18.06 Quiz 1 March 1, 2010 Professor Strang

Your PRINTED name is: _____ **1.**

Your recitation number or instructor is _____ **2.**

3.

4.

1. Forward elimination changes $A\mathbf{x} = \mathbf{b}$ to a row reduced $R\mathbf{x} = \mathbf{d}$: the complete solution is

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + \mathbf{c_1} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathbf{c_2} \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

(a) **(14 points)** What is the 3 by 3 reduced row echelon matrix R and what is $\mathbf{d}$?

(b) **(10 points)** If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects R and $\mathbf{d}$ to the original A and $\mathbf{b}$? Use this matrix to find A and $\mathbf{b}$.

2. Suppose A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}.$$

(a) **(16 points)** Find all special solutions to $Ax = 0$ and describe in words the whole nullspace of A.

(b) **(10 points)** Describe the column space of this particular matrix A. "All combinations of the four columns" is not a sufficient answer.

(c) **(10 points)** What is the reduced row echelon form $R^* = \text{rref}(B)$ when B is the 6 by 8 block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}$$ using the same A?

3. **(16 points) Circle the words** that correctly complete the following sentence:

(a) Suppose a 3 by 5 matrix A has rank $r = 3$. Then the equation $Ax = b$

(always / sometimes but not always)

has (a unique solution / many solutions / no solution).

(b) What is the column space of A? Describe the nullspace of A.

4. Suppose that A is the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 2 & 4 \end{bmatrix}.$$

(a) **(10 points)** Explain in words how knowing all solutions to $A\mathbf{x} = \mathbf{b}$ decides if a given vector $\mathbf{b}$ is in the column space of A.

(b) **(14 points)** Is the vector $\mathbf{b} = \begin{bmatrix} 8 \\ 28 \\ 14 \end{bmatrix}$ in the column space of A?

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