Set 10: Section 4.2, Expectation and Variance

We explore *expectation* in more detail.

Proposition: For a discrete rv X with pmf $p_X(x)$

$$\mathrm{E}[g_1(x) + \cdots + g_k(x)] = \mathrm{E}[g_1(x)] + \cdots + \mathrm{E}[g_k(x)]$$

Definition: The *variance* of a discrete rv X with pmf $p_X(x)$ is

$$\sigma^2 \equiv \sigma_X^2 \equiv \mathrm{Var}(X) \equiv E\{[X - \mathrm{E}(X)]^2\}$$

- **we call σ or σ_X the *standard deviation* of X**
- **σ and σ^2 are measures of spread of $p_X(x)$**
- **contrast sample quantities $(\bar{x}, s)$ with popln quantities (μ, σ)**

Example:

Computation Formula for Variance:

$$\sigma^2 = \mathrm{Var}(X) = E(X^2) - (\mu)^2$$

Example:

Laws of Variance: (a, b **are constants)**

1. $\mathrm{Var}(b) = 0$

2. $\mathrm{Var}(X + b) = \mathrm{Var}(X)$

3. $\mathrm{Var}(aX) = a^2\mathrm{Var}(X)$

Example: Consider the experiment consisting of three flips of a coin. Let $X \equiv$ **the number of heads. Obtain** $\mathrm{Var}(X)$**.**

Example: Let X **be the average January temperature in degrees Celsius where** $\mathrm{E}(X) = 5\ {}^oC$ **and** $\mathrm{Var}(X) = 3\ {}^oC^2$**. Find the expected value and the variance of** Y **where** Y **is the average January temperature in degrees Fahrenheit.**

Problem: Calculate σ and $\mathrm{E}(3X + 4X^2)$ corresponding to the rv X with pmf $p_X(x)$ where

x	4	8	10
$p_X(x)$	0.2	0.7	0.1

Example: In a game of chance, I bet x dollars. With probability p, I win y dollars. What should x be for this to be a fair game?