Micro III Notes

Parker Howell

Last Updated: 2024-08-16

Contents

1 General Equilibrium

1.0.1 Def "GE: what resources available?"

1. L goods, $l = 1, ..., L$ (finite).

  i. consumption: apples, leisure time, location, etc.
  ii. inputs: labor, land, capital
  iii. dated goods: apple today vs apple tomorrow
  iv. contingent goods: umbrella w rain vs umbrella w/o rain

2. Initial (total) Endowment: $\omega = (\omega_1, ..., \omega_L) \in \mathbb{R}^L_+$
3. Production

  ii. Production set $Y \subseteq \mathbb{R}_L$.
  iii. Call $y \in \mathbb{R}^L$ a "feasible production plan" if $y \in Y$.

4. Feasibility Set/Resource Set: $R = \{x \in \mathbb{R}^L : x = \omega + y \text{ for some } y \in Y\}$.

1.0.2 Def "GE: Properties on Y"

1. Y is closed and nonempty
2. Possible to do nothing: $(0, ..., 0) = \vec{0} \in Y$
3. No free lunch: $Y \cap \mathbb{R}^L_+ = \emptyset$
4. Free disposal: $y \in Y, \hat{y} \leq y \Rightarrow \hat{y} \in Y$

1.0.3 Def "feasible"

x is *feasible* if:

1. $x_i \in X_i, \forall i = 1, ..., I$ and
2. $\sum_{i=1}^I x_i \in R$ (i.e., $\exists y \in Y : \sum_{i=1}^I x_i = \omega + y$)

1.0.4 Def "Pareto Efficient"

A feasible consumption plan, x is *Pareto Efficient* (Pareto Optimal) if there does not exist any other feasible $\hat{x}$ such that

1. $\hat{x_i} \succsim_i x_i \forall i = 1, 2, ..., I$ and
2. $\hat{x_i} \succ_i x_i$ for at least one $i \in \{1, ..., I\}$.

If such $\hat{x}$ exists, we say $\hat{x}$ *Pareto-dominates* x.

1.0.4.1 Remark

1. firms are not people (agents).
2. Pareto efficiency is not about equity.

1.0.5 Def "PE algorithm"

Steps:

1. compute R from ω and Y to figure out feasible consumption plans.
2. $U_i = \{u_i(x_i) : x \text{ is feasible}\}, \forall i = 1, ..., I$. (set of all possible utility levels)
3. Fix $\bar{u_i} \in U_i, i = 2, ..., I$.
    a. $\max_x u_1(x_1)$ subject to $u_i(x_i) \geq \bar{u_i}, i = 2, ..., I$.
4. Change values: $u_i \in U_i, i = 2, ..., I$ and repeat step 3.

1.0.6 Remark "the PE algorithm is not sufficient UNLESS"

1. u_i is continuous and strongly monotone, and
2. $Y = \{y \in \mathbb{R}^L : \vec{y} \leq \vec{0}\}$.

The other way to show that the solution fo the UMP is PE is to show that the utility frontier is down-sloping:

$$\frac{\partial u_2^*}{\partial u_1^*} = \frac{\frac{\partial u_2^*}{\partial x_{11}^*}}{\frac{\partial u_1^*}{\partial x_{11}^*}} < 0$$

1.0.7 Prop "Pareto $\Rightarrow x^*$ solution"

If x^* is Pareto efficient, then x^* solves step 3 above for some $\bar{u_2}, ..., \bar{u_I}, \bar{u_i} \in U_i \forall i = 2, ..., I$.

2 Walrasian Equilibrium

2.0.1 Def "WE Primitives"

1. Individual firms
   - $j = 1, ..., J, Y_j \in \mathbb{R}^L$ such that
   - $$Y = \sum_{j=1}^{J} Y_j = \{y \in \mathbb{R}^L : \exists y_j \in Y_j, \forall j = 1, ..., J \text{ s.t. } y = \sum_{j=1}^{J} y_j\}$$
2. Individual endowment
   - $\omega_i \in \mathbb{R}^L_+, \forall i = 1, ..., I$ such that $\omega = \sum_{i=1}^{I} \omega_i$ (previously we had only ω and not ω_i)
3. Consumer i share in firm j's profits
   - $\underbrace{\theta_i^j \in [0,1]}_{\text{share of firm j's profits to consumer i for consumption}}$ $, \forall j$ such that $\sum_{i=1}^{I} \theta_i^j = 1$ (consumers own 100% of firms)

2.0.2 Def "WE Allocation"

An allocation is

$$(\underbrace{x_1, ..., x_I}_{\text{consumption plans}}, \underbrace{y_1, ..., y_J}_{\text{production plans}}) \subseteq \mathbb{R}^{IL+JL}, \underbrace{\forall i, x_i \in X_i}_{\text{consumption is feasible}}, \underbrace{\forall j, y_j \in Y_j}_{\text{production is feasible}}.$$

2.0.3 Def "Walrasian Equilibrium"

$p^*, (x^*, y^*)$ are a Walrasian Equilibrium if the following are satisfied:

1. Profit Maximization: each firm j maximizes profits given prices: $p^* \cdot y_j^* \geq p^* \cdot y_j, \forall y_j \in Y_j, \forall j = 1, ..., J$
2. Preference maximization:
   i. $\max_{x_i} u_i(x_i)$ s. to $\underbrace{p^* \cdot x_i^*}_{\text{expenditure}} \leq \underbrace{p^* \cdot w_i}_{\text{endowment income}} + \underbrace{\sum_{j=1}^{J} \theta_i^j (p^* \cdot y_j^*)}_{\text{income from profits}}$ $, \forall i = 1, ..., I$ and
   ii. $x_i^* \succsim_i x_i$ for all $x_i \in X_i$ s. to $\underbrace{p^* \cdot x_i^* \leq p^* \cdot w_i + \sum_{j=1}^{J} \theta_i^j (p^* \cdot y_j^*)}_{\text{budget set}}$
3. All markets clear: $\sum_{i=1}^{I} x_{i,l}^* = \sum_{i=1}^{I} \omega_{i,l}^* + \sum_{j=1}^{J} y_{j,l}^*, \forall l = 1, ..., l.$

   Don't forget: the budget set is NOT related to what is available on the market - this is why 0 prices create infinite demand and can't exist in WE

2.0.4 Obs 1 "normalizing prices"

If (p^*, x^*, y^*) is a Walrasian Equilibrium, then $(\lambda p^*, x^*, y^*)$ is a Walrasian Equilibrium $\forall \lambda > 0$

2.0.5 Obs 2 "Walras' law"

If $\succsim_i$ is locally nonsatiated and (p^*, x^*, y^*) satisfies profit maximization then

$$\underbrace{p^* \cdot \sum_{i=1}^{I} x_i^*}_{\text{agg demand}} = \underbrace{p^* \cdot \sum_{i=1}^{I} \omega_i + p^* \cdot \sum_{j=1}^{J} \overbrace{\sum_{i=1}^{I} \theta_i^j}^{=1} (y_j^*)}_{\text{agg supply}}.$$

2.0.6 Obs 3 "markets clear for all but one, then markets clear for all"

If

1. $\succsim_i$ is locally nonsatiated,
2. (p^*, x^*, y^*) satisfies profit maximization,
3. $p_L^* \neq 0$, and
4. $\sum_{i=1}^I x_{i,l}^* = \sum_{i=1}^I \omega_{i,l} + \sum_{j=1}^J (y_{j,l}^*), \forall l = 1, ..., L-1,$

Then $\sum_{i=1}^I x_{i,L}^* = \sum_{i=1}^I \omega_{i,L} + \sum_{j=1}^J (y_{j,L}^*)$

In words: if market clearing holds for all other goods, and the price of the last good is nonzero, then I have market clearing for the last good.

2.0.7 Thm "First Welfare Theorem (WE $\Rightarrow$ Pareto-efficient)"

Suppose $\succsim_i$ is locally nonsatiated $\forall i = 1, ..., I$ and (p^*, x^*, y^*) is a Walrasian Equilibrium. Then (x^*, y^*) is Pareto-efficient.

2.0.7.1 Remark: "Implicit assumptions First Welfare Thm"

1. firms are price-taking
2. no externalities
3. markets are complete (less relevant to this course, but an important assumption in finance theory.)

2.0.8 Def "system of transfers"

A system of transfers, T is a vector $T = (T_1, ..., T_I) \in \mathbb{R}^I$ such that $\sum_{i=1}^I T_i = 0$.

Note:

- $T_i > 0 \Rightarrow$ "receiving transfer"
- $T_i < 0 \Rightarrow$ "paying transfer (tax)"

2.0.9 Def "WE with transfers"

(p^*, x^*, y^*) is a Walrasian equilibrium with transfers T if it satisfies

1. Profit maximization (no change here),
2. Preference maximization: x_i^* maximizes $\succsim_i$ (represented by u_i subject to $p^* \cdot x_i^* \leq p^*\omega_i + \sum_{j=1}^J \theta_i^j p^* \cdot y_j^* + T_i$), and
3. Market clearing (no change here either).

Note: we always add T_i even if $T_i < 0$.

2.0.10 Thm "First Welfare Theorem with transfers"

Suppose T is a system of transfers, $\succsim_i$ is locally nonsatiated $\forall i = 1, ..., I$ and (p^*, x^*, y^*) is a Walrasian Equilibrium with T. Then (x^*, y^*) is Pareto-efficient.

2.0.11 Thm "Second Welfare Theorem (PE $\Rightarrow$ WE)"

Under assumptions 1-4, if (x^*, y^*) is Pareto-Efficient then there exists a system of transfers T and $p^* \neq \vec{0}$ such that (p^*, x^*, y^*) is a Walrasian Equilibrium with transfers T.

Assumptions:

1. $\forall$ consumers i, X_i is convex and $\succsim_i$ satisfies
   i. convexity: $x_i \succsim x_i' \Rightarrow \lambda x_i + (1-\lambda)x_i' \succsim x_i', \forall \lambda \in (0,1)$, and

    ii. something slightly weaker than strict convexity: $\forall x_i, x_i', \hat{x}_i \in X_i : [x_i \succ_i \hat{x}_i, x_i' \succsim_i \hat{x}_i] \Rightarrow \lambda x_i + (1 - \lambda) x_i' \succ_i \hat{x}_i, \forall \lambda \in (0,1)$.
2. Y_j is convex $\forall j = 1, ..., J$.
3. $\succsim_i$ is locally nonsatiated
4. $\forall i, \succsim_i$ is continuous, and $X_i \in \mathbb{R}^L_{++}$.
    - This one is VERY STRONG – means that EVERY individual has a nonzero amount of EVERY good.

Note: assumption 4 is weaker in MWG and other places, but we will always use this version in this class.

3 Existence of Walrasian Equilibria

3.0.1 Def "set of price vectors"

$\Delta = \{p \in \mathbb{R}^L : \forall l, p_l \geq 0 \text{ and } \sum_{l=1}^{L} p_l = 1\}$

3.0.2 Def "excess demand function"

$\varphi : \Delta \to \mathbb{R}^L$ such that $\forall p \in \Delta$:

$$\varphi(p) = \sum_{i=1}^{I} x_i^*(p) - \sum_{i=1}^{I} \omega_i - \sum_{j=1}^{J} y_j^*(p)$$

is called the excess demand function.

- $\varphi(p) = \vec{0} \Rightarrow$ WE exists
  - (in WE: demand(p) = supply(p) $\iff$ demand(p) - supply = 0 $\iff$ excess demand(p) = 0).
- And, under some assumptions we can get the converse:
  - WE exists $\Rightarrow \varphi(p) = \vec{0}$.

3.0.3 Note "Implicit assumptions of excess demand fn"

1. We can focus on $p \in \Delta$. That is, in WE, $p_l \geq 0 \forall l$ and for at least one good $l, p_l > 0$.
2. There is a unique solution to the profit and preference maximizing problems (i.e., φ is a function, not a correspondence.). This one isn't really a problem though because we can always generalize later and use Kakutani's instead of Brouwer's fixed point theorem.
3. Excess demand for any good is finite (even for $0-priced goods).

3.0.4 Thm "sufficient conditions on φ"

Suppose $\varphi : \Delta \to \mathbb{R}^L$ is well-defined (i.e., satisfies implicit assumptions 1-3) and satisfies

1. φ is continuous,
2. φ satisfies Walras' law ($p \cdot \varphi(p) = 0, \forall p \in \Delta$), and
3. if $p_l = 0$ for some l then $\varphi(p_l) > 0, \forall p \in \Delta$,

Then, there exists at least one $p \in \Delta$ such that $\varphi(p) = \vec{0}$

Note (exercise): $f(p) = p \iff \varphi(p) = 0$

3.0.5 Thm "sufficient conditions on primitives"

Suppose $\forall i, \forall j$:

1. $X_i \in [0, m]^L$ where m is "large" (i.e, $m = \max_{l=1,\ldots,L}\{\max_{(x,y)}\{x_l\}\} + 1$)
2. $\omega_{i,l} > 0, \forall l, i$
3. $\succsim_i$ is continuous, strongly monotone, and strictly convex, and
4. $\vec{0} \in Y_j, Y_j$ is convex and satisfies $\forall y_j, y_j' : y_j \neq y_j' \Rightarrow \exists \hat{y}_j \in Y_j$ such that $\hat{y}_{j,l} > \lambda \hat{y}_{j,l} + (1-\lambda)\hat{y}_{j,l}', \forall l$.

Then, φ is well-defined and satisfies 1-3 of Thm "sufficient conditions on φ".

4 Arrow's Impossibility Theorem

4.0.1 Def "setting for Arrow's Impossibility Theorem"

1. Finite set of "alternatives," $A = \{a, b, c, ...\}$
2. Finite set of individuals, $N = \{1, ..., N\}$
3. $\mathcal{R}$ is the set of all rational and antisymmetric preferences over A, ruling out indifferences ($R_i \in \mathcal{R}$ is the preference of individual i).
4. A preference profile is $R = (R_1, ...R_n) \in \mathcal{R}^n$

Goal: $f : \mathcal{R}^n \to \mathcal{R}$

4.0.2 Thm "Arrow's Impossibility Theorem"

Assumptions:

1. Universal domain: any ranking is possible (so long as it is complete, transitive, and anti-symmetric)
2. society's preference must be complete, transitive, and anti-symmetric
   - antisymmetric: for all $x, y \in X, [x \succsim y$ AND $x \neq y] \Rightarrow y \not\succsim x$
   - i.e., $[xRy$ and $yRx] \Rightarrow x = y$
3. $|A| \geq 3$

Axioms:

1. Pareto: $aR_ib, \forall i \Rightarrow af(R)b$
2. Independence of irrelevant alternatives: $\forall i[aR_ib \iff a\hat{R}_ib] \Rightarrow [af(R)b \iff af(\hat{R})b]$
3. No dictator: $\nexists i \in N : \forall R \in \mathcal{R}^n, f(R) = R_i$

There is no $f : \mathcal{R}^n \to R$ that satisfies all three axioms at the same time.

4.0.3 Def "winning coalition"

Denote $\mathcal{P}(N)$ to be the set of all nonempty subsets of N $(2^N \setminus \{\emptyset\})$.

We say $\mathcal{G} \subseteq \mathcal{P}(N)$ is a set of winning coalitions if

1. $\forall G \subseteq \mathcal{P}(N)$ either $G \in \mathcal{G}$ or $N \setminus G \in \mathcal{G}$, but not both. And
2. $[G \in \mathcal{G}, G' \supseteq G] \Rightarrow G' \in \mathcal{G}$

4.0.4 Def "Generalized majority voting"

$f : \mathcal{R}^n \to R$ is a GMV if there exists a set of winning coalitions $\mathcal{G}$ such that $\forall a, b \in A, \forall R \in \mathcal{R}^n$:

$$af(R)b \iff \{i \in N : aR_ib\} \in \mathcal{G}.$$

4.0.4.1 Term "Condorcet Winner/Condorcet cycle" the pairwise champion or beats-all winner, is formally called the Condorcet winner. The head-to-head elections need not be done separately; a voter's choice within any given pair can be determined from the ranking.

Some elections may not yield a Condorcet winner because voter preferences may be cyclic—that is, it is possible (but rare) that every candidate has an opponent that defeats them in a two-candidate contest. (This is similar to the game rock paper scissors, where each hand shape wins against one opponent and loses to another one).

4.0.4.2 Term "Social Welfare Function (SWF)" Function with preferences as inputs and a complete ranking as outputs.

4.0.4.3 Term "Collective Choice Rule (CCR)" Function with preferences as inputs and a winner as output.

5 Quasilinear Economy

5.0.0.1 Term "Quasilinear economy"

$$U(x_i, m_i) = g_i(x_i) + m_i, \forall i \text{ where } g_i \text{ is strictly concave.}$$

Called quasilinear because utility is linear in money but not in the consumption good.

A utility function is quasilinear in commodity 1 if it is in the form

$$u(x_1, \ldots, x_L) = x_1 + \theta(x_2, ..., x_L)$$

where θ is an arbitrary function.

5.0.1 Def "cost function"

$c_j : \mathbb{R}_+ \to \mathbb{R}_+$ is a cost function. $c_j(y_j)$ is the amount of money needed to produce y_j units of consumption good l. c_j must be

1. smooth,
2. increasing, $c_j'(y_j) > 0, \forall y_j > 0$,
3. convex $c_j''(y_j) > 0 \forall y_j > 0$ (i.e., the more you produce, the harder it becomes to produce more.)

5.0.2 Def "primitives on quasilinear economy"

1. Goods:
   1. $l = 1$ is "money," $l = 2$ is consumption good
2. Consumers $i = 1, ..., I$:
   1. $\forall i = 1, ..., I, X_i = \mathbb{R} \times \mathbb{R}_+$ (means negative amounts of money are allowed!)
   2. Initial endowment of $(\omega_i, 0)$.
   3. $u_i(m_i, x_i) = m_i + g_i(x_i)$
      1. money has constant marginal utility and MRS does not depend on m_i.
      2. Assumptions on g_i is smooth and $g_i'(x_i) > 0, g_i''(x_i) < 0, \forall x_i > 0$ (this is concave, but more restrictive - *log* satisfies, but x^2 does not). No need for Inada conditions.
3. Firms $j = 1, ..., J$:
   1. firm j's production set: $Y_j = \{(-z_j, y_j) : y_j \geq 0, z_j \geq c_j(y_j)\}$
      1. $z_j > c_j$ would mean that free disposal is employed
      2. $z_j \geq 0$ - we can write this separately, it's just redundant since c_j maps to/from $\mathbb{R}_+$ and $z_j \geq c_j(y_j)$.
4. Consumer i's share of firm j's profits is as before: θ_i^j

5.0.3 Def "allocation"

A vector $(m, x, z, y) \in \mathbb{R}^I \times \mathbb{R}_+^{I+2J}$ where

1. m_i: money of consumer i
2. x_i: consumption good of consumer i
3. z_j: input of money firm j uses
4. y_j: consumption good firm j produces.

An allocation is *feasible* if

1. Plans are feasible:
   1. $\forall i, (m_i, x_i) \in X_i$
   2. $\forall j, (-z_j, y_j) \in Y_j$
2. Markets clear:
   1. $\sum_{i=1}^I m_i = \sum_{i=1}^I \omega_i - \sum_{j=1}^J z_j$ (money is all used up)
   2. $\sum_{i=1}^I x_i = \sum_{j=1}^J y_j$ (demand = supply)

5.0.4 Def "Walrasian Allocations"

1. Simplifications without loss:
    1. Normalize price of money to 1, price of $l = 2$ to $p > 0$.
        1. always works in a quasilinear economy since utility is strictly increasing in both goods.
        2. focus on the market for the consumption good (since $p > 0$ we apply observation 3)
2. Solve for Demand

UMP is:

$$\max_{m_i \in \mathbb{R}, x_i \in \mathbb{R}_+} m_i + g_i(x_i) \text{ s to } m_i + px_i \leq \omega_i + \sum_{j=1}^{J} \theta_i^j (py_j - c_j(y_j))$$

But we can assume that $m_i + px_i = \omega_i + \sum_{j=1}^{J} \theta_i^j (py_j - c_j(y_j))$, isolate, and plug m_i into the objective function to get the unconstrained maximization problem:

$$\max_{x_i \geq 0} \omega_i + \sum_{j=1}^{J} \theta_i^j (py_j - c_j(y_j)) - px_i + g_i(x_i)$$

Which is the same as

$$\max_{x_i \geq 0} g_i(x_i) - px_i$$

So, the UMP for demand in a quasilinear economy is unaffected by money

By FOC and SOCs we get the Econ 101 result that consumers will demand more until MU = MC (unless demand is 0):

$$\begin{cases} MU = g_i'(x_i) = p = MC \text{ if } p \leq g_i'(0) \\ 0, \text{ otherwise} \end{cases}$$

$p \leq g_i'(0)$ since MU is decreasing, so MU is highest at zero

Caution: watch out for corner solutions since x_i can equal 0!

This is why the partial equilibrium from intermediate micro leads to the same conclusions as general equilibrium - we just always studied quasilinear economies in intermediate micro

3. Solve for Supply

Profit max problem:

$$\max_{y_j \geq 0} py_j - c_j(y_j)$$

which is the same as

$$\begin{cases} MC = c_j'(y_j) = p \text{ if } p \geq c_j'(0) \\ 0 \text{ otherwise.} \end{cases}$$

The above assumes that MC of production is increasing, so $c_j'(0)$ is the lowest marginal cost

Econ 101: MC = p, unless supply = 0.

5.0.4.1 Remark: Market supply = sum of all firm's supply

5.1 Pareto Efficient Allocations

Allocation of money does not matter

Summing utilities is okay to do in a quasilinear economy - in general it is not!

5.1.1 Prop 1 "PE iff max welfare"

In the quasilinear model a feasible allocation (m, x, z, y) is Pareto Efficient if and only if (x, y) maximize $\sum_{i=1}^{I} g_i(x_i) - \sum_{j=1}^{J} c_j(y_j)$ among all feasible allocations.

That is, $\forall i, j, x_i \geq 0, y_j \geq 0$:

1. $\sum_{i=1}^{I} x_i = \sum_{j=1}^{J} y_j$ and
2. $\sum_{i=1}^{I} m_i = \sum_{i=1}^{I} \omega_i - \sum_{j=1}^{J} z_j$, where $z_j = c_j(y_j)$.

5.1.2 Prop 2 "PE iff max sum of utility"

In the quasilinear model a feasible allocation (m, x, z, y) is Pareto Efficient if and only if (x, y) maximizes the sum of consumer utilities, $\sum_{i=1}^{I}(m_i + g_i(x_i))$, among all feasible allocations.

Econ 101: surplus = sum of utilities of consumption good - sum of firms' costs

6 Aggregation

6.0.0.1 Term “summation of sets”

$$M_1 + M_2 = \{x_1 + x_2 : x_1 \in M_1, x_2 \in M_2\}$$

6.0.1 Def “Supply correspondence of firm j”

The supply correspondence of firm j with production set Y_j is S_j, where $\forall p \in \mathbb{R}^L$:

$$S_j(p) = \{y_j \in Y_j : p \cdot y_j \geq p \cdot \hat{y}_j, \forall \hat{y}_j \in Y_j\}$$

- We assume $S_j(p)$ is well defined $\forall p$ – that is, profit maximization has a solution
- In general, $S_j(p)$ is NOT a singleton set.

6.0.2 Def “Aggregate supply correspondence”

$$S^*(p) = (S_1 + \cdots + S_J)(p) = S_1(p) + \cdots + S_J(p)$$

> “S looks like the supply correspondence of just one firm”

Denote production set, $Y = Y_1 + \cdots Y_J$ and S is the supply correspondence of a firm with production set Y. Then $S = S_1 + \cdots S_J$.

6.0.3 Prop “Supply Aggregates If”

Denote $Y = Y_1, ..., Y_J$ and S is the supply correspondence of a firm with production set Y. Then $S = S_1, ..., S_J$

6.0.4 Def “Setting for demand aggregation”

- $X_i \in \mathbb{R}^L_+, p \in \mathbb{R}^L_{++}, m_i \geq 0$, budget set: $p \cdot x_i \leq m_i$
- demand function: $D_i : \underbrace{\mathbb{R}^L_{++}}_{p} \times \underbrace{\mathbb{R}^L_+}_{m_i} \to \mathbb{R}^L_+$
- D_i continuous

6.0.5 “representative consumer”

There is a representative consumer if her demand function D maximizes her preference and

$$D(p, \sum_i m_i) = \sum_i D_i(p, m_i), \forall p \in \mathbb{R}^L_{++}, m_i \geq 0, i = 1, ..., I$$

6.0.6 “Necessary Conditions to have a representative consumer”

1. Income aggregation
   - $\forall p \in \mathbb{R}^L_{++}, m_i \geq 0, \hat{m}_i \geq 0, \forall i$:
     - $\sum_i m = \sum_i \hat{m}_i \Rightarrow \sum_i D_i(p, m_i) = \sum_i D_i(p, \hat{m}_i)$
2. Preference Maximization
   - there exists a preference $\succsim$ such that $D(p, m)$ maximizes $\succsim$ on $\{x \in \mathbb{R}^L_+ : p \cdot x \leq m\}$
   - this is “WARP for demand”: $\forall (p, m), (p', m')$:

$$[p \cdot D(p', m') \leq m \text{ and } D(p, m) \neq D(p', m')] \Rightarrow p' \cdot D(p, m) > m'$$

   same as WARP from 601 when we had a singleton!

   if your choice in the first menu is also contained in the second menu

6.0.7 Prop "demand fns allow aggregation iff"

Suppose $D_i(p, 0) = \vec{0}, \forall i = 1, ..., I$ and $p \in \mathbb{R}^L_{++}$. Then income aggregation for $D_1, ..., D_I$ if and only if

1. $D_i = D_j, \forall i, j$, and
    i. i.e., all consumers have the same demand function
2. there exists a function $\alpha : \mathbb{R}^L_{++} \to \mathbb{R}^L_{+}$ s.t. $D_i(p, m_i) = m_i \alpha(p), \forall i, \forall p, \forall m_i \geq 0$
    i. i.e., fixing p, the proportion of income spent on any good does not depend on the income level

    technically a positive result, but we interpret it as a negative result because the assumption are so hard to satisfy IRL

7 Misc

7.0.1 Def "numeraire"

a tradable economic entity in terms of whose price the relative prices of all other tradables are expressed (i.e., the price that is normalized to [1 or p - need to figure this out!])

w/o free disposal, have to consume everything. w/ free disposal, need strongly monotone preferences to guarantee that consumers consume the entire endowment (budget constraint will bind).

7.0.2 Def "Marginal rate of substitution - MRS"

$$MRS_1^{\text{consumer } i} \equiv \frac{\frac{\partial u_i}{\partial x_{i1}}}{\frac{\partial u_i}{\partial x_{i2}}}$$

In equilibrium we know that

$$MRS_1 = -\frac{p_1}{p_2}$$

We also know that given $I = L = 2$ this holds for any consumer:

$$MRS_1^{\text{consumer } 1} = -\frac{p_1}{p_2} = MRS_1^{\text{consumer } 2}$$

If $L \neq 2$ then MRS is not helpful. If $L = 2$ and $I > 2$ I think this will hold for all i, but I'm not sure...

> MRS is the absolute value of the slope of the indifference curve at whichever commodity bundle quantities are of interest.

7.0.3 "Common Question Types"

1. GE
   - find all PE consumption plans (with and without production)
   - show that x^* is PE
2. WE
   - find all WE (with and without Edgeworth)
   - find one WE
   - show that PE allocations are on the edges of the Edgeworth box (2x)
3. Existence of WE
   - show no WE
   - convert PE to WE
   - is a given WE also PE?
4. Arrow
   - given aggregation rule, which assumptions/axioms (not) satisfied
5. Q-linear economies
   - find all PE
   - find all WE
6. Aggregation
   - is there a representative consumer? (for which values of α_i is there a rep cons)

7.0.4 "Algorithm Find ALL PE consumption plans"

0. if possible/helfpul draw Edgeworth box. Do ICE
   a. "Checking for interior solutions:" set MRS...
   b. check edges separately: 4 cases: (1) $x_{11} = 0, \ldots$ (4) $x_22 = 0$
      i. for each case draw point B on the edge and point A on the interior and check which is PE
      ii. without drawing, say: "to increase u_1, want to increase x_{11} which requires a decrease in x_{21} which lowers u_2."

c. check corners: 4 cases again. just do them.

1. identify feasible consumption plans, R
2. figure out $U_i = \{u_i(x_i) : x \text{ is feasible}\}$
   a. this will usually be something like $[0, 250]$ if, e.g. $\omega = (5, 10), u_2 = x_{21}^2 x_{22}$
3. if production
   a. say "if we want to preserve x_l of good l, then the max of good $-l$ we can produce is $\sqrt{\omega_l - x_l}$"
   b. say "by free disposal: consider only points along the frontier, not inside the frontier."
4. Check corners:
   - $\bar{u}_2 = 0 \Rightarrow ((5, 10), (0, 0))$
   - $\bar{u}_2 = 250 \Rightarrow ((0, 0), (5, 10))$
5. Solve Umax (Fix some $\bar{u} \in U_2$)
   a. check SOCs!
   b. can also set $MRS_1^{cons1} = MRS_1^{cons2}$ and solve for interior solution.
6. Write or describe the set
   a. often easier to describe–certainly better to describe if you're unsure of answer or notation since you may get partial credit for a good explanation but not for the wrong set.
7. Explain why the set is complete. One of:
   - "utility is continuous and strongly monotone (Cobb-Douglas satisfies if excluding 0's), so by thm all solutions to the maximization are PE."
   - utility frontier is down-sloping: $\frac{\partial u_2^*}{\partial u_1^*} = \frac{\frac{\partial u_2^*}{\partial x_{11}^*}}{\frac{\partial u_1^*}{\partial x_{11}^*}} < 0$

Common Edgeworth Box Solutions:

- Convex: path from corner to corner
- Linear symmetric: any point in the Edgeworth box is a solution
- Linear nonsymmetric: i must have no good 2 to give out, or $-i$ must have no good 1 to give out
- symmetric Leontief: range in the center over which vertical portions of ICs overlap

prices irrelevant to PE!!

initial endowment to one player is irrelevant – PE is what the planner looks at, doesn't matter if PE allocations are worse than initial endowment!

if production is IRS it probably makes sense to allocate EVERYTHING to production before allocating to consumers. . . . (unless, of course, utility of a specific non-producible good is increasing even faster than the returns to production)

7.0.5 "Algorithm Solve for WE"

0. Think through which possible solutions make sense BEFORE doing math
   a. e.g., is producing ever worthwhile? should we produce everything possible?
1. assert prices nonzero. Normalize $p_L = 1$.
2. Solve profit max
3. Calculate π
4. derive budget constraint for each consumer
5. Solve Umax
   a. if nondifferentiable, use budget constraint in Edgeworth box
      i. recall: slope of budget line is $-p_1/p_2$ (or just $-p$ since $p_2 = 1$)
         - you'll need to solve for p (using logic) BEFORE trying to plot this
      ii. you'll never actually get Edgeworth right by drawing it. . . so plot budget line and then solve for actual quantities naively:
      iii. set $u_1 = BC$ and $u_2 = BC$ to see where intersections happen
         - e.g., $x_{11}^2 + x_{12}^2 = \underbrace{-1}_{p_1/p_2} * x_{12} + 3$ and $x_{11} + x_{22} = -1 * x_{22} + 3$
      iv. rule out infeasible intersections

    b. check SOCs
    c. if convex: the "critical" price is $\hat{p} = \frac{m}{f(m)}$
6. Use goods market clearing condition(s) to relate x^* to p
    a. discard negative prices
7. plug p back in to get actual values for x^*, y^*
    a. sanity check final answer!
        i. e.g., sum of of good l consumption should equal the inputs of good l to production
8. If transfers:
    a. you know x^* and p^* at this point, so plug into budget constraint for each consumer i and solve for T_i.
9. Triple-check that you have the correct range of prices supporting the x^*, y^* you so painstakingly found.

    if you have labor just fully allocate it and leave it out of the budget constraint!

    Ask: do any prices satisfy the equilibrium conditions?

7.0.6 "Algorithm maximize convex/linear functions"

Given $f(x)$ subject to $px = m$.

You'll have three cases:

1. $x^* = 0$ if $p < \hat{p}$
2. $x^* = f(m)$ if $p > \hat{p}$
3. If linear: $x^* = [0, f(m)]$ if $p = \hat{p}$
4. If str convex: $x^* = \{0, f(m)\}$ if $p = \hat{p}$

Picking $\hat{p}$:

- In production settings: usually works to always pick p such that $p = \frac{m}{f(m)}$
- If utility is perfect substitutes (e.g., $u = x_{11}^2 + x_{12}^2$) then pick $\hat{p} = 1$ since $p > 1$ is the same as saying $p_2 > p_1$ –if a good is more expensive and a perfect substitute then obviously demand will be at corner.

7.0.7 "Algorithm Arrow's Impossibility"

- Universal Domain
    - Prove:
    - Counterexamples:
- Antisymmetric
    - Prove:
    - Counterexamples:
    - Notes:
        * anti-symmetry rules out indifference.
- Completeness
    - Prove:
    - Counterexamples:
    - Notes:
        * indifferent $\not\Rightarrow$ incomplete
- Transitivity
    - Prove:
    - Counterexamples:
- Pareto
    - Prove:
        * ??
    - Counterexamples:
        * ??
- No dictator

- Prove:
    - Suffices to show: for every i there exists $R = (R_1, ..., R_n) \in \mathcal{R}^n$ such that $f(R) \neq R_i$
- Counterexamples:
    - find the dictator

- IIA
  - Prove:
    - "Our rule considers only two alternatives whose relative ranking does not affect society's choice"
  - Counterexamples:
    - Borda counts

7.0.8 "Algorithm Quasilinear Questions"

Find Set of PE Allocations

1. Invoke Prop 1 or Prop 2
2. If Prop 1: Setup welfare max:
    a. $\max_{x_i \geq 0}(\sum_i m_i + \sum_i g_i(x_i))$ s. to feasibility
        i. when you see feasibility, think: two market clearing constraints. can often plug those in to solve unconstrained.
3. If Prop 2: Setup max of sum of utilities:
    a. $\max_{x_1,...,x_n} \sum_i (m_i + g(x_i))$
    b. DON'T plug-in budget constraint!!! use market clearing instead if you want to rewrite the problem: $\iff \max_{x_1,...,x_n} \sum_i [\omega_i + c_j(y_j) + g(x_i)]$
4. After solving the max problem you should have x_i^* and maybe y_j^*. If no y, get it. Then, plug back into market clearing conditions to solve for m_i.
5. Write final set in terms of x_i, m_i.

    don't use budget constraint for PE allocations

Find WE

1. Same steps as regular algorithm. Things to pay attention to:
    a. m_i is part of the final answer - remember: just because it's not a consumption good doesn't mean it's not part of the consumption allocation.
    b. often easier to solve if you plug the constraints in

7.0.9 "Algorithm Aggregation Questions"

Prove Demand Doesn't Aggregate

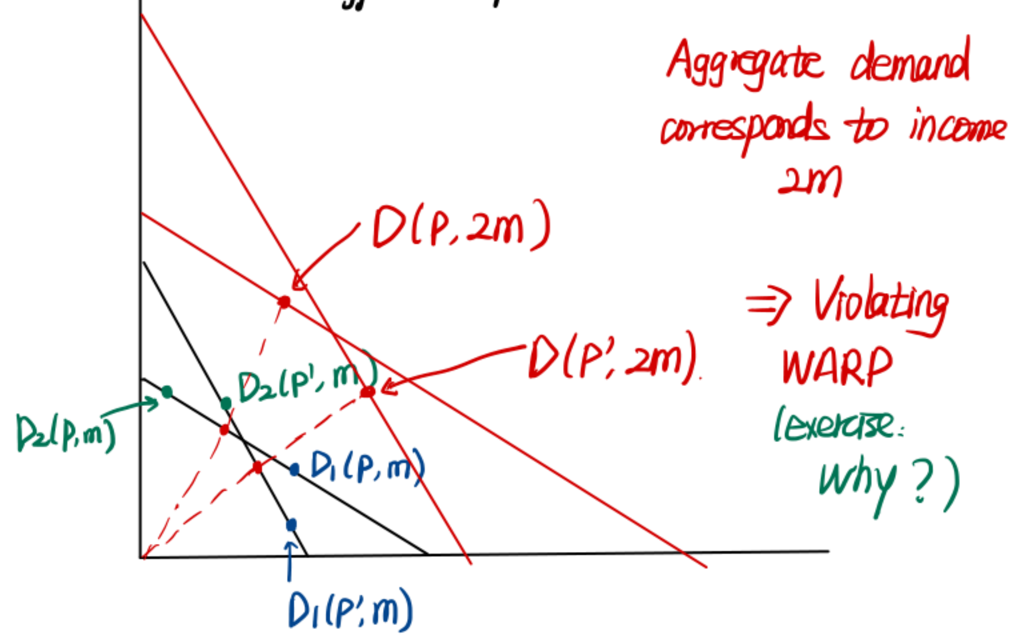
1. Decide whether you want to violate (1) income aggregation, (2) pref maximization
2. To violate income aggregation: WTS: $\exists p \in \mathbb{R}^L_{++}$ s.t. $D_1 \neq D_2$
    i. solve for demand for two consumers in terms of p and m_1, m_2.
    ii. plug in arbitrary prices, such that $p > 0, \forall l$ (just do $p_1 = 1, p_2 = 2$)
    iii. If there is representative consumer, it shouldn't matter how we allocate money.
    iv. define $m = m_1 + m_2$. First, give all of m to i (and 0 to j), then give all of m to j.
    v. should see that $D_i \neq D_j$ for those two consumers. if not, try different numbers.
3. To violate pref maximization
    i. WARP for demand... draw:

②. Violation of WARP.

Assume $I=2$, fix m the same for two consumers

⇒ they have the same budget line.

We consider two different price vectors.



7.0.10 "Mistakes to avoid"

- consumer always likes 2 sugars (i.e., 2 units of x) with each coffee (i.e., for every 1 y). Might be tempted to think that if the consumer wants 2x for every 1y, then the ICs in this case are kinked on the line y = 2x. Wrong!
  - Think about which bundles have the "right" proportions of the goods in this case. We know that if the consumer has 1 coffee (1y) she wants 2 sugars (2x). If she has 2 coffees (2x) she wants 4 sugars (4y).
- don't use duality unless utility is *strictly* convex!
- when writing GMC for good l: use TOTAL endowment for that good, not just one consumer's!
- Mu writes inputs to production super weird. Be careful!
  - e.g., "Firm 2 can use $y_{2,1}$ units of good 1 to produce $y_{2,3} = 2y_{2,1}$ units of good 3" means the marginal cost is 1/2, not 2!!
- when drawing Edgeworth with concave ICs–make sure to draw the right indifference curves
- quasilinear: is only quasilinear environment if we let $m_i < 0$. if the utility function is quasilinear, that doesn't mean we're in quasilinear environment unless the linear good can go negative!
- $\frac{d}{dx_i}(\sum_i x_i)^2 = 2\sum_i x_i \neq 2x_i$
- in q-linear: an allocation includes the allocation of m. remember: m is another good, even if it's not a consumption good!
- In WE: don't stop once you have x^*, y^*. You need to find the (range) of p such that these hold
- Pareto-efficient is NOT necessarily welfare-maximizing!
- Linear production: profits must be zero! Don't waste time trying to solve for profits. They are zero.
- PE set - almost always should have all x solutions in terms of one of the 4 x's. E.g, $\{x_{11} = x_{11}, x_{12} = \frac{9x_{11}}{1+4x_{11}}, x_{21} = 2 - x_{11}, 2 - \frac{9x_{11}}{1+4x_{11}}, |x_{11} \in [0, 2]\}$