MTH 230 (Kilner) – Final Exam Information – Fall 2022

The Final Exam must be taken on Monday, December 19 or Tuesday, December 20.

- Students testing at MCC may choose from one of the options below:

  - Monday, 12/19 from 5pm – 7pm in Building 8, Room 200
  - Tuesday, 12/20 from 5pm – 7pm in Building 8, Room 300

  *Note: the final exams are scheduled in large lecture halls where online students from various courses will be testing and will be proctored by multiple faculty (including myself). **You do NOT have to sign-up using Registerblast**, just let me know whether you plan to take the exam Monday or Tuesday. Please let me know by the end of the day on Thursday, December 15.

- Students taking an exam with an off-campus proctor may schedule to take the exam any time on Monday or Tuesday that fits your proctor's schedule. Once you have scheduled a day/time with your proctor, please let me know or I won't send them the exam. Please let me know by the end of the day on Thursday, December 15.

The final exam is comprehensive, but with an emphasis on the material from Testing Unit 4. The exam will be completed on paper and you will only be allowed to use a writing utensil, a calculator, and scrap paper (no books, notes, internet, etc…).

You will be given up to **2 HOURS** to complete the exam.

*****On the next page you will find the topics for the final exam. I have identified sample problems for the "procedural/computational" problems from UNIT 4 ONLY to give you a rough idea of the kind of problem to expect. You can review your past exams and exam information documents for the previous unit topics. Looking at the specific problem that I have identified is not by itself intended to prepare you to solve the exam problem, so you are encouraged to look at similar problems in the book, notes, and/or videos.**

Final Exam Topics

*Calculators ARE allowed on this exam though sufficient work must be shown to support your answers.

Definitions, Theorems, and Concepts:

You are responsible for knowing the definitions, theorems, and concepts presented in class.

You may be tested on these items in the following ways:

- True/False questions
- Concept questions which require very little work, but require you to understand the concepts to answer a question.

Computation/Solving/Procedural Problems:

*These problems will be chosen from topics covered in Units 2, 3, and 4 with more **emphasis on Unit 4**. Several topics covered in units 2 and 3 can be tested with problems in Unit 4.

*Problems involving vector spaces will be chosen to include only $\mathbb{R}^n$, $\mathbb{P}_n$, function spaces such as $C[a,b]$, and/or M_{mn}. Make sure you are familiar with these spaces.

Computational/Solving/Procedural Problems will be selected from the following:

- Determine whether or not a collection of vectors are linearly independent or linearly dependent. Be able to support your answer.
- Determine whether or not a given collection of vectors form a basis for a specified vector space. Be able to support your answer.
- Given a set of vectors S, determine whether or not a particular vector is in the span of S. If the vector is in the span of S, be able to express it as a linear combination of the vectors in S.
- Given a basis for a vector space, write the coordinate vector relative to the basis for a specified vector within the vector space.
- Obtain bases for the row space, column space, and nullspace of a matrix. You should also be aware of the connection between the range/kernel of a linear transformation $T_A: \mathbb{R}^n \to \mathbb{R}^m$ and the column space/nullspace of the matrix A.
- Determine the rank, and nullity of a matrix.
- Given the dimensions and rank of a matrix A, determine the dimensions of the row space of A, the column space of A, the nullspace of A, and the nullspace of A^T.
- Given an inner product space (not necessarily Euclidean), be able to calculate the inner product of two vectors, the norm of a vector, the distance between two vectors, and be able to identify whether or not two vectors are orthogonal relative to the inner product.
- Be able to show whether or not a given set of vectors is orthogonal or orthonormal and also whether or not the set of vectors are linearly independent.

- Given an orthonormal basis $\mathfrak{B}$ for an inner product space, express a vector $\mathbf{v}$ in the space as a linear combination of the basis vectors and give the coordinate vector of $\mathbf{v}$ relative to $\mathfrak{B}$, i.e $[\mathbf{v}]_{\mathfrak{B}}$.
- Given an orthogonal or orthonormal basis for a subspace W of an inner product space V, and given a vector $\mathbf{v} \in V$, find the projection of $\mathbf{v}$ onto W and the component of $\mathbf{v}$ orthogonal to W. That is, find: $\text{proj}_W \mathbf{v}$ and $\text{proj}_{W^\perp} \mathbf{v}$.
- Use the Gram-Schmidt process to transform a basis with 3 vectors into an orthogonal or orthonormal basis (relative to the Euclidean Inner Product).
- Determine whether or not a given matrix is orthogonal.
- Determine the eigenvalues of a square matrix. For a triangular matrix, this can be done by inspection, for all others, you will be expected to obtain and solve the characteristic equation.
- Given the eigenvalues for a matrix, obtain bases for the corresponding eigenspaces.
- Determine whether or not a square matrix A is diagonalizable. If so, be able to find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$

Unit 4 Topics:

- Find the standard matrix $[T]$ for a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$. [Ex. 1.8 #14]
- Find the matrix $[T]_{\mathfrak{B}',\mathfrak{B}}$ for a linear transformation $T: V \to W$ relative to the bases $\mathfrak{B}'$ and $\mathfrak{B}$. [Ex. 8.4 #7 and the example in Unit 4.4 videos titled "2. Matrix for transformation #2"]
- Obtain the standard matrix for any of the following geometric linear operators on $\mathbb{R}^2$: reflection about the x-axis, reflection about the y-axis, contraction/dilation by a factor of k, counterclockwise rotation about the origin by an angle θ, orthogonal projection onto the x-axis, orthogonal projection onto the y-axis, shear in the x-direction by a factor of k, shear in the y-direction by a factor of k. Also be able to use matrix multiplication to obtain the standard matrix for a composition of these operators. [Ex. 8.6 #7]
- Find bases for the kernel and range of a given linear transformation. Then be able to indicate the rank and nullity of the linear transformation. [Ex. 8.1 #27]
- Determine whether or not a linear transformation is 1-to-1 and furthermore whether or not it is invertible. Be able to justify your response. [Ex 8.2 #3, #7, and #19]
- Given a linear operator $T: V \to V$ obtain a basis $\mathfrak{B}$ for which the matrix for T relative to $\mathfrak{B}$ (i.e. $[T]_{\mathfrak{B}}$) is diagonal. [Unit 4.5 Supplemented Practice Problem #2]
- Potential Bonus Problem: Be able to give the matrix form of the equation of an ellipse by representing the quadratic form in the equation as $\mathbf{x}^T A\mathbf{x}$. Furthermore, given the eigenvalues of A and the corresponding matrix P that orthogonally diagonalizes A, carry out a rotation of the coordinate axes to obtain a change-of-variable that eliminates the cross product term. Be able to sketch the ellipse. [See Unit 4.6 videos and examples]