Micro I Notes

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Contents

1 Preference

1.0.1 Def "binary relation"

A binary relation R on X is a subset of $X \times X$.

The ordered pair $(x, y) \in R$ means x is (weakly) preferred to y.

1.0.2 Remark "binary relation notation"

We use $\succsim$ to denote R but all these methods are the same thing:

$$(x, y) \iff xRy \iff x \succsim y$$

1.0.3 Def "attributes of binary relations"

A binary relation $\succsim$ is

1. complete if for all $x, y \in X$ either $x \succsim y$ or $y \succsim x$
2. transitive if for all $x, y, z \in X, [x \succsim y, y \succsim z] \Rightarrow x \succsim y$
3. reflexive if for all $x \in X, x \succsim x$
4. symmetric if for all $x, y \in X, x \succsim y \iff y \succsim x$
5. irreflexive if for all $x \in X, x \not\succsim x$
6. asymmetric if for all $x, y \in X, x \succsim y \Rightarrow y \not\succsim x$
7. antisymmetric if for all $x, y \in X, [x \succsim y \text{ AND } x \neq y] \Rightarrow y \not\succsim x$
   - i.e., xRy and yRx implies $x = y$

1.0.4 "Lemma 1 (handout) rational implies"

1. $\succsim$ rational $\Rightarrow \succ$ asymmetric and transitive
2. $\succsim$ rational $\Rightarrow \sim$ symmetric and transitive
3. $\succsim$ rational $\Rightarrow x \succsim y \succsim z$ and either $x \not\sim y$ or $y \not\sim z$ implies $x \succ z$

1.0.5 Def "rational"

A binary relation $\succsim$ is *rational* if it is complete and transitive

2 Choice

2.0.1 Def "Power Set and Budget Constraint"

$$\mathcal{P}(X) \equiv \{Y \subseteq X : Y \neq \emptyset\} = 2^X \setminus \{\emptyset\}$$

$$\mathcal{B} \subseteq \mathcal{P}(X) \Rightarrow \mathcal{B} \neq \emptyset$$

2.0.2 Def "choice function"

A function $c : \mathcal{B} \to \mathcal{P}(X)$ is called a *choice function* if for all $A \in \mathcal{B}, c(A) \subseteq A$.

1. If c is a choice function, then $c(A) \neq \emptyset, \forall A \in \mathcal{B}$
2. $|c(A)| \geq 1$

2.0.3 Def "choice structure"

The pair $(\mathcal{B}, c)$ is called a *choice structure*.

2.0.4 Def "WARP"

Weak Axiom of Revealed Preference. The following are equivalent for all $A, B \in \mathcal{B}$:

- $c(A) \cap B \neq \emptyset \Rightarrow c(B) \cap A \subseteq c(A)$
- $[c(A) \cap B \neq \emptyset \text{ AND } c(B) \cap A \neq \emptyset] \Rightarrow [c(A) \cap B \subseteq c(B) \text{ AND } c(B) \cap A \subseteq c(A)]$
- $x, y \in A \cap B, x \in c(A), y \in c(B) \Rightarrow x \in c(B), y \in c(A)$
- Sen's α and β both hold

2.0.5 Def Sen's α and β

For all $A, B \in \mathcal{B}$:

1. Sen's α: If $x \in B \subseteq A$ and $x \in c(A)$ then $x \in c(B)$
2. Sen's β: If $x, y \in c(B), B \subseteq A$ and $y \in c(A)$ then $x \in c(A)$

2.0.6 Prop 1 "WARP $\iff$ α and β"

2.0.7 Term "single-valued"

We say c is single-valued if $\forall A \in \mathcal{B}, |c(A)| = 1$

2.0.8 Prop 2 "α and single-valued $\Rightarrow$ WARP"

If $(\mathcal{P}(X), c)$ satisfies α and c is single-valued, then it satisfies WARP.

3 Choice and Preference

3.0.1 Def "$c_{\succsim}$"

For any binary relation $\succsim$ define

$$c_{\succsim}(A) = \{x \in A : x \succsim y, \forall y \in A\}, \forall A \in \mathcal{P}(X)$$

3.0.2 Def "$\succsim_c$"

For a choice structure $(\mathcal{P}(X), c)$ define a binary relation $\succsim_c$ such that

$$x \succsim_c y \iff x \in c(\{x, y\})$$

3.0.3 Prop "satisfies WARP $\iff \succsim$ rational"

If $\succsim$ is a binary relation on a finite set X then $(\mathcal{P}(X), c_{\succsim})$ is a choice structure that satisfies WARP if and only if $\succsim$ is rational.

3.0.4 Prop "WARP $\Rightarrow c = c_{\succsim}$"

If $(\mathcal{P}, c)$ is a choice structure that satisfies WARP then there exists a unique rational binary relation $\succsim$ such that $c = c_{\succsim}$ (i.e., for all $A \in \mathcal{P}, c(A) = c_{\succsim}(A)$).

4 Utility

4.0.1 Def "utility function"

A function $U : X \to \mathbb{R}$ represents $\succsim$ if and only if, for all $x, y \in X$

$$x \succsim y \iff U(x) \geq U(y).$$

> Note: we'll use the same letter, U, in expected utility, but the U in expected utility will take in lotteries (e.g., p) instead of alternatives (e.g., x).

4.0.2 Prop "U represents $\succsim \Rightarrow$ rational"

If $U : X \to \mathbb{R}$ represents $\succsim$, then $\succsim$ is rational.

> Note: this proposition does NOT require finiteness!

4.0.3 Prop "X finite + $\succsim$ rational $\Rightarrow \exists U$

If X is finite and $\succsim$ is rational then there exists $U : X \to \mathbb{R}$ that represents $\succsim$.

4.0.4 Prop "$V \iff \phi$ and ϕ strictly increasing"

Suppose $U : X \to \mathbb{R}$ represents $\succsim$. Then $V : X \to \mathbb{R}$ represents $\succsim$ if and only if there exists $\phi : U(X) \to V(X)$ such that $V = \phi \circ U$ and ϕ is strictly increasing.

5 Expected Utility

5.0.1 Def "lottery"

Suppose X is a non-empty and finite set of prizes (consequences). Then a *lottery*, p is a probability distribution over X:

$$p : X \to [0,1], \text{ such that } \sum_{x \in X} p(x) = 1.$$

We denote $\mathcal{L}$ as the set of all lotteries.

5.0.2 Term "degenerate lottery"

If $p(x) = 1$ for some $x \in X$ then we write δ_x.

5.0.3 Remark: $\mathcal{L}$ is the set of alternatives, and unlike X is infinite!

5.0.4 Def "vNM expected utility function/utility index"

$U : \mathcal{L} \to \mathbb{R}$ is an expected utility function if there exists $u : X \to \mathbb{R}$ such that $U(p) = \sum_{x \in X} u(x)p(x), \forall p \in \mathcal{L}$.

Called a "von Neumann–Morgenstern utility index" or a "Bernoulli index."

5.0.5 Def "Mixture operation"

For all $p, q \in \mathcal{L}, \alpha \in [0,1]$:

- let $\alpha p + (1-\alpha)q$ denote a lottery $r \in \mathcal{L}$ such that for all $x \in X, r(x) = \alpha p(x) + (1-\alpha)q(x)$.

5.0.6 Def "expected utility representation"

$\succsim$ on $\mathcal{L}$ has an expected utility representation if there exists $u : X \to \mathbb{R}$ such that for $p, q \in \mathcal{L}$

$$p \succsim q \iff U(p) = \sum_{x \in X} p(x)u(x) \geq U(q) = \sum_{x \in X} q(x)u(x)$$

5.0.7 Def "linear function"

A function $U : \mathcal{L} \to \mathbb{R}$ is *linear* if for all $\alpha \in [0,1]$ and $p, q \in \mathcal{L}$:

$$U(\alpha p + (1-\alpha)q) = \alpha U(p) + (1-\alpha)U(q).$$

5.0.8 Lemma "linear $\iff$ U is an expected utility function"

A function $U : \mathcal{L} \to \mathbb{R}$ is *linear* if and only if U is an expected utility function.

5.0.9 Def "vNM Axioms"

The von Neumann–Morgenstern axioms are:

- A1. (Rationality): $\succsim$ is complete and transitive.
- A2. (Independence): For all $p, q, r \in \mathcal{L}, \alpha \in (0,1)$ we have $[p \succ q] \Rightarrow [\alpha p + (1-\alpha)r \succ \alpha q + (1-\alpha)r]$.
- A3. (Continuity): For all $p, q, r \in \mathcal{L}$, if $p \succ q \succ r$ then there exists $\alpha, \beta \in (0,1)$ such that $\alpha p + (1-\alpha)r \succ q$ and $q \succ \beta p + (1-\beta)r$.
- We don't use A2* in this class, but some literature uses it:
  - A2*. (Independence, strengthened): $[p \succsim q] \iff [\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r]$

5.0.10 Lemma "applying vNM Axioms"

Suppose $\succsim$ satisfies A1, A2, and A3 of vNM axioms.

(i) $\forall \alpha \in (0,1), p, q \in \mathcal{L}$:
  (a) $p \succ q \Rightarrow p \succ \alpha p + (1-\alpha)q \succ q$
  (b) $p \sim q \Rightarrow p \sim \alpha p + (1-\alpha)q \sim q$
(ii) $p \succ q, 1 \geq \alpha > \beta \geq 0 \Rightarrow \alpha p + (1-\alpha)q \succ \beta p + (1-\beta)q$
(iii) $p \sim q, \alpha \in (0,1) \Rightarrow \alpha p + (1-\alpha)r \sim \alpha q + (1-\alpha)r$
(iv) $p \succsim q, r \succsim s, \alpha \in (0,1) \Rightarrow \alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)s$

5.0.11 Thm "A1-A3 $\iff$ EU representation"

Suppose X is finite and $\succsim$ is a binary relation on $\mathcal{L}$. Then

$$[\succsim \text{ satisfies A1, A2, and A3 }] \iff \succsim \text{ has an Expected Utility representation.}$$

5.0.12 Thm "V linear representation $\iff V(p) = aU(p) + b$"

Suppose U is a linear representation of $\succsim$, and $V : \mathcal{L} \to \mathbb{R}$ is a linear function. Then

$$[V \text{ is a linear representation of } \succsim] \iff [\exists a > 0, b \in \mathbb{R} \text{ such that } V(p) = aU(p) + b, \forall p \in \mathcal{L}]$$

5.0.12.1 Term "affine" Basically a line, but the intercept doesn't have to be zero. "A function composed of a linear function and a constant and its graph is a straight line."

5.0.13 Def "simple lottery"

$p : X \to \mathbb{R}$ is a *simple lottery* if

1. $\{x \in X : p(x) > 0\}$ is finite
2. $\displaystyle\sum_{\{x \in X : p(x) > 0\}} p(x) = 1$

5.0.14 Def "simple improvement"

For all (simple) lotteries $p, q \in \mathcal{L}$, p is a simple improvement of q if there exists $\lambda \in (0,1), m, m' \in X, m > m';, r \in \mathcal{L}$ such that

$$p = \lambda \delta_m + (1-\lambda)r \text{ and } q = \lambda \delta'_m + (1-\lambda)r.$$

5.0.15 Def "FOSD"

For $p, q \in \mathcal{L}$, p first-order stochastically dominates (FOSD) q if

1. $p \neq q$, and
2. $F_p(x) \leq F_q(x), \forall x \in X$

   caution: CDF of p SMALLER than CDF of q means p dominates q, not the other way around

5.0.16 Fact "p FOSD $q \iff p$ is a complex improvement of q"

5.0.17 Thm "EU representation means p FOSD $q \Rightarrow p \succ q$ and u strictly increasing"

Suppose $\succsim$ has an expected utility representation. Then the following are equivalent:

1. p FOSD $q \Rightarrow p \succ q$
2. p is a simple improvement of $q \Rightarrow p \succ q$
3. Every EU representation of $\succsim$ has a strictly increasing utility index ($U(p) = \sum_{x \in supp(p)} \underline{u(x)} p(x)$)

5.0.18 Def "Betweenness Axiom"

$$\forall p, q \in \mathcal{L}, \lambda \in (0,1):$$
$$p \sim q \Rightarrow \lambda p + (1-\lambda)q \sim p$$

5.0.19 Def "Allais paradox"

Individuals prefer certainty over a risky outcome even if this defies the expected utility axiom

5.1 Risk Attitudes

5.1.1 Lemma

Suppose $\succsim$ has an expected utility representation with a strictly increasing and continuous index u. Then $\forall p \in \mathcal{L}, \exists$ unique $CE(p) \in X$ such that $p \sim \delta_{CE(p)}$. In this case, $CE(p)$ is the *certainty equivalent* of p.

$$CE(p) = u^{-1}(U(p)) = u^{-1}(\sum_{x \in supp(p)} u(x)p(x)) \in X \iff u(CE(p)) = U(p)$$

5.1.2 Def "risk attitudes"

Let $\bar{p} = \sum_{x \in supp(p)} p(x)x$ be the expected value of p. Then p is riskier than $\delta_{\bar{p}}$. For all $p \in \mathcal{L}$, $\succsim$ is

1. *risk averse* if $\delta_{\bar{p}} \succsim p$
2. *risk neutral* if $\delta_{\bar{p}} \sim p$
3. *risk seeking* if $p \succsim \delta_{\bar{p}}$

5.1.3 Def "simple Mean-Preserving Spread (MPS)"

Say $\hat{p} = \lambda p + (1-\lambda)r$ is a simple mean-preserving spread of $\hat{q} = \lambda\delta_{\bar{p}} + (1-\lambda)r$. $\hat{p}$ is riskier than $\hat{q}$.

5.1.4 Thm "risk-averse $\iff$ u concave $\iff$ MPS"

$\succsim$ has an EU representation such that its utility index, u, is strictly increasing and continuous. Then the following are equivalent:

1. $\succsim$ is risk-averse / -seeking / -neutral
2. [p is a simple MPS of q] $\Rightarrow q \succsim p$
3. u is concave / convex / affine

5.1.5 Def "more risk averse"

$\succsim_1$ is more risk averse than $\succsim_2$ if for all $m \in X, p \in \mathcal{L}$ we have

$$\delta_m \succsim_2 p \Rightarrow \delta_m \succsim_1 p.$$

5.1.5.1 Term "Absolute risk aversion" Call $-\frac{u_1''}{u_2'}$ Arrow-Pratt's measure of absolute risk aversion (ARA).

5.1.6 Thm "more risk averse $\iff$ concave composition"

$\succsim_1, \succsim_2$ have an EU representation with u_1, u_2 strictly increasing and smooth. Then the following are equivalent:

1. $\succsim_1$ is (always) more risk averse than $\succsim_2$.
2. there exists a concave and strictly increasing function $f: u_2(X) \to u_1(X)$ such that $u_1 = f \circ u_2 = f(u_2(x)), \forall x \in X$
3. $ARA_{u_1} \geq ARA_{u_2}$

5.1.6.1 Term "smooth"

Twice continuously differentiable

5.1.7 Def "CARA"

$$ARA = -\frac{u''(x)}{u'(x)} = a(\text{const} \in \mathbb{R})$$

Constant Absolute Risk Aversion (CARA) means risk attitude does not depend on the initial wealth level.

5.1.8 Def "functional form for CARA"

$$u(x) = \begin{cases} \frac{1-e^{-ax}}{a} \text{ if } a \neq 0 \\ x \text{ if } a = 0 \end{cases}$$

- $a > 0 \Rightarrow$ risk-averse
- $a = 0 \Rightarrow$ risk-neutral
- $a < 0 \Rightarrow$ risk-seeking

  Can compare the "a-level" of individuals to see who is more risk averse (e.g., $a_1 > a_2 \Rightarrow$ person 1 more risk-averse)

5.1.9 Def "CRRA"

$$RRA = -x\frac{u''(x)}{u'(x)} = \text{const} = r$$

Constant Relative Risk Aversion (CRRA) means risk attitude does not depend on inflation.

$$u(x) = \begin{cases} \frac{x^{1-r}}{1-r} \text{ if } r \neq 1 \\ \ln x \text{ if } r = 1 \end{cases}$$

person is risk-neutral when $u(x) = x$.

6 Demand Theory

6.0.1 Def "Well-behaved" preference

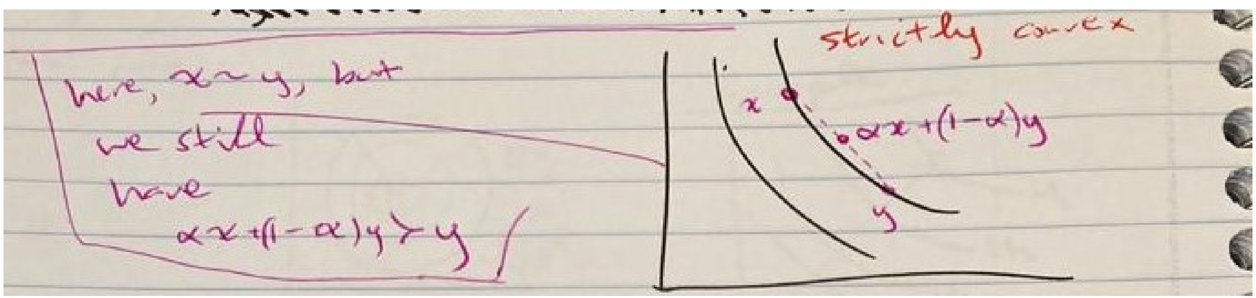
1. Monotone
$$[x_i \geq y_i, \forall i] \Rightarrow [x \succsim y]$$
2. Strongly Monotone
$$[x_i \geq y_i, \forall i \text{ and } x \neq y] \Rightarrow [x \succ y]$$
3. MWG Monotone
$$[x_i > y_i, \forall i] \Rightarrow [x \succ y]$$
4. Locally non-satiated
$$\forall x \in X, \forall \varepsilon > 0, \exists y \in B_\varepsilon(x) \text{ such that } y \succ x$$
5. Continuity $x^{(m)} \to x$ and $y^{(m)} \to y, \forall m \geq 1$ and $x^{(m)} \succsim y^{(m)} \Rightarrow x \succsim y$

   $2 \Rightarrow 3 \Rightarrow 4$ (but 3 alone does not imply 4)

6.0.2 Def "convex relation"

For all $x, y \in X$ and for all $\alpha \in (0,1)$:

- $\succsim$ is *convex* if
$$x \succsim y \Rightarrow \alpha x + (1-\alpha)y \succsim y$$
- $\succsim$ is *strictly convex* if
$$x \succsim y, x \neq y \Rightarrow \alpha x + (1-\alpha)y \succ y$$



6.0.3 Claim "$\succsim$ is rational $\not\Rightarrow \succsim$ has a utility representation"

6.0.4 Term "lexicographic preferences"

1. $\succsim$ is rational
2. $\succsim$ is not continuous
3. $\succsim$ does not have a utility representation

   An agent prefers any amount of one good (X) to any amount of another (Y). Specifically, if offered several bundles of goods, the agent will choose the bundle that offers the most X, no matter how much Y there is. Only when there is a tie between bundles with regard to the number of units of X will the agent start comparing the number of units of Y across bundles.

6.0.5 Def "economic preference / well-behaved preference"

A binary relation $\succsim$ is an *economic preference* (well-behaved preference) if it is

1. rational,
2. continuous,
3. monotone, and
4. locally non-satiated.

6.0.6 Thm "economic preference $\Rightarrow$ represented by continuous U"

$\succsim$ is an economic preference $\Rightarrow \succsim$ can be represented by a continuous utility function U.

6.0.7 Remark U is also monotone and MWG monotone, but not strong

7 Demand

7.0.0.1 Term "price vector" $P \in \mathbb{R}^n_{++}$. $p \in P : p = (p_1, ..., p_n), p_i > 0 \forall i$

7.0.0.2 Term "Budget constraint"

$$B(p,m) = \{\vec{x} \in X : p \cdot \vec{x} = \sum_{i=1}^{n} p_i \cdot x_i \leq m\}, \forall p \in P, m \geq 0.$$

$B(p,m)$ is compact, non-empty (and convex, but that isn't important for now.)

7.0.1 Thm "economic preference $\Rightarrow$ solution exists"

If $\succsim$ is an economic preference then $\forall p, m \geq 0, \exists x^* \in B(p,m)$ such that

1. $x^* \succsim y, \forall y \in B(p,m)$ and
2. $p \cdot x^* = m$.

7.0.2 Def "demand"

$$D : \mathcal{B} \to \mathcal{P}(X) \text{ such that } D(B(p,m)) = \{x^* \in B(p,m) : x^* \succsim y \forall y \in B(p,m)\}$$

Where $\mathcal{B} = \{B(p,m) : p \in P, m \geq 0\} \subsetneq \mathcal{P}(X)$. We can verify that $(\mathcal{B}, D)$ is a choice structure.

7.0.3 Remark "$(\mathcal{B}, D)$ is a choice structure"

$(\mathcal{B}, D)$ is a choice structure:

1. $D(B(p,m)) \subseteq B(p,m)$
2. $D(B(p,m)) \neq \emptyset$

7.0.4 Def "demand function"

If $D(B(p,m))$ is always a singleton we can define:

$$x : P \times \mathbb{R}_+ \to X$$

such that $x(p,m)$ is the only element of $D(B(p,m))$. We call $x(\cdot)$ the demand function.

7.0.5 Thm "strictly convex $\Rightarrow$ demand fn well-defined"

If $\succsim$ is a strictly convex economic preference then $x(\cdot)$ is a well-defined and continuous function.

7.0.6 Def "indirect utility function"

$V : P \times \mathbb{R}_+ \to \mathbb{R}$ such that $V(p,m) = U(x(p,m))$.

7.0.7 Def "neoclassical utility function"

$U : X \to \mathbb{R}$ is a *neoclassical utility function* if U

1. is continuous, and
2. represents a strictly convex economic preference.

7.0.8 Thm "neoclassical $U \Rightarrow$ properties of V"

Suppose U is neoclassical and V is the associated indirect utility function. Then,

1. V is continuous,
2. V is strictly increasing in m, and nonincreasing in each p_i.
3. V is quasiconvex ($V(p,m) \geq V(p',m') \Rightarrow V(p,m) \geq V(\alpha p + (1-\alpha)p', \alpha m + (1-\alpha)m')$, and
4. V is homogeneous of degree 0 ($V(p,m) = V(\lambda p, \lambda m)$)

7.1 Hicksian Demand

7.1.1 Thm "neoclassical $\Rightarrow$ unique minimization solution"

Suppose U is neoclassical. Then for all $p \in P, u \in U(X), \exists$ unique $x^* \in X$ such that

1. $U(x^*) = u$ and
2. $\forall x \in X, x \neq x^* : U(x) \geq u \Rightarrow p \cdot x > p \cdot x^*$

7.1.2 Def "Hicksian demand function

Let $x^h(p,u)$ denote $x^* \forall p \in P, u \in U(X)$. Then $x^h : P \times U(X) \to X$ is called the *Hicksian demand function*.

7.1.3 Def "expenditure function"

Define the *expenditure function*, $e : P \times U(X) \to \mathbb{R}$ such that $\forall p \in P, u \in U(X)$:

$$e(p,u) = p \cdot x^h(p,u).$$

7.1.4 Thm "duality theorem"

Suppose U is neoclassical. Then

1. $e(p, V(p,m)) = m$
2. $V(p, e(p,u)) = u$
3. $x(p, e(p,u)) = x^h(p,u)$
4. $x^h(p, V(p,m)) = x(p,m)$.

Three important results:

7.1.5 Thm "relating $e(\cdot)$ and $x^h(\cdot)$"

Suppose U is neoclassical. Then

1. $e(\cdot)$ is continuous,
2. $e(\cdot)$ is strictly increasing in u and non-decreasing in p_i,
3. $e(\cdot)$ is concave in p for fixed $u \in U(X)$,
4. $e(\lambda p, u) = \lambda e(p,u), \forall \lambda > 0$, and
5. $\frac{\partial}{\partial p_i} e(p,u) = x_i^h(p,u), \forall i = 1, ..., n$.
   - This is Shephard's Lemma. It's the counterpart to Roy's identity, just on the Hickisan side (and simpler).

7.1.6 Thm "relating $x(\cdot)$ and $V(\cdot)$ (Roy's identity)"

Suppose U is neoclassical, $V(p,m)$ is differentiable at (p,m) and $x(p,m)$, is in the interior of X. Then

$$x_i(p,m) = -\frac{\frac{\partial}{\partial p_i} V(p,m)}{\frac{\partial}{\partial m} V(p,m)}$$

7.1.7 Thm "relating $x(\cdot)$ and $x^h(\cdot)$ (Slutsky equation)"

Suppose U is neoclassical, $x(\cdot)$ is continuously differentiable, and $x(p, m)$ is in the interior of X. Then

$$\left.\frac{\partial x_i^h}{\partial p_k}(p,u)\right|_{(p,u)=(p,V(p,m))} = \underbrace{\frac{\partial x_i}{\partial p_k}(p,m)}_{\text{substitution effect}} + \underbrace{\frac{\partial x_i}{\partial m}(p,m)x_k(p,m)}_{\text{income effect}}, \forall i,k \in \{1,...,n\}$$

i, k can be same (sensitivity of demand to price of own good) or different (sensitivity of demand to price of another good)

8 Misc

8.0.1 "Utility Maximization"

Utility is maximized when

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

Note that the objective function is linear in t. Thus, maximization occurs at a "corner," where either $t = 0$ or $t = 1$.If $a_1 > a_2$, then it is best to set $t = 1$; otherwise, it is best to set $t = 0$.

8.1 Algorithms

8.1.1 "Typical 601 questions"

1. Choice/ Decision theory
   - silly transitivity/completeness proofs
   - WARP and choice structures (proofs)
2. Utility
   - what amount of wealth to make you indifferent
   - what is expected value of bet
   - is U neoclassical?
3. Demand theory
   - write down indirect utility function
   - derive demand / use duality

8.1.2 Algorithm "Does this choice function satisfy WARP?"

1. Assume violation
   i. try minimal counterexample (3-4 elements, + WARP def)
   ii. can try to violate just α or just β
2. No violation
   i. Choose appropriate WARP def
   ii. define $A, B, c(A), c(B), x, y, ...$
   iii. prove, usually with cases

8.1.3 Algorithm "expected utility questions"

Prove A3

- You have to actually come up with the "values" of α and β (remember, $\alpha > \beta$ even though the def doesn't say this) by saying:
- "Consider compound lottery, γ, s.t. $\gamma U(p) + (1 - \gamma)U(p) = U(q)$"
- Solve for γ and notice $\gamma \in (0, 1)$.
- Now you can pick any $\alpha \in (\gamma, 1)$ and any $\beta \in (0, \gamma)$ and directly show A3.

8.1.4 Algorithm "Hicksian. . . "

finish

8.1.5 "Random tips/facts"

- continuity (A3) is hard to find a counterexample for–usually a question involving continuity will have you prove that it holds.
- to show/find counterexamples for A1-A3 it's often helpful to set $q = r$
- to always satisfy WARP: set $u(x) = c$.
- use degenerate lotteries in counterexamples/proofs!

- easier to do CDFs in a table than a graph. Denote them $F_p(x)$ and $F_q(x)$.
- U monotone $\Rightarrow$ budget constraint binds at the maximizer

8.1.6 "Mistakes to avoid"

- when you calculate $U(x)$ your output shouldn't have δ_a in it... you need to plug in a! You want to get utility out, not a lottery.
- Leontief $\min\{2x_1, x_2\} \Rightarrow 2x_1 = x_2$, so when you calculate $V(p,m)$, you STILL HAVE THE 2 IN THERE!! DON'T LEAVE IT OUT!!
- when verifying Slutsky: don't plug $V(p,m)$ in for u until AFTER you differentiate (you'll often get 0 since Hicksian demand often doesn't have p in the solution.)

8.1.7 Algorithm "Plugging in lotteries"

DON'T JUST PLUG IN! YOU'LL GET IT WRONG. Instead, write out:

$$U(p) = \sum_{x \in supp(p)} p(x)u(x)$$

and then calculate carefully. Try to do it both the right and wrong ways if possible to compare, because it's very easy to mess up and still get a "reasonable" answer.