

The Laws of Logic

Idempotence: $p \vee p \Leftrightarrow p$

$$p \wedge p \Leftrightarrow p$$

Commutativity: $p \vee q \Leftrightarrow q \vee p$

$$p \wedge q \Leftrightarrow q \wedge p$$

Associativity: $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$$

Distributivity: $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

Double Negation: $\neg(\neg p) \Leftrightarrow p$

De Morgan's laws: $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

Let **1** denote a tautology and **0** a contradiction.

Identity: $p \vee \mathbf{0} \Leftrightarrow p$

$$p \wedge \mathbf{1} \Leftrightarrow p$$

Dominance: $p \vee \mathbf{1} \Leftrightarrow \mathbf{1}$

$$p \wedge \mathbf{0} \Leftrightarrow \mathbf{0}$$

The following are not formally known as “laws of logic” but they are known logical equivalences that you can use in your proofs.

Known Tautology: $p \vee (\neg p) \Leftrightarrow \mathbf{1}$

Known Contradiction: $p \wedge (\neg p) \Leftrightarrow \mathbf{0}$

$$\neg \mathbf{1} \Leftrightarrow \mathbf{0}$$

$$\neg \mathbf{0} \Leftrightarrow \mathbf{1}$$

Contrapositive: $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$

Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Biconditional: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Absorption: $(p \vee q) \wedge q \Leftrightarrow q$

$$(p \wedge q) \vee q \Leftrightarrow q$$