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Final Project

4)

a) • Slope of Y on X_1

$$\hat{b}_1 = \frac{S_{x_1 y}}{S_{x_1 x_1}}$$

$S_{x_1 y}$ is the Covariance between X_1 & Y

$S_{x_1 x_1}$ is the variance of X_1

• Slope of Y on X_2

$$\hat{b}_2 = \frac{S_{X_2 Y}}{S_{X_2 X_2}}$$

$S_{X_2 Y}$ is the Covariance between X_2 & Y

$S_{X_2 X_2}$ is the Covariance of X_2

• Slope of regression for X_2 on X_1

$\because$ X_1 & X_2 have sample correlation equal to 0.

$$\therefore \hat{b}_{x_2|x_1} = \frac{S_{x_2 x_1}}{S_{x_1 x_1}} = 0$$

b) • Residuals of regression of Y on X_1 :

$$e_i = y_i - \hat{B}(Y|X_1) * (X_{1i} - \bar{X}_1)$$

• Residual of the regression of X_2 on X_1 :

$$e_i = X_{2i} - \hat{B}_{(X_2|X_1)} * (X_{1i} - \bar{X}_1)$$

c) • Slope of regression corresponding to the added variable plot for the regression of Y on X_2 after X_1 :

$$\hat{B}(Y|X_2, X_1) = \frac{Se2Y}{Se2X_2}$$

• $Se2Y$ is the sum of products of the residuals from the regression of Y on X_1 & Y.
• SeX_2 is the sum of products of the residuals from the regression of X_2 on X_1 & X_2

$(SX_1e2 = 0)$

$$\therefore Se2Y = SXY - \hat{B}(Y|X_1) * SX_1Y$$

&

$$Se2X_2 = SX_2X_2 - \hat{B}(X_2|X_1) * SX_1X_2$$

$$\hat{\beta}(y \mid X_2, X_1) = \frac{(Sxy - \hat{\beta}(y \mid X_1) \cdot SX_1y)}{(SX_2X_2 - \hat{\beta}(X_2 \mid X_1) \cdot SX_1X_2)}$$

$\because SX_1X_2 = 0$ &

$\hat{\beta}(X_2 \mid X_1) = 0$

$$\hat{\beta}(y \mid X_2, X_1) = \frac{Sxy}{SX_2X_2} = \hat{\beta}(y \mid X_2)$$

$\therefore$ The slope of the added-variable plot is the same as the slope for simple regression Y on X_2 ignoring X_1.

5) $H = X(X'X)^{-1}X'$

$X_{n \times (p+1)}$ is of full rank & contains 1^{st} Column of 1's.

$X = [1, X']$, Where X' is an $n \times p$ matrix

$H = X(X'X)^{-1}X' = [1, X']([1, X']'[1, X'])^{-1}[1, X']'$

$P = [1, X']'[1, X']$ ... $(p+1) \times (p+1)$ matrix

$$Tr(H) = Tr([1, H']P^{-1}[1, H']')$$
$$Tr(H) = 1 + Tr(X'P^{-1}X')$$

$$Tr(X'P^{-1}X') = \sum_{i=1}^{n} \lambda_i \rightarrow \text{eigenvalues}$$

$$\therefore Tr(H) = 1 + \sum_{i=1}^{n} \lambda_i$$

$\because \lambda_i$ is non-negative,

$$Tr(H) \geq 1$$
$$\frac{1}{n} Tr(H) = \frac{1}{n} + \frac{1}{n} \sum_{i=1}^{n} \lambda_i$$

$\because 1/n$ is a positive term,

$$1/n \leq \frac{1}{n} Tr(H)$$

$$\therefore \frac{1}{n} \leq Tr(H) \leq 1$$

$$= \frac{1}{n} \leq h_{ii} \leq 1 \quad //$$