1. (8 marks) Suppose the number of goals attempted by a football team in a match follows a Poisson distribution with mean λ. Each goal attempt by the team has a probability p of resulting in a goal, independently of other attempts. Given that the team had n goal attempts in a match, the number Y of goals scored follows a binomial distribution with parameters n and p.

   (a) (2 marks) Find the joint probability mass function $f_{N,Y}(n, y; \lambda, p)$ for the number N of goal attempts and the number Y of goals scored.

   (b) (6 marks) A sport analyst records n_i, the number of goal attempts, and y_i, the number of goals scored, for $i = 1,2,. . . . , K$ matches. Find the log-likelihood function and determine the joint Maximum Likelihood Estimates (MLE) $(\hat{\lambda}, \hat{p})$ for K matches' worth of data.
   You don't need to show the second derivative conditions for this exercise.