Stat 261 Assignment 1

Jianping (Joy) Yu

Due Date: May 23, 2023, 11:59 pm

Answer the questions (handwritten on paper or on a tablet or computer file). Create a PDF file of your answers (scan handwritten notes or save tablet notes to pdf). Upload your PDF file to Brightspace.

NOTE: jpeg files are not acceptable.

1. (4 points) In November of each year, a walk-in clinic allows people to walk in to get a flu shot. Let X be the number of people who come to the clinic for a flu shot on a randomly selected day (in November). Suppose X has the following distribution:

x	0	1	2	3	4	5
$P(X = x)$	0.3	0.1	0.2	0.1	0.1	0.2

   If at least 3 people walk in for a flu shot on a particular day, what is the probability that there are 4 or fewer who walk in for a flu shot that day?

2. (6 points) Daily sales at a gas station are thought to be independent of one another with daily mean \$5000 and standard deviation \$700. Approximate the probability that the average daily sales over one year (i.e. 365 days) is greater than \$5,100.

3. (10 points) Suppose that diseased trees are distributed randomly and uniformly throughout a large forest with an average of λ per acre. Let X denote the number of diseased trees in a randomly chosen one-acre forest plot where the range of X is, $\mathcal{X} = \{0, 1, 2, ...\}$.

   (a) We will use the Poisson distribution with mean λ to model X. Write down the probability mass function (p.m.f.) for X. Why would this distribution be suitable for modelling the number of diseased trees in a randomly chosen one-acre plot of forest?

   (b) Suppose that we observe the number of diseased trees on n randomly chosen one-acre parcels, $X_1, X_2, ..., X_n$. The random variables $X_1, X_2, ..., X_n$ can be assumed to be independent. Write down the JOINT probability mass function for $X_1, X_2, ..., X_n$. Simplify this expression which is a function of λ and the $X's$. Use this joint distribution for the remainder of Question 1.

(c) We are going to use the Method of Maximum Likelihood to estimate λ. Write down the Likelihood function $L(\lambda)$.

(d) Write down the Log-likelihood function $\ell(\lambda)$.

(e) Write down the Score Function $S(\lambda)$.

(f) Derive the maximum likelihood estimate of λ.

(g) Write down the Information Function $I(\lambda)$.

(h) Use the second derivative test to show that you have found a maximum.

(i) Suppose that the numbers of diseased trees observed in $n = 4$ randomly chosen one-acre parcels were: 5, 8, 9, 2. Compute the maximum likelihood estimate of λ using this data.