$= 1.85$

Decision:-

$F_{obs} > 1.85$

$2.77 > 1.85$

$\therefore$ we Reject Ho.

Conclusion: There is sufficient evidence to conclude Ha.

3.) a.) Randomized block design.

b.) Given that TSS = 682.25

$$SST = \sum_{i=1}^{t} \frac{y_i^2}{b} - \frac{y^2}{bt}$$

$$= \frac{(90.4)^2 + (74.3)^2 + (89)^2}{4} - \frac{(253.7)^2}{4 \times 3}$$

$$= 5403.413 - 5363.641$$

$$\boxed{SST = 39.772}$$

$$SSB = \sum_{i=1}^{b} \frac{y_i^2}{t} - \frac{y^2}{bt}$$

$$= 5800.703 - 5363.641$$

$$\boxed{SSB = 437.062}$$

$$SSE = TSS - SSt - SSB$$

$$= 682.25 - 39.772 - 437.062$$

$$\boxed{SSE = 205.416}$$

Calculating $MST = \frac{SST}{(t-1)} = 19.886$

$$MSB = \frac{SSB}{(b-1)} = \frac{437.062}{3} = 145.68$$

$$MSE = \frac{SSE}{(b-1)(t-1)} = \frac{205.416}{3 \times 2} = 34.236$$

F_{obs} (treatments) $= \frac{MST}{MSE} = \frac{19.886}{34.236} = 0.58$

F_{obs} (Blocks) $= \frac{MSB}{MSE} = \frac{145.68}{34.236} = 4.25$

ANOVA table:-

Source Due to	Sum Squares	df	Mean Squares	F
Treatment	39.772	2	19.886	0.58
Blocks	437.062	3	145.6873	4.25
error	205.416	6	34.236	
Total	682.25	11		

c) Hypothesis:-

$H_0: \alpha_A = \alpha_B = \alpha_C$

H_a: Not all means are equal.

Test Statistics:-

F_{obs} (treatments) $= 0.58$

P-value:-

$P(F_{t-1, (b-1)(t-1)} > F_{obs})$

$P(F_{2,6} > 0.58)$

> 0.25

Decision:-

P-value $> \alpha (0.05)$

$\therefore$ we fail to reject H_0.

onclusion:- There is no sufficient evidence to conclude that all means are not equal.

d.)

Hypothesis:

$H_0: B_1 = B_2 = B_3 = B_4$

H_a : Not all of them are equal.

Test Statistics:-

F_{obs} (Blocks) = 4.25

P-value:-

$P(F_{b-1, (b-1)(t-1)} > F_{obs})$

$P(F_{3,6} > 4.25)$

< 0.10 & > 0.05

P-value > α (0.05)

Decision:-

We fail to reject H_0.

Conclusion:-

We don't have enough evidence to conclude there is a significance difference in between block effects.

e.) Relative efficiency $= \dfrac{MSE_{CR}}{MSE_{RB}}$

$$= \frac{(b-1)MSB + b(t-1)MSE_{RB}}{(bt-1)MSE_{RB}}$$

$$= \frac{(3)(145.68) + (4)(2)(34.236)}{(3(4)-1)\,34.236} = 1.88$$

4) a) This is Completely randomized design.

b) $TSS = 6.93$

$$SSB = \sum_{i=1}^{t} \frac{y_i^2}{n_i} - \frac{y_{..}^2}{n_T}$$

$n_T = 12$

$$SSB = \frac{10.6^2 + 15.9^2 + 17.3^2}{4} - \frac{43.8^2}{12}$$

$= 6.24$

$SSW = TSS - SSB = 6.93 - 6.24 = 0.69$

$t = 3$

$s_B^2 = SSB / t-1 = 6.24 / 3-1 = 3.12$

$$s_W^2 = SSW / n_T - t = \frac{0.69}{12-3} = 0.07$$

$$F_{obs} = \frac{s_B^2}{s_W^2} = 3.12 / 0.07 = 44.57$$

ANOVA table:-

Source Due to	Sum of Squares (SS)	df	Mean Square (MS)	F
Between Samples	6.24	2	3.12	44.57
within Samples	0.69	9	0.07	
Totals	6.93	11		

c.) Hypothesis:-

$H_0: \mu_A = \mu_B = \mu_C$

H_a: Not all the means are equal.

Test Statistics:-

$F_{obs} = 44.57$

P-value:-

$\text{P-value} = P(F_{t-1, n_T - t} > F_{obs})$

$= P(F_{2,9} > 44.57)$

< 0.01

Decision:-

$\alpha = 0.05$

P-value $< \alpha$, therefore we reject H_0.

Conclusion:-

There is sufficient evidence to conclude H_a.

5.) a.) Dependent Variable - Price - y

Independent Variable - Mileage - x.

b.) Given that $S_{xx} = 20$, $S_{xy} = 60$.

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{60}{20} = 3$$

$$\bar{y} = \beta_0 + \hat{\beta}_1 \bar{x}$$

$$14 = \beta_0 + (3)(4)$$

$$\beta_0 = 2$$

$\therefore$ equation is $\boxed{\bar{y} = 2 + 3x}$

c.) First we need to find the value of S_E

$$S_E = \sqrt{\frac{S_{SE}}{n-2}}$$

$$S_{SE} = S_{yy} - \frac{S^2_{xy}}{S_{xx}}$$

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} = 1204 - 980 = 224$$

$$S_{SE} = S_{yy} - \frac{S^2_{xy}}{S_{xx}} = 224 - \frac{60^2}{20} = 224 - 180 = 44$$

$$S_E = \sqrt{\frac{S_{SE}}{n-2}} = \sqrt{\frac{44}{3}} = 3.82$$

Hypothesis:-

$H_0: \cancel{\beta_1 = \beta_{10}}$ $\Rightarrow$ $H_0: \beta_1 = 0$

$H_a: \cancel{\beta_1 \neq \beta_{10}}$ $\quad$ $H_a: \beta_1 \neq 0$

Test Statistics:-

$$t_{obs} = \frac{\hat{\beta}_1 - \hat{\beta}_{10}}{SE/\sqrt{S_{xx}}} = \frac{3-0}{3.82/\sqrt{20}} = \frac{3}{3.82/4.43} = \frac{3}{0.85}$$

$$= 3.52$$

Rejection Region:-

$|t_{obs}| > t_{n-2, \alpha/2}$

$|3.52| > t_{3, 0.025}$

$= 3.182$

Decision:-

$|t_{obs}| > t_{3, 0.025}$ $\quad$ We Reject H_0.

$= 3.52 > 3.182$ ✓ (True)

Conclusion:- There is sufficient evidence to conclude H_a.

6.) a) To choose between Pooled P-test and Seperate t-test, we need to perform F-test

Hypothesis:-

$H_0 : \sigma^2_M = \sigma^2_c$

$H_a : \sigma^2_M \neq \sigma^2_c$

Test Statistics:-

$$F_{obs} = \frac{S_i^2}{S_j^2} \quad \text{when } S_i^2 > S_j^2$$

$$= \frac{S_c^2}{S^2_M} = \frac{480^2}{350^2} = 1.88$$

Rejection Region:-

$$F_{obs} > F_{n_c-1, n_M-1, 0.025} = F_{8,9,0.025} = 4.10$$

(or)

$$< \frac{1}{F_{n_M-1, n_c-1, 0.025}} = \frac{1}{F_{9,8,0.025}} = \frac{1}{4.36} = 0.22$$

$F_{obs} > 4.10$ (or) < 0.22

Decision:- $F_{obs} = 1.88$

It is not greater than 4.10 or less than 0.22,

$\therefore$ we fail to reject H_0.

Conclusion:- There is no sufficient evidence to conclude H_a.

Therefore we conduct pooled t-test.

b.) Hypothesis:-

$H_0: \mu_C - \mu_M = 0$

$H_a: \mu_C - \mu_M > 0$

Test - Statistics:-

$$t_{obs} = \frac{(\bar{y}_C - \bar{y}_M) - D_0}{s_p\sqrt{\frac{1}{n_C} + \frac{1}{n_M}}}$$

$$s_p^2 = \frac{(n_C - 1)s_C^2 + (n_M - 1)s_M^2}{n_C + n_M - 2} = \frac{8 \times 486^2 + 9 \times 350^2}{19 - 2}$$

$$= 173276.47$$

$s_p = 416.26$

$$t_{obs} = \frac{(8540 - 7358) - 0}{416.26\sqrt{\frac{1}{9} + \frac{1}{10}}} = 6.18$$

Rejection Region:-

$t_{obs} > t_{n_C + n_M - 2, \alpha} = t_{17, 0.05} = 1.740$

$t_{obs} > 1.740$

Decision:-

Reject H_0 Since $6.18 > 1.740$

Conclusion:-

There is sufficient evidence to Conclude H_a.