18.06 Quiz 2 April 7, 2010 Professor Strang

Your PRINTED name is: ______________________ **1.**

Your recitation number or instructor is ______________________ **2.**

3.

1. (30 points)

(a) Find the matrix P that projects every vector b in R^3 onto the line in the direction of $a = (2, 1, 3)$.

(b) What are the column space and nullspace of P? Describe them geometrically and also *give a basis for each space.*

(c) What are *all* the eigenvectors of P and their corresponding eigenvalues? (You can use the geometry of projections, not a messy calculation.) The diagonal entries of P add up to ________ .

2. (30 points)

(a) $p = A\widehat{x}$ is the vector in $C(A)$ nearest to a given vector b. If A has independent columns, what equation determines $\widehat{x}$? What are all the vectors perpendicular to the error $e = b - A\widehat{x}$? What goes wrong if the columns of A are dependent?

(b) Suppose $A = QR$ where Q has orthonormal columns and R is upper triangular invertible. Find $\widehat{x}$ and p in terms of Q and R and b (not A).

(c) (Separate question) If q_1 and q_2 are any orthonormal vectors in R^5, give a formula for the projection p of any vector b onto the plane spanned by q_1 and q_2 *(write p as a combination of q_1 and q_2).*

3. (40 points) This problem is about the n by n matrix A_n that has zeros on its main diagonal and all other entries equal to -1. In MATLAB $A_n = \text{eye}\,(n) - \text{ones}\,(n)$.

(a) Find the determinant of A_n. Here is a suggested approach:

Start by adding all rows (except the last) to the last row, and then factoring out a constant. (You could check $n = 3$ to have a start on part b.)

(b) For any invertible matrix A, the $(1, 1)$ entry of A^{-1} is the ratio of ________________ .

So the $(1, 1)$ entry of A_4^{-1} is ________________ .

(c) Find *two orthogonal eigenvectors* with $A_3\, x = x$. (So $\lambda = 1$ is a double eigenvalue.)

(d) What is the third eigenvalue of A_3 and a corresponding eigenvector?