

Set 11: Section 4.3, Binomial Distribution

Bernoulli Trials: An experiment with n trials such that:

1. Each trial has exactly two outcomes, called success and failure.
2. The probability of success/failure for each trial is independent of the outcome of all other trials.
3. For each trial, the probability of success is a constant, p .

Binomial Random Variable: X =number of successes in n Bernoulli trials with success probability p . We write $X \sim \text{binomial}(n, p)$,

Example: Suppose a coin is flipped 20 times. Let X denote the number of tails.

Example: On a multiple choice test, there are 10 questions. Each question has 8 possible responses. A person completes the test by randomly selecting answers. Let X denote the number of questions the person gets correct.

Example: Three people have their blood typed. Each person has a 40% probability of having type A blood. Let Y denote the number of people in the group with type A blood.

Binomial Probability Mass Function (pmf):

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x} \text{ for } x = 0, 1, 2, \dots, n$$

We denote the pmf by $b(x; n, p)$.

Example: For the multiple choice test example, what is the probability that the person gets 2 or 3 questions correct?

Example: In the manufacture of lithium batteries, it is found that 7% of all batteries are defective. We test 6 randomly selected batteries. What is the probability that at least two batteries are defective?

Expected Value and Variance: If $X \sim \text{binomial}(n, p)$, then:

$$E(X) = np \text{ and } \text{Var}(X) = np(1 - p)$$

Example: What is the expected number of defective lithium batteries per batch of 6?

Example: For the battery-testing experiment, what is the variance?

Cumulative Distribution Tables: These tables give $P(X \leq r) = \sum_{x=0}^r b(x; n, p)$ for “nice” values of n and p . pages 343-348

Example: 20 people have their blood typed. What is the probability that no more than 15 will have type A blood.

Example: What is the probability that exactly 13 people will have type A blood?

Example: What is the probability that at least 5 people will have type A blood?

Example: It is known that 20% of all people have more than one bank account. Fifteen people are randomly selected and asked about the number of accounts they have. If at least 2 have more than one account, what is the probability that no more than 5 have more than one account?