

# Math 415 Homework 1

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## 1 Find all the zero divisors of $\mathbb{Z}/20\mathbb{Z}$ .

Notice that the additive identity of  $\mathbb{Z}/20\mathbb{Z}$  is  $\mathbb{Z} = 1\mathbb{Z}$ . Therefore a zero divisor is any  $a, b \in \mathbb{Z}$  such that  $ab \equiv 1 \pmod{20}$ . This is the set of all integers that are coprime to 20 or  $\{[1], [3], [7], [9], [11], [13], [17], [19]\}$ . ( $[x]$  means the equivalence class of  $x$  modulo 20)

## 2 Determine $U(\mathbb{Z}[i])$ where $\mathbb{Z}[i] \subseteq \mathbb{C}$ is the ring of Gaussian Integer.

Notice  $1 \in \mathbb{Z}[i]$  is its own inverse so  $1 \in U(\mathbb{Z}[i])$ . Likewise  $-1 \in \mathbb{Z}[i]$  is its own inverse so  $-1 \in U(\mathbb{Z}[i])$ . Notice  $i(-i) = 1 = (-i)i$  so  $i, -i \in U(\mathbb{Z}[i])$ .

I claim there are no more units in the Gaussian Integers. Assume  $x \in U(\mathbb{Z}[i])$  and  $x \neq 1, -1, i, -i$ .

So  $x = a + bi$  and there exists a  $y = \alpha + \beta i$  such that  $xy = 1$ .

## 3 Let $G$ be a group and let $R$ be a ring. We denote by $RG$ the set of all functions $f: G \rightarrow R$ whose support $\{x \in G \mid f(x) \neq 0\}$ is finite. Note that if $G$ is finite then this condition is automatically satisfied.

### 3.1 Show that $RG$ is a ring under the operations defined by

$$(f + g)(x) = f(x) + g(x)$$

$$(fg)(x) = \sum_y f(y)g(y^{-1}x)$$

for all  $f, g \in RG$  and  $x \in G$ . Note the sum on the right makes sense as it has only finitely many non-zero terms. The product just defined is usually called the convolutional product and we call  $RG$  the group ring.

$$(f+g)(x) = f(x) + g(x) \quad \text{equation*} \quad (fg)(x) = \sum_y f(y)g(y^{-1}x) \quad \text{equation*}$$

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