MATH 200 - Summer 2023: Assignment 1

Due: Upload your solutions to Crowdmark BEFORE 4pm (PT) Tuesday May 23

You may upload and change your files at any point up until the due date of Tuesday May 23 at 4pm.

Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. You may hand write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work.

For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation.*** Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Math 200 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

A 5% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 4pm to 4:59pm will have 5% deducted, assignments submitted 5pm-5:59pm will have 10% deducted, etc.

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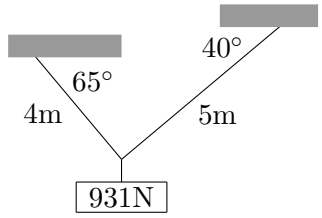
1. **[5 marks]** A city wants to hang a new sign over a popular tourist alley that runs between two old buildings. Due to the different roof heights on each side of the alley, the wires used to fasten the sign are attached at different heights; the wires are 4m and 5m long, and their angles with the rooftops are 65° and 40°, respectively. The sign has a mass of 95kg (that is, a weight of 95 kg $\times$ 9.8 m/s^2= 931N). The tensions in the wires are $\mathbf{T}_1$ and $\mathbf{T}_2$, respectively.

   Determine $\mathbf{T}_1$ and $\mathbf{T}_2$, and their magnitudes.



Answer:

$\mathbf{T_1}$ = _____, $|\mathbf{T}_1|$ = _____

$\mathbf{T_2}$ = _____, $|\mathbf{T}_2|$ = _____

2. **[5 marks]** Let $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$. We can express $\mathbf{u}$ as the sum of two vectors, $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ (that is, $\mathbf{u} = \mathbf{v}_\perp + \mathbf{v}_\parallel$), where $\mathbf{v}_\perp$ is a vector perpendicular to the vector $\mathbf{v}$, and $\mathbf{v}_\parallel$ is a vector parallel to $\mathbf{v}$. Determine $\mathbf{v}_\perp$ and $\mathbf{v}\|$ (*hint: start by finding* $\mathbf{v}_\parallel$ *- a projection might help*). Once you have $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$, confirm that $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ are perpendicular to one another.

Answer:

$\mathbf{v}_\perp =$ _____

$\mathbf{v}_\parallel =$ _____

3. **[5 marks]** Use the scalar triple product to determine the volume of the pyramid with corner points $A(1, 5, -1)$, $B(-2, 4, -1)$, $C(7, 4, 3)$, and $D(2, 0, 3)$. The volume of a pyramid is $V = \frac{1}{3}ah$ where a is the area of the base, and h is the height. (Hint: What shape is the base of the pyramid? How does the area of this shape relate to the area of a parallelogram?)

Answer:

Volume:

4. **[9 marks]** Let L_1 be the line of intersection of the planes $3x - 2y + 4z = 10$ and $5x + 3y - 2z = 15$, and let L_2 be the line defined parametrically below.

$$L_2 : \quad x = 7 + 2t, \quad y = 11 - 5t, \quad z = 13 + 6t.$$

(a) Determine the parametric form of L_1.

Answer:

L_1: $x =$ _____, $y =$ _____, $z =$ _____, for $-\infty < t < \infty$.

(b) Prove that L_1 and L_2 are (i) not parallel and (ii) not intersecting (and hence they are skew).

(c) Determine the (minimum) distance between L_1 and L_2.

Answer:

Distance: