Parker

Wednesday, October 26, 2022 9:07 AM

61.5

10/21@ 11Am

2hr 15m

Submission Name
MTH 230 Exam #2 OL Accommodated

Submission Date
Wed, Oct 19, 2022 8:13 AM

Additional Instructions
Students may take the exam Thursday, 10/20 or Friday, 10/21

Associated Exam(s)
Accommodated Testing at the Brighton Campus - 6-207 > Accommodated Test - Brighton Campus - 6-207

Associated File(s)
230Exam2F22.pdf

Student Name(s)
Madison Burke, Jacob Lee, Cory Parker, Anne Pyrak

Is a Calculator allowed?
Yes - Basic, Yes - Scientific, Yes - Scientific Graphing

Professor Name
Kilner, Steve

TWRD
X1

What is the student allowed to bring into the test?
Scrap Paper/ Graph Paper

Completed Exam Return
Email Exam to Professor

Access Code Required?

Duration of Exam
90 min – 135m

Requested Date Window

Exam Completed
Friday, October 21, 2022 12:00 am

return (/monroecc/Professor)

Start: 10:57am

End: 1:12pm

Actual: 12:38pm

230 Ex2 Page 1

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MTH 230 (Kilner) – Exam 2

Read all instructions on this coversheet before completing and submitting your exam.

Failure to follow any of these directions may result in a penalty.

- **You MAY use a calculator of any kind on this exam as long as it doesn't have internet capabilities (no cell phones). When using a calculator to perform row reductions and/or matrix operations, you must indicate on your paper, what you entered into the calculator as well as the result. In general show enough work to support your answers when appropriate.**
- **You may NOT access the textbook or any other resources (formula sheets, notes, websites, etc...) aside from a calculator. Treat this as you would a test that would be taken in a regular classroom, where you have nothing available to you other than the exam, paper to write on, and something to write with. Scrap paper is permitted.**
- **You may NOT obtain assistance from anyone in completing this exam.**

*****Anyone found violating any of the above rules will receive a 0 on the exam and will be reported to the appropriate offices at MCC.**

- Only methods covered in this course up to the current unit may be used on this exam.
- Where appropriate you must show sufficient work to support your final answers. All such work must be done in this test booklet.

Make sure your name is on the next page.

You must show work to support your answers when appropriate.

You have up to 1 hour and 30 minutes to complete the exam.

Print Your Name (first and last): Cory Parker

[A] Consider the matrix A given below

$$A = \begin{bmatrix} 3 & -6 & 4 & 2 & 9 \\ 4 & -8 & 5 & 2 & 12 \\ 2 & -4 & 2 & 0 & 6 \\ -1 & 2 & 1 & 0 & -7 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & -2 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(redundant)

(A.1) Find a basis for the column space of A.

Column Space Basis: $\left\{ \begin{bmatrix} 3 \\ 4 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$

(A.2) Let $\mathcal{B}$ be the basis you obtained in (A.1) for the column space of A and let $\mathbf{c}_5$ be the fifth column of A. Determine $(\mathbf{c}_5)_{\mathcal{B}}$, the coordinate vector for $\mathbf{c}_5$ relative to $\mathcal{B}$. $[A\vec{x}=\vec{b}]$

$$\vec{C}_5 = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} = C_1 \begin{bmatrix} 3 \\ 4 \\ 2 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 4 \\ 5 \\ 2 \\ 1 \end{bmatrix} + C_3 \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{(\vec{C}_5) = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}}$$

Just 3-coordinates

-2 pts

(A.3) Find a basis for the row space of A.

$$\boxed{B = \{(1,-2,0,0,5), (0,0,1,0,-2), (0,0,0,1,1)\}}$$

✓

(A.4) Find a basis for the nullspace of A.

$x_1 = 2t - 5s$
$x_2 = t$
$x_3 = 2s$
$x_4 = -s$
$x_5 = s$

$$\Rightarrow \boxed{\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \right\}}$$

✓

(A.5) Find the rank and nullity of A. rank(A) = 3   nullity(A) = 2

✓ (−2)

[B] Let B be a 7×9 matrix.

(B.1) What is largest possible value for rank(B)? 7

(B.2) Suppose that rank(B) = 5. Use this information and the fact that B is a 7×9 matrix to fill in the blanks below with the appropriate dimensions.

The dimension of the row space of B is: 5

The dimension of the column space of B is: 5

The dimension of the nullspace of B is: 4

The dimension of the nullspace of B^T is: 2

[C] Determine whether or not the following collections of vectors are linearly independent or linearly dependent. Justify your answers.

(C.1) $\{(1,-2,5,3),(3,-6,15,9)\}$ in the vector space $\mathbb{R}^4$.

$$3\begin{bmatrix}1\\-2\\5\\3\end{bmatrix}=\begin{bmatrix}3\\-6\\15\\9\end{bmatrix}$$ ∴ Linearly Dependent

(C.2) $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4\}$ in the vector space $\mathbb{R}^3$. (your justification should apply for any collection of four vectors in $\mathbb{R}^3$)

Linearly Dependent as $\underbrace{\text{you will have atleast one free variable.}}_{\text{? Why?}}$

-2 pts Insufficient

(C.3) $\{x^2-2x+3,\ 2x^2+x+1,\ 6x^2+3x-2\}$ in the vector space $\mathbb{P}_2$.

$$\Rightarrow \begin{bmatrix}3 & 1 & -2\\-2 & 1 & 3\\1 & 2 & 6\end{bmatrix} \xRightarrow{\text{(RRef)}} \underbrace{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}}_{\text{Explain what this shows}}$$ (Linearly Independent)

-2

[D] Does the following set of 2×2 matrices form a basis for M_{22}? ~~YES~~ NO (circle one)

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

doesn't span M_{22}

Justify your response.

$$\cancel{a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} a & c \\ c & b \end{bmatrix}}} \neq A \quad \det(A) = ab - c^2 \neq 0$$

? $-4pts$

[E] Determine the dimensions of the following subspaces of $\mathbb{R}^4$. No work is needed here.

(E.1) all vectors of the form $(a, b, c, 0)$ Dimension: 3

(E.2) all vectors of the form (a, b, c, d), where $a = b = c = d$ Dimension: ~~2~~ 1

$-2pts$

(E.3) all vectors of the form (a, b, c, d), where $d = a + b$ and $c = a - b$ Dimension: 2

[F] The set $S = \{(1,1,0), (0,1,1), (1,0,1)\}$ forms a basis for $\mathbb{R}^3$. The coordinates for $\mathbf{v}$ relative to the basis S is given by $(\mathbf{v})_S = (2, 7, -1)$. Determine the coordinates of $\mathbf{v}$ relative to the standard basis for $\mathbb{R}^3$.

$$\begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \therefore (\vec{v})_S = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$$

$-4pts$

Answer: $(\mathbf{v}) = \cancel{\begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}}$

$(1, 9, 6)$

-10

[G] Consider the following bases for $\mathbb{R}^2$: $\mathfrak{B} = \{(1,2), (2,3)\}$ and $\mathfrak{B}' = \{(-1,1), (-2,3)\}$.

(G.1) Find the transition matrix from $\mathfrak{B}'$ to $\mathfrak{B}$ which we denote $P_{\mathfrak{B}' \to \mathfrak{B}}$. Support your answer.

(G.1) Find the transition matrix from $\mathfrak{B}'$ to $\mathfrak{B}$ which we denote $P_{\mathfrak{B}'\to\mathfrak{B}}$. Support your answer.

$$[\vec{v}]_{\mathcal{B}} = P_{\mathcal{B}'\to\mathcal{B}}[\vec{v}]_{\mathcal{B}'} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} (-a+b) & (-2a+3b) \\ (-c+d) & (-2c+3d) \end{bmatrix}$$

?

-9 pts

(G.2) Let $\mathbf{v}$ the vector in $\mathbb{R}^2$ whose coordinate vector relative to $\mathfrak{B}'$ is: $(\mathbf{v})_{\mathfrak{B}'} = (5,4)$. Determine the coordinate vector for $\mathbf{v}$ relative to $\mathfrak{B}$ (i.e. find $(\mathbf{v})_{\mathfrak{B}}$). Express your answer as an ordered pair.

[H] Let W be the subspace of $\mathbb{P}_2$ consisting of all polynomials $p(x) = a + bx + cx^2$, which satisfy $p(1) = 0$. Obtain a basis for W and indicate the dimension of W.

?

$P(1) = a + b + c^2 = 0$

$$\boxed{c = \pm\sqrt{(-a-b)}}$$

3

$[-1$

-7 pts

Basis: dim(W) = ______

-16

(I) Indicate whether each of the following statements are (always) True or (sometimes) False.

(I.1) Any set of vectors containing the zero vector is linearly dependent. True False

(I.2) The solutions of the linear system $A\mathbf{x} = \mathbf{b}$ form a vector space. True False

(I.3) The dimension of the nullspace of a matrix is always less than the dimension of the column space of the matrix. True False

(I.4) All bases for a given finite dimensional vector space have the same number of vectors. True False

(I.5) If A is invertible, the nullspace of A contains only the zero vector. True False

(I.6) The row space of A is equivalent to the column space of A^T. True False

(I.7) The nullspace of A is equivalent to the nullspace of A^T. True False

(I.8) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are any two vectors in a vector space V and if $a, b \in \mathbb{R}$, then $a\mathbf{v_1} + b\mathbf{v_2}$ is also in V. True False

(I.9) The matrices A and $R = \text{rref}(A)$ have the same row space. True False

(I.10) The matrices A and $R = \text{rref(A)}$ have the same column space. True False

-4.5pt

For the remaining True/False problems, assume that S and T are sets of vectors where $S \subseteq T$ (i.e. S is a subset of T).

(I.11) If S is linearly independent, then T is linearly independent. True False

(I.12) If T is linearly independent, then S is linearly independent. True False

(I.13) If S is linearly dependent, then T is linearly dependent. True False

(I.14) If T is linearly dependent, then S is linearly dependent. True False

-4pt

-8.5