# CSC 226

## Algorithms and Data Structures: II

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One surprising application of network flow is called the ***baseball elimination*** problem

- Let $T$ be a set of teams in a sports league
  - originally applied to baseball
- At any point in the season team $i \in T$, will have $w_i$ wins with $g_i$ games left to play
- The problem: is it possible for team $i$ to finish in first place?
- Turns out it can depend on the number of head-to-head games between the competing teams
  - denoted $g_{i,j}$ = number of games left between teams $i$ and $j$

# Baseball elimination problem

Q. Which teams have a chance of finishing the season with the most wins?

| i | team | | wins | losses | to play | ATL | PHI | NYM | MON |
|---|---|---|---|---|---|---|---|---|---|
| 0 |  | Atlanta | 83 | 71 | 8 | – | 1 | 6 | 1 |
| 1 |  | Philly | 80 | 79 | 3 | 1 | – | 0 | 2 |
| 2 |  | New York | 78 | 78 | 6 | 6 | 0 | – | 0 |
| 3 |  | Montreal | 77 | 82 | 3 | 1 | 2 | 0 | – |

Montreal is mathematically eliminated.

- Montreal finishes with $\leq 80$ wins.
- Atlanta already has $83$ wins.

# Baseball elimination problem

Q. Which teams have a chance of finishing the season with the most wins?

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Philadelphia is mathematically eliminated.

- Philadelphia finishes with $\leq 83$ wins.
- Either New York or Atlanta will finish with $\geq 84$ wins.

Observation. Answer depends not only on how many games already won and left to play, but on whom they're against.

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|---|---|---|---|---|---|---|---|---|
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# Baseball elimination problem

Q. Which teams have a chance of finishing the season with the most wins?

| i | team | wins | losses | to play | NYY | BAL | BOS | TOR | DET |
|---|---|---|---|---|---|---|---|---|---|
| 0 | New York | 75 | 59 | 28 | – | 3 | 8 | 7 | 3 |
| 1 | Baltimore | 71 | 63 | 28 | 3 | – | 2 | 7 | 4 |
| 2 | Boston | 69 | 66 | 27 | 8 | 2 | – | 0 | 0 |
| 3 | Toronto | 63 | 72 | 27 | 7 | 7 | 0 | – | 0 |
| 4 | Detroit | 49 | 86 | 27 | 3 | 4 | 0 | 0 | – |

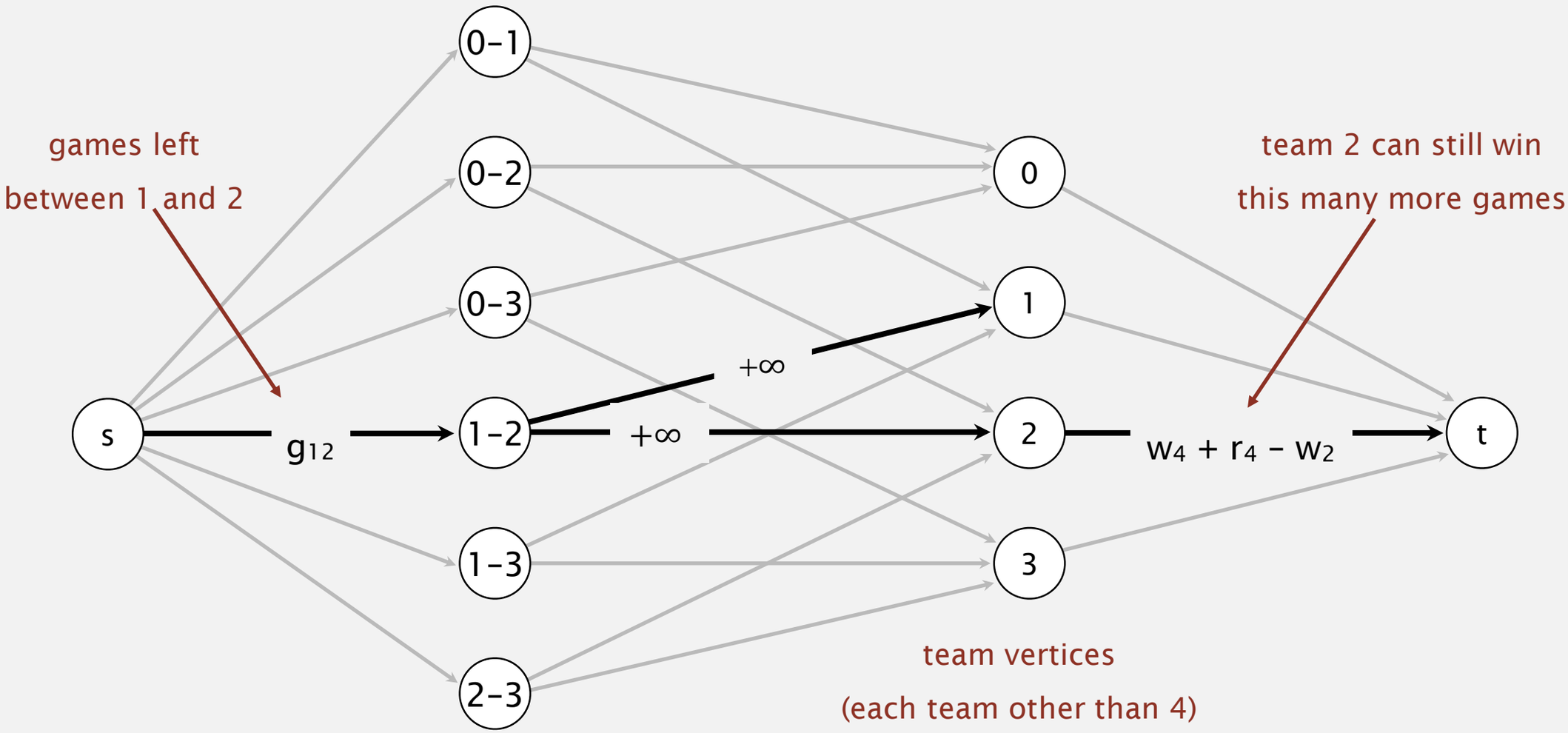
AL East (August 30, 1996)

Is Detroit mathematically eliminated?

- Seems like they still have a chance.
- Detroit finishes with $\leq 76$ wins.

# Baseball elimination problem: maxflow formulation

Intuition. Remaining games flow from $s$ to $t$.




Fact. Team 4 not eliminated iff all edges pointing from $s$ are full in maxflow.

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| 1 | Baltimore | 71 | 63 | 28 | 3 | – | 2 | 7 | 4 |
| 2 | Boston | 69 | 66 | 27 | 8 | 2 | – | 0 | 0 |
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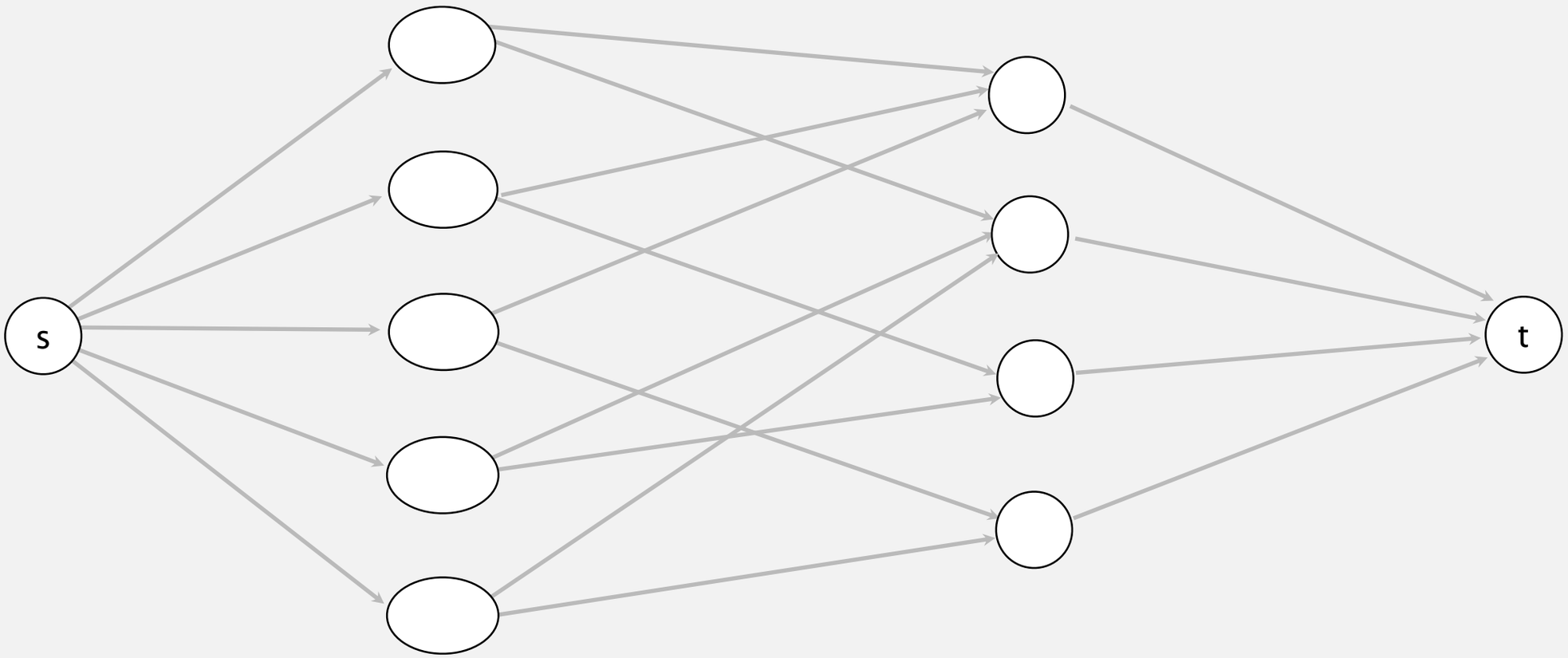
Is Detroit mathematically eliminated?



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Is Toronto mathematically eliminated?