Set 8, Section 4.1: Discrete Random Variables

A *Random Variable* is a function which maps each outcome of an experiment to a number.

Example: 1) Toss a fair coin until a H is observed. Let X be the number of tails until a head is observed.

Example: 2) Suppose two dice are rolled. Let the random variable X denote the sum of the rolls.

$X =$ **Sum of 2 dice**

		2'nd					
	Roll	1	2	3	4	5	6
1'st	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

The rolls can have sum $2, 3, \ldots, 12$. Thus, X takes on values $2, 3, \ldots, 12$.

$P(X = 3)$ **is equivalent to finding the probability that the sum of the rolls is 3.**

A *Bernoulli random variable* takes values 0 and 1.

Examples:

***Discrete* random variables: range of X is discrete.**

***Finite discrete*: possible values for X is finite.**

***Infinite discrete*: values for X can be "lined up" in a sequence (countable).**

We use X to denote a random variable; lower case x to denote a value.

The *Probability Mass Function* or *pmf* of a discrete random variable $p(x)$ is defined by $p(x) = P(X = x)$ for every number x.

Example: Construct a table of the pmf for the random variable X, where X is the sum of the rolls of two dice.

Referring to the dice chart we saw earlier, we have that the possible values for X are $2, 3, \ldots, 12$.

Since there is one roll where the sum is 2, then $P(X = 2) = \frac{1}{36}$.

Since there are two rolls where the sum is 3, then $P(X = 3) = \frac{2}{36}$.

Display $p(x) = P(X = x)$ for the sum of rolls of 2 dice:

x	2	3	4	5	6	7	8	9	10	11	12
$p(x) = P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Note: By default, for any value of x not specified in the table, $p(x) = 0$.

Example: Refer to the probability distribution table we have just constructed. What is the probability that the sum of the two rolls is 8 or more?

This is equivalent to finding $P(X \geq 8)$.

Example: Suppose we know that the sum is at least 7. What is the probability that the sum is no more than 10?

GIVEN that $X \geq 7$, what is the probability that $X \leq 10$. In symbols, we want $P(X \leq 10|X \geq 7)$.

Using our conditional probability formula:

$$P(X \leq 10|X \geq 7) = \frac{P(X \leq 10 \cap X \geq 7)}{P(X \geq 7)}$$

The probability mass function may depend on a parameter, which may be assigned any one of a number of different values.

Each value of the parameter will give rise to a different pmf.

The collection of all pmf's for all the different values of the parameter is called a *family* of probability distributions.

Example: A machine produces steering wheels which are acceptable or defective.

Let θ be the probability that a single steering wheel is defective.

Let the random variable X count the number of defective steering wheels in a sample of three.

We find X has the following distribution:

x	0	1	2	3
$p(x;\theta)$	$(1-\theta)^3$	$3\theta(1-\theta)^2$	$3\theta^2(1-\theta)$	θ^3

Here, θ is the parameter which takes values between 0 and 1.

The *Cumulative Distribution Function*, $F(x)$, of a discrete random variable X with probability mass function $p(x)$ is defined as:

$$F(x) = P(X \leq x) = \sum_{y \leq x} p(y).$$

Example: Construct the cumulative distribution table for the random variable X, where X is the sum of the rolls of two dice.

x	2	3	4	5	6	7	8	9	10	11	12
$F(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	1