Set 9: Section 4.2, Expectation and Variance

Definition: The ***cumulative distribution function*** **(cdf) of a random variable** X **with probability mass function** p_X **is given by**

$$F_X(x) = P(X \le x) = \sum_{y \le x} p_X(y)$$

Example: **Consider three flips of a coin and let** X **be the number of heads. Obtain the cdf of** X**.**

Properties of a cdf F**:**

(1) $F(-\infty) = 0$, $F(\infty) = 1$

(2) F **is monotone increasing**

(3) F **is right continuous**

Given a cdf corresponding to a discrete distribution, you should be able to determine the pmf.

Example:

Let X=number of dots on a toss of a fair die
Toss the die 6,000 times and obtain $x_1, x_2, x_3, ..., x_{6000}$.

What do you EXPECT $\bar{x}$ to be?

Definition: The *expectation* of a discrete rv X with pmf $p_X(x)$ is given by

$$\mu \equiv \mathrm{E}(X) \equiv \sum_x x \; p_X(x)$$

The expectation can be thought of as the long run average of the random variable over hypothetical repetitions of the experiment.

Example: Consider the experiment consisting of three flips of a coin. Let $X \equiv$ the number of heads. Obtain $\mathrm{E}(X)$.

Example: Consider a Bernoulli random variable X. Obtain $\mathrm{E}(X)$.

Consider a coin which has $P(H) = \theta$. Toss the coin until a H is obtained. Let X be the number of T's. What is $E(X)$?

Proposition: The expectation of a function $g(X)$ corresponding to the discrete random variable X with pmf $p_X(x)$ is given by

$$\mathrm{E}[g(X)] = \sum_x \; g(x) \; p_X(x)$$

Example: Consider the experiment of tossing a coin three times and let X be the number of heads. Obtain $\mathrm{E}(X^2)$.

Proposition: $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ **where a, b are constants.**

Problem: A store orders copies of a weekly magazine for its magazine rack. Let X be the weekly demand for the magazine with pmf

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

The store owner pays \$1 for each magazine and the customer price is \$2. If leftover magazines at the end of the week have no value, what is the expected profit if the owner orders 6 magazines.

Is expectation always a reasonable criterion?

Problem for discussion: Suppose that you are given the chance to play a game a single time where the entrance fee is \$1 million dollars. With probability 0.99, you lose and receive nothing. With probability 0.01, you win and receive \$1 billion dollars. Should you play the game?