ENGINEERING MATHEMATICS - I
18MA1BSEM1

DEPARTMENT:- CSE

USN:- 1BM19CS088

Q1

(a) Prove that $\tan\phi = \frac{r\,d\theta}{dr}$

→ Let P be the point on the curve C
PT be the tangent to the curve at P
PT meets the X-axis at the point T

$\therefore \psi = \theta + \phi$

→ Slope of the tangent PT = $\tan\psi$

$\Rightarrow \frac{dy}{dx} = \tan\psi$ —— (I)

$\therefore x = r\cos\theta$

$\rightarrow \frac{dx}{d\theta} = -r\sin\theta + \frac{dr}{d\theta}\cos\theta$ —— (II)

$\therefore y = r\sin\theta$

$\rightarrow \frac{dy}{d\theta} = r\cos\theta + \sin\theta\left(\frac{dr}{d\theta}\right)$

$\therefore \frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$ —— (III)

→ Dividing eq (III) by $\frac{dr}{d\theta}\cos\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{r\,d\theta}{dr} + \tan\theta}{1 - \frac{r\,d\theta}{dr}\tan\theta}$

$\psi = \theta + \phi$

$\rightarrow \tan\psi = \tan(\theta + \phi)$

$$\therefore \frac{dy}{dx} = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$$

$$\therefore \boxed{\tan\phi = \frac{r\,d\theta}{dr}}$$

Q1

(c) Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^3}{24} + \cdots$

$\rightarrow f(x) = \sqrt{1+\sin 2x} = \sqrt{\cos^2 x + \sin^2 x + 2\sin x\cos x}$

$= \sqrt{(\sin x + \cos x)^2}$

$= \sin x + \cos x$

$\rightarrow f(x) = \sin x + \cos x \qquad f(0) = 1$

$f'(x) = \cos x - \sin x \qquad f'(0) = 1 - 0 = 1$

$f''(x) = -\sin x - \cos x \qquad f''(0) = 0 - 1 = -1$

$f'''(x) = -\cos x + \sin x \qquad f'''(0) = -1$

$f^{IV}(x) = \sin x + \cos x \qquad f^{IV}(0) = 1$

$\rightarrow f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{IV}(0) + \cdots$

$= 1 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(1) + \cdots$

$$\boxed{f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \cdots}$$

Q 2

(a) If $U = f(x-y, y-z, z-x)$

then prove that $U_x + U_y + U_z = 0$

$\rightarrow u = f(\cancel{\xi}, s, t)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \cancel{\xi}} \cdot \frac{\partial \cancel{\xi}}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{d t}{\partial x}$$

$$= f'(\xi, s, t)(1) + f'(\xi, s, t)(0) + f'(\xi, s, t)(-1)$$

$$\rightarrow \frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial \xi}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0)$$

$$\rightarrow \frac{\partial u}{\partial y} = -\frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial s}$$

$$\therefore \frac{\partial u}{\partial z} = \frac{\partial u}{\partial \cancel{\xi}}\frac{\partial \xi}{\partial \cancel{z}} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

$$= \frac{\partial u}{\partial \xi}(0) + \frac{\partial u}{\partial s}(-1) + \frac{\partial u}{\partial t}(1)$$

$$\therefore \frac{\partial u}{\partial z} = \frac{\partial u}{\partial z} - \frac{\partial u}{\partial s}$$

$\rightarrow U_x + U_y + U_z = 0$ //

Q2

(b) $f(x,y) = f(0,0) + [x f(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)] +$

$+ \frac{1}{3!}[x^3 f_{xxx}(0,0) + 3x^2y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0)$

$+ y^3 f_{yyy}(0,0)] + \dots$

$\rightarrow f(x,y) = e^{ax+by} \longrightarrow 1$

$f_x = a e^{ax+by} \longrightarrow a$

$f_y = b e^{ax+by} \longrightarrow b$

$f_{xx} = a^2 e^{ax+by} \longrightarrow a^2$

$f_{xy} = ab e^{ax+by} \longrightarrow ab$

$f_{yy} = b^2 e^{ax+by} \longrightarrow b^2$

$f_{yyy} = b^3 e^{ax+by} \longrightarrow b^3$

$f_{xxx} = a^3 e^{ax+by} \longrightarrow a^3$

$f_{xyy} = ab^2 e^{ax+by} \longrightarrow ab^2$

$f_{xxy} = a^2 b e^{ax+by} \longrightarrow a^2 b$

$\rightarrow f(x,y) = 1 + (ax+by) + \frac{(ax+by)^2}{2!} + \frac{(ax+by)^3}{3!} + \dots$

Q2

(c) $f(x,y) = 1 + \sin(x^2+y^2)$

$\rightarrow f_x = 2x\cos(x^2+y^2)$

$f_y = 2y\cos(x^2+y^2)$

$\rightarrow 2x\cos(x^2+y^2) = 0$ & $2y\cos(x^2+y^2) = 0$

$\therefore x = 0,\ y = 0$ & $(0,0)$ is stationary point

$\rightarrow A = f_{xx} = -4x^2\sin(x^2+y^2) + 2\cos(x^2+y^2)$

$B = f_{xy} = -4xy\sin(x^2+y^2)$

$C = f_{yy} = -4y^2\sin(x^2+y^2) + 2\cos(x^2+y^2)$

At $(0,0)$: $A = 2,\ B = 0,\ C = 2$

$\therefore AC - B^2 = 4 > 0$

$\therefore AC - B^2 > 0$

$\therefore A = 2 > 0$, $(0,0)$ is a minimum point

$\rightarrow$ Thus, the minimum value of $f(x,y) = f(0,0) = 1$

Q 4

(a) Relation b/w Beta and Gamma function:-

$$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

$$\rightarrow \Gamma(m) = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$$

$$\Gamma(n) = 2\int_0^\infty e^{-y^2} y^{2m-1}\,dy$$

$$\Gamma(m+n) = 2\int_0^\infty e^{-r} r^{2m+2n-1}\,dr$$

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

→ Let

$$\Gamma(m)\Gamma(n) = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2n-1} y^{2m-1}\,dx\,dy$$

$$= 4\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2} (r\cos\theta)^{2m+1} (r\sin\theta)^{2n-1}\, r\,dr\,d\theta$$

$$= 2\int_0^\infty e^{-r^2} r^{2(m+n)-1}\,dr \times 2\int_0^{\pi/2} (\sin\theta)^{2n-1} (\cos\theta)^{2m-1}\,d\theta$$

$$\Gamma m\,\Gamma n = \Gamma(m+n) \times \beta(m,n)$$

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Q 4

(b) $$\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$$

$$=\int_{-c}^{c}\int_{-b}^{b}\left(x^2z+y^2z+\frac{z^3}{3}\right)_{-a}^{a}dy\,dx$$

$$=\int_{-c}^{c}\int_{-b}^{b}\left(x^2a+y^2a+\frac{a^3}{3}+x^2a+y^2a+\frac{a^3}{3}\right)dy\,dx$$

$$=2\int_{-c}^{c}\int_{-b}^{b}\left(x^2a+y^2a+\frac{a^3}{3}\right)dy\,dx$$

$$=2\int_{-c}^{c}\left[x^2ay+\frac{y^3a}{3}+\frac{a^3}{3}y\right]_{-b}^{b}dx$$

$$=2\int_{-c}^{c}\left(2x^2ab+\frac{2b^3a}{3}+\frac{a^3b}{3}\right)dx$$

$$=4\left[\frac{2abc^3}{3}+\frac{2ab^3c}{3}+\frac{2a^3bc}{3}\right]$$

$$=\frac{8abc}{3}(a^2+b^2+c^2)$$ //

Q4

(c) $r = a(1-\cos\theta)$

$\rightarrow \int r\,d\theta = \int a(1-\cos\theta)\,d(\theta)$

$\rightarrow$ Between $\theta = 0$ to $\theta = 2\pi$

$\left|\left(0 \text{ and } \frac{\pi}{2}\right)\right| \quad \left|\left(\frac{\pi}{2} \text{ and } \pi\right)\right| \quad \left|\left(\pi \text{ and } \frac{3\pi}{2}\right)\right| \quad \left|\frac{3\pi}{2} \quad 2\pi\right|$

$$= |a(\theta - \sin\theta)|$$

$$= \left|a\left(\frac{\pi}{2} - \sin\left(\frac{\pi}{2}\right)\right) - a(0 - \sin 0)\right|$$
$$+ \left|a(\pi - \sin\pi) - a\left(\frac{\pi}{2} - \sin\left(\frac{\pi}{2}\right)\right)\right|$$
$$+ \left|a\left(\frac{3\pi}{2} - \frac{\sin(\pi)}{2}\right) - a(\pi - \sin\pi)\right|$$
$$+ \left|a(2\pi - \sin 2\pi) - a\left(\frac{3\pi}{2} - \sin\left(\frac{3\pi}{2}\right)\right)\right|$$

$$= \left|a\left(\frac{\pi}{2} - 1\right) - a(0-0)\right|$$
$$+ \left|a(\pi - 0) - a\left(\frac{\pi}{2} - 1\right)\right|$$
$$+ \left|a\left(\frac{3\pi}{2} - (-1)\right) - a(\pi - 0)\right|$$
$$+ \left|a(2\pi - 0) - a\left(\frac{3\pi}{2} - (-1)\right)\right|$$

$$a\left\{\left(\frac{\pi}{2} - 0\right) + \left(\pi - \frac{\pi}{2} + 1\right) + \left(\frac{3\pi}{2} + 1 - \pi\right) + \left(2\pi - \frac{3\pi}{2} + 1\right)\right\}$$

$$a\left\{\frac{\pi}{2} + \frac{\pi}{2} + 1 + \frac{5\pi}{2} + 1 + \frac{\pi}{2} + 1\right\}$$

$\rightarrow$ Area enclosed by the curve $r = a(1-\cos\theta)$ is $A = a\{2\pi + 3\}$.

Q 6

(a) $2y\,dx + (2x\log x - xy)dy = 0$

$m = 2y$

$n = 2x\log x - xy$

$\rightarrow \frac{\partial m}{\partial y} = 2$

$\rightarrow \frac{\partial n}{\partial y} = 2(1+\log x) - y$

$\rightarrow \dfrac{\frac{\partial m}{\partial y} - \frac{\partial n}{\partial y}}{n}$

$= \dfrac{2 - [2(1+\log x) - y]}{2x\log x - xy}$

$\rightarrow \frac{-1}{x} = f(x)$

$I.F = e^{\int f(x)dx}$

$= e^{-\log x}$

$= e^{\log x^{-1}}$

$= x^{-1}$

$= \frac{1}{x}$

$\rightarrow$ Multiplying given eq with $1/x$

$= \frac{1}{x} 2y\,dx + \frac{1}{x}(2\log x - y)\,dy = 0$

$\int \frac{2y}{x} dx + \int -y\,dy = c$

$\therefore 2y\log x - \frac{1}{2}y^2 = C$

Q 6

(b) $z\sin\theta - \cos\theta \frac{dz}{d\theta} = z^2$

$\therefore \cos\theta \frac{dz}{d\theta} + z^2 = z\sin\theta$

$\therefore \cos\theta \frac{dz}{d\theta} - z\sin\theta = -z^2$ ——— (I)

$\rightarrow$ Dividing eq (I) by $\cos\theta$

$\therefore \frac{dz}{d\theta} - z\tan\theta = -z^2\sec\theta$ ——— (II)

$\rightarrow$ Dividing eq (II) by $-z^2$

$\therefore \frac{-1}{z^2}\frac{dz}{d\theta} + \frac{1}{z}\tan\theta = \sec\theta$

But $\frac{1}{z} = t$

$\therefore -\frac{1}{z^2}\frac{dz}{d\theta} = \frac{dt}{d\theta}$

$\therefore \frac{dt}{d\theta} + (t)\tan\theta = \sec\theta$

$\rightarrow I.F = e^{\int P\,d\theta}$

$I.F = e^{\int \tan\theta\,d\theta}$

$= e^{\log \sec\theta}$

$= \sec\theta$

$\therefore I.F(t) = \int Q\,(I.F)\,d\theta$

$\sec\theta\left(-\frac{1}{z}\right) = \int \sec^2\theta\,d\theta$

$-\frac{\sec\theta}{z} = \tan\theta + c$ //

Q 6

(c) $\left(\xi + \frac{k^2}{\xi}\right)\cos\theta = a$

$\rightarrow$ Taking log on both sides

$\therefore \log\left(\xi + \frac{k^2}{\xi}\right) + \log\cos\theta = \log a$

$\rightarrow$ Differentiating w.r.t. θ

$\therefore \frac{1}{\left(\xi + \frac{k^2}{\xi}\right)}\left(1 - \frac{k^2}{\xi^2}\right)\frac{d\xi}{d\theta} + \left(-\frac{\sin\theta}{\cos\theta}\right) = 0$

$\therefore \frac{\xi}{\xi^2 + k^2} \cdot \frac{\xi^2 - k^2}{\xi^2}\frac{d\xi}{d\theta} = \tan\theta$

$\rightarrow$ Replacing $\frac{d\xi}{d\theta}$ by $-\xi^2\frac{d\theta}{d\xi}$

$\therefore \frac{\xi}{\xi^2 + k^2} \cdot \frac{\xi^2 - k^2}{\xi^2}\left(-\xi^2\frac{d\theta}{d\xi}\right) = \tan\theta$

$\therefore \frac{\xi(\xi^2 - k^2)}{\xi^2 + k^2}\left(\frac{-d\theta}{d\xi}\right) = \tan\theta$

$\therefore \frac{-d\theta}{\tan\theta} = \frac{\xi^2 + k^2}{\xi(\xi^2 - k^2)}d\xi$

$\therefore \int \frac{(\xi^2 + k^2)}{\xi(\xi^2 - k^2)}d\xi + \int \cot\theta\, d\theta = c$

$\rightarrow$ But $\frac{\xi^2 + k^2}{\xi(\xi - k)(\xi + k)} = \frac{-1}{\xi} + \frac{1}{\xi - k} + \frac{1}{\xi + k}$

$$\therefore \int \frac{(\xi^2+k^2)\,d\xi}{\xi(\xi^2-k^2)} = -\log\xi + \log(\xi-k) + \log(\xi+k)$$

$$\therefore \int \frac{(\xi^2+k^2)\,d\xi}{\xi(\xi^2-k^2)} = \log\left(\frac{\xi^2-k^2}{\xi}\right)$$

$$\rightarrow \log\left(\frac{\xi^2-k^2}{\xi}\right) + \log \sin\theta = c$$

$$\log\left(\frac{\xi^2-k^2}{\xi}.\sin\theta\right) = \log b$$

$$\therefore \left(\xi - \frac{k^2}{\xi}\right)\sin\theta = b$$

Q7

(a) $\frac{d^3y}{dx^3} + \frac{3d^2y}{dx^2} + \frac{3dy}{dx} + y = 8e^x + \sin x$

So $(D^3 + 3D^2 + 3D + 1)y = 8e^x + \sin x$

$\rightarrow m^3 + 3m^2 + 3m + 1 = 0$

$\therefore m = -1, -1, -1$

$\rightarrow y_c = (C_1 + C_2 + C_3 x)e^{-x}$

$\rightarrow y_p = \frac{1}{D^3+3D^2+3D+1}(8e^x + \sin x)$

$= \frac{8e^x}{D^3+3D^2+3D+1} + \frac{\sin x}{D^3+3D^2+3D+1}$

$= \frac{8e^x}{1+3+3+1} + \frac{\sin x}{-D-3+3D+1}$

$= e^x + \frac{(D-1)\sin x}{2(D^2-1)}$

$= e^x + \frac{\cos x + \sin x}{-4}$

$\therefore y = y_c + y_p \Rightarrow (C_1 + C_2 + C_3)e^{-4} + e^x + \frac{\cos x + \sin x}{-4}$ //

Q7

(b) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log_e x$

$\rightarrow$ Let $x = e^t$

$t = \log x$

$\therefore$ $x\frac{dy}{dx} = Dy$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$

where $D = \frac{d}{dt}$

$\therefore$ $[D(D-1) - D + 1]y = t$

$\rightarrow$ A.E is $(D-1)^2 = 0$

$D = 1, 1$

$\rightarrow$ C.F. $= (c_1 + c_2 t)e^t$

$P.I = \frac{1}{(D-1)^2} t = (1-D)^{-2} t = (1 + 2D + 3D^2 + \dots)t = t + 2$

Q7

(c) $(D^2+1)y = \sec x$

$m^2+1=0$

$m = \pm i$

$\rightarrow CF = C_1\cos x + C_2 \sin x$

$u = \cos x$

$v = \sin x$

$\rightarrow W(u,v) = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$

$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= 1$

$\rightarrow$ Let $y = Au + Bv$

where $A = -\int \frac{vR\,dx}{W}$

$B = \int \frac{vR\,dx}{W}$

$\rightarrow A = -\int \frac{(\sin x)(\sec x)}{1}\,dx$

$= -\log(\sec x) + C_3$

$B = \int \frac{(\cos x)(\sec x)}{1}\,dx$

$= x + C_4$

$y = [-\log(\sec x) + C_3] + [x + C_4]$

~~import~~

```java
import java.util.Scanner;
public class Main{
        private static Scanner sc;
        public static void main(String[] args)
        {
            int Size, i;
            int positiveCount=0, negativeCount=0, zerocount=0;
            sc= new Scanner(System.in);
            System.out.println("Please Enter Number of elements in
                              an array: ");
            Size = sc.nextInt();
            int[] a = new int [Size];
            System.out.println("Please enter"+Size+"elements of an Array:");
            for(i=0; i<Size; i++)
            {
                a[i]=sc.nextInt();
            }
            for (i=0; i<Size; i++)
            {
                    if(a[i] >0)
                    {
                        postiveCount++;
                    }
                    else if (a[i]==0)
                    {
                        zerocount++;
                    }
```

```
        else
        {
            negativeCount++;
         }
    }
    System.out.println("\n Total number of Positives = "+positiveCount);
    System.out.println("\n Total number of Negatives = "+ negativeCount);
    System.out.println("\n Total number of Zeros = "+ zerocount);
  }
}
```