ECON 753
University of Michigan
Winter 2025

Assignment 2 – Count Model
Due 2/6/2025, 5PM

In 1969, the popular magazine *Psychology Today* published a 101-question survey on extramarital affairs. Professor Ray Fair (1978) extracted a sample of 601 observations on men and women who were currently in their first marriage and analyzed their responses to the survey. He used the "Tobit" model as his estimation framework for this study. The dependent variable is a count of the number of affairs, so instead of a Tobit, a standard Poisson model may be a better choice. Download the data set `HW4.csv`, and estimate the parameters to the model below using nonlinear least squares and maximum likelihood using the algorithms that I suggest below.

Data Description

- y - count data: number of affairs in the past year.
- $\boldsymbol{x}$ - constant term=1, age, number of years married, religiousness (1-5 scale), occupation (1-7 scale), self-rating of marriage (1-5 scale)

Assingment

The following is the data generating assumptions for the Poisson model, where j is the number of affairs:

$$Pr[y_i = j] = \frac{e^{-\lambda_i}\lambda_i^j}{j!} \tag{1}$$
$$log(\lambda_i) = \boldsymbol{x}_i'\boldsymbol{\beta} \tag{2}$$
$$E(y_i \mid x_i) = e^{\boldsymbol{x}_i'\boldsymbol{\beta}} \tag{3}$$

for some $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)'$.

The log-likelihood function is:

$$\begin{aligned}
\ln L &= \sum_{i=1}^{n} \ln f\left(y_i \left| x_i\right., \beta\right) \\
&= \sum_{i=1}^{n} \ln \frac{e^{-\lambda_i}\lambda_i^j}{j!} \\
&= \sum_{i=1}^{n} \left[-\lambda_i + y_i \ln \lambda_i - \ln y_i!\right] \\
&= \sum_{i=1}^{n} \left[-e^{\mathbf{x}_i{}'\beta} + y_i x_i'\beta - \ln y_i!\right]
\end{aligned}$$

The residual sum of squares is:

$$S(\beta) = \sum_{i=1}^{n} \left(y_i - e^{\beta' x_i} \right)^2$$

1. Estimate the parameter vector $\boldsymbol{\beta}$ using maximum likelihood. Use as the starting value a vector of zeros. Use four algorithms,

1. Quasi-Newton with BFGS and a numerical derivative.
2. Quasi-Newton with BFGS and a analytical derivative.
3. Nelder-Mead.
4. The BHHH algorithm we went over in class. (This is the only one you should code up yourself. You can use packages for the others.)

Report, in a table, the estiamted parameters, number of iterations, number of function evlauations, and time, for each method.

2. Report the eigenvalues for the Hessian approximation for the BHHH MLE method from the last question. Report the eigenvalues for the initial Hessian approximation, and the Hessian at the estimated parameters.

3. Now estimate the model using the NLLS method we went over in class, starting from the same initial point. Report the results in the table as well.