

MATH 200 - Summer 2023: Assignment 3

Due: Upload your solutions to Crowdmark BEFORE 4pm (PT) Thursday June 22

You may upload and change your files at any point up until the due date of Thursday June 22 at 4pm.

A 5% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 4pm to 4:59pm will have 5% deducted, assignments submitted 5pm-5:59pm will have 10% deducted, etc.

For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation.*** Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Math 200 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

1. [6 marks] For each of the following, sketch their domains, and determine if they are open/closed and bounded/unbounded. Note, you may need to refer back to your §12.6 guide to surfaces.

(a) $f(x, y) = \arccos(y - x^2)$.

Answer:

Open/Closed: _____

Bounded/Unbounded: _____

(b) $g(x, y, z) = \ln(x^2 + y^2 - z^2 - 6)$

Answer:

Open/Closed: _____

Bounded/Unbounded: _____

2. [9 marks] Determine the following limits, if they exist. If they do exist, write the limit value in the box provided; otherwise, write “DNE”.

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{5x^3y^3 - 5}{xy - 1}.$

Answer:

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy(x+y)\sin(y-x)}{x^4 + y^4}$

Answer:

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}.$

Hint: Try finding $\lim_{(x,y) \rightarrow (0,0)} \left| \frac{xy}{\sqrt{x^2 + y^2}} \right|$ and $\lim_{(x,y) \rightarrow (0,0)} - \left| \frac{xy}{\sqrt{x^2 + y^2}} \right|$ by Squeezing.

What is $\left| \frac{y}{\sqrt{x^2 + y^2}} \right|$ bounded by?

Answer:

3. [8 marks] Consider the function

$$f(x, y) = \begin{cases} \frac{x^2+y^2}{xy}, & xy \neq 0 \\ 0, & xy = 0. \end{cases}$$

(a) Is $f(x, y)$ continuous at $(0, 0)$? Demonstrate why or why not.

(b) Do both partial derivatives exist at $(0, 0)$? Demonstrate why or why not. If they exist, use the limit definition to compute them.

(c) Is $f(x, y)$ differentiable at $(0, 0)$? Demonstrate why or why not.

4. **[5 marks]** Use the Chain Rule to determine $\frac{dw}{dt}$ at $t = 0$ for $w = f(x, y, t) = xy^2 + tx + t^2y$ with $x = \sin 2t$ and $y = \cos 2t$. Be sure to explicitly show your chain rule formula, including labelled diagrams.

Answer:

$$\left. \frac{dw}{dt} \right|_{t=0} =$$