

# Math 415 Homework 1

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## 1 Find all the zero divisors of $\mathbb{Z}/20\mathbb{Z}$ .

Notice that the additive identity of  $\mathbb{Z}/20\mathbb{Z}$  is  $\mathbb{Z} = 1\mathbb{Z}$ . Therefore a zero divisor is any  $a, b \in \mathbb{Z}$  such that  $ab \equiv 1 \pmod{20}$ . This is the set of all integers that are coprime to 20 or  $\{[1], [3], [7], [9], [11], [13], [17], [19]\}$ . ( $[x]$  means the equivalence class of  $x$  modulo 20)

## 2 Determine $U(\mathbb{Z}[i])$ where $\mathbb{Z}[i] \subseteq \mathbb{C}$ is the ring of Gaussian Integer.

Notice  $1 \in \mathbb{Z}[i]$  is its own inverse so  $1 \in U(\mathbb{Z}[i])$ . Likewise  $-1 \in \mathbb{Z}[i]$  is its own inverse so  $-1 \in U(\mathbb{Z}[i])$ . Notice  $i(-i) = 1 = (-i)i$  so  $i, -i \in U(\mathbb{Z}[i])$ .

I claim there are no more units in the Gaussian Integers.