Confidence Intervals Summary

Case 0: X is normal, σ^2 is known. Then $\bar{X} \sim \text{Normal}(\mu, \sigma^2/n)$.

$$\bar{x}_{\text{obs}} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

is a $(1-\alpha)100\%$ CI for μ.

Case 1: σ^2 is known, n is large. By the CLT $\bar{X} \approx \text{Normal}(\mu, \sigma^2/n)$. In this case,

$$\bar{x}_{\text{obs}} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

is an approximate $(1-\alpha)100\%$ CI for μ.

Case 2: σ^2 is unknown, n is large. By the CLT $\bar{X} \approx \text{Normal}(\mu, \sigma^2/n)$. In this realistic case,

$$\bar{x}_{\text{obs}} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

is an approximate $(1-\alpha)100\%$ CI for μ where s is the sample standard deviation.

% Confidence	Critical Value
90	$Z_{\alpha/2} = Z_{0.05} = 1.645$
95	$Z_{\alpha/2} = Z_{0.025} = 1.96$
99	$Z_{\alpha/2} = Z_{0.005} = 2.576$
99.9	$Z_{\alpha/2} = Z_{0.0005} = 3.29$

If we are given a desired *width* of the confidence interval, we can solve for the unknown, required n.

<u>Case 3</u>: X is normal, σ^2 is unknown, n is small.

$$\bar{x}_{\text{obs}} \pm t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$(1-\alpha)$**100% confidence interval for μ.**

<u>Case 4</u>: X is binomial(n,p), $\hat{p}_{\text{obs}} = x_{\text{obs}}/n$, $n\hat{p}_{\text{obs}} \geq 5$ and $n(1-\hat{p}_{\text{obs}}) \geq 5$

$$\hat{p}_{\text{obs}} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_{\text{obs}}(1-\hat{p}_{\text{obs}})}{n}}$$

is an approximate $(1-\alpha)100\%$ CI for p.

In practice, we get lazy in our notation and drop the subscript 'obs' from $\bar{x}_{\text{obs}}$ and $\hat{p}_{\text{obs}}$.