# MATH 200 - Summer 2023: Assignment 1

Due: Upload your solutions to Crowdmark BEFORE 4pm (PT) Tuesday May 23

---

You may upload and change your files at any point up until the due date of Tuesday May 23 at 4pm.

Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. You may hand write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work.

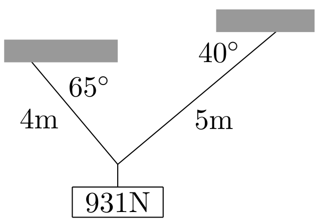
For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation***. Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Math 200 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

A 5% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 4pm to 4:59pm will have 5% deducted, assignments submitted 5pm-5:59pm will have 10% deducted, etc.

---

1. **[5 marks]** A city wants to hang a new sign over a popular tourist alley that runs between two old buildings. Due to the different roof heights on each side of the alley, the wires used to fasten the sign are attached at different heights; the wires are 4m and 5m long, and their angles with the rooftops are 65° and 40°, respectively. The sign has a mass of 95kg (that is, a weight of 95 kg × 9.8 m/s$^2$= 931N). The tensions in the wires are $\mathbf{T}_1$ and $\mathbf{T}_2$, respectively.

   Determine $\mathbf{T}_1$ and $\mathbf{T}_2$, and their magnitudes.




$T_1 = \langle -|T_1|\cos 65°, |T_1|\sin 65° \rangle$, $T_2 = \langle |T_2|\cos 40°, |T_2|\sin 40° \rangle$

$T_1 + T_2 = \langle 0, 95 \rangle$

$\Rightarrow -|T_1|\cos 65° + |T_2|\cos 40° = 0$

$|T_1|\sin 65° + |T_2|\sin 40° = 95$

$\Rightarrow$ $|T_2| = \dfrac{|T_1|\cos 65°}{\cos 40°}$

$|T_1|(\sin 65° + \cos 65° \tan 40°) = 95$

$|T_1| = \dfrac{95}{\sin 65° + \cos 65° \tan 40°}$

$\Rightarrow |T_2| = \dfrac{95 \cos 65°}{\cos 40° \sin 65° + \cos 65° \sin 40°}$

$$\Rightarrow |T_1| = \left\langle -\left(\frac{95}{\sin 65° + \cos 65° \tan 40°}\right)\cos 65°, \left(\frac{95}{\sin 65° + \cos 65° \tan 40°}\right)\sin 65° \right\rangle$$

$$|T_2| = \left\langle \left(\frac{95\cos 65°}{\cos 40° \sin 65° + \cos 65° \sin 40°}\right)\cos 40°, \left(\frac{95\cos 65°}{\cos 40° \sin 65° + \cos 65° \sin 40°}\right)\sin 40° \right\rangle$$

**Answer:**

$\mathbf{T}_1 = \langle -31.8407, 68.2825 \rangle$, $|\mathbf{T}_1| = 75.3414$

$\mathbf{T}_2 = \langle 31.8407, 26.7175 \rangle$, $|\mathbf{T}_2| = 41.5650$

2. **[5 marks]** Let $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$. We can express $\mathbf{u}$ as the sum of two vectors, $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ (that is, $\mathbf{u} = \mathbf{v}_\perp + \mathbf{v}_\parallel$), where $\mathbf{v}_\perp$ is a vector perpendicular to the vector $\mathbf{v}$, and $\mathbf{v}_\parallel$ is a vector parallel to $\mathbf{v}$. Determine $\mathbf{v}_\perp$ and $\mathbf{v}\|$ (*hint: start by finding* $\mathbf{v}_\parallel$ *- a projection might help*). Once you have $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$, confirm that $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ are perpendicular to one another.

$u = V_\perp + V_{\parallel}$

$V_\perp = u - V_{\parallel}$

$V_{\parallel} = \text{proj}_v u = \left(\frac{u\cdot v}{|v|^2}\right)v$

$|V| = \sqrt{4^2 + (-6)^2 + 3^2} = \sqrt{61}$

$u \cdot v = (5)(4) + (3)(-6) + (-2)(3) = -4$

$= \left(\frac{-4}{61}\right)v$

$V_{\parallel} = \left\langle \frac{-16}{61}, \frac{24}{61}, \frac{-12}{61} \right\rangle$

$V_\perp = \left\langle 5 - \left(\frac{-16}{61}\right), 3 - \left(\frac{24}{61}\right), -2 - \left(\frac{-12}{61}\right) \right\rangle$

$V_\perp = \left\langle \frac{321}{61}, \frac{159}{61}, \frac{-110}{61} \right\rangle$

check

$V_\perp \cdot V_{\parallel} = \left(\frac{-16}{61}\right)\left(\frac{321}{61}\right) + \left(\frac{24}{61}\right)\left(\frac{159}{61}\right) + \left(\frac{-12}{61}\right)\left(\frac{-110}{61}\right) = ?$

$= \left(\frac{-5136}{3721}\right) + \left(\frac{3816}{3721}\right) + \left(\frac{1320}{3721}\right)$

$= \left(\frac{-5136}{3721}\right) + \left(\frac{5136}{3721}\right)$

$= 0$

$\Rightarrow V_\perp$ and $V_{\parallel}$ are perpendicular

**Answer:**

$\mathbf{v}_\perp = \frac{321}{61}i + \frac{159}{61}j - \frac{110}{61}k$

$\mathbf{v}_\parallel = \frac{-16}{61}i + \frac{24}{61}j - \frac{12}{61}k$

3. **[5 marks]** Use the scalar triple product to determine the volume of the pyramid with corner points $A(1,5,-1)$, $B(-2,4,-1)$, $C(7,4,3)$, and $D(2,0,3)$. The volume of a pyramid is $V = \frac{1}{3}ah$ where $a$ is the area of the base, and $h$ is the height. (Hint: What shape is the base of the pyramid? How does the area of this shape relate to the area of a parallelogram?)

Find area of base:

$\vec{AB} = (-3,-1,0)$, $|\vec{AB} \times \vec{AC}| = \left| \begin{vmatrix} i & j & k \\ -3 & -1 & 0 \\ 6 & -1 & 4 \end{vmatrix} \right| = |i(-4)+j(12)+k(9)| = \sqrt{241}$

$\vec{AC} = (6,-1,4)$ $\quad n = \langle -4,12,9 \rangle$

$\sqrt{241}$ is area for a square, area for triangle is $\frac{1}{2}\sqrt{241} = a$

Find height

$\vec{AD} = (1,-5,4)$

$h$ is $\left|\vec{AD} \cdot \frac{n}{|n|}\right| = \left| \langle 1,-5,4 \rangle \cdot \frac{\langle -4,12,9 \rangle}{\sqrt{241}} \right| = \left| (1)\left(\frac{-4}{\sqrt{241}}\right) + (-5)\left(\frac{12}{\sqrt{241}}\right) + (4)\left(\frac{9}{\sqrt{241}}\right) \right|$

$= \left| \frac{-28}{\sqrt{241}} \right| = \frac{28}{\sqrt{241}} = h$

$V = \frac{1}{3}ah = \frac{1}{3}\left(\frac{1}{2}\cancel{\sqrt{241}}\right)\left(\frac{28}{\cancel{\sqrt{241}}}\right) = \frac{28}{6} = \frac{14}{3}$

**Answer:**

**Volume:** $\frac{14}{3}$ cubic units

$\vec{n}_1 = \langle 3,-2,4\rangle$ $\quad n_2 = \langle 5,3,-2\rangle$

4. **[9 marks]** Let $L_1$ be the line of intersection of the planes $3x - 2y + 4z = 10$ and $5x + 3y - 2z = 15$, and let $L_2$ be the line defined parametrically below.

$$L_2: \quad x = 7 + 2t, \quad y = 11 - 5t, \quad z = 13 + 6t.$$

(a) Determine the parametric form of $L_1$.

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ 5 & 3 & -2 \end{vmatrix} = i(-8) + j(26) + k(-1) = \langle -8, 26, -1 \rangle$

if $z = 0$,

$\begin{matrix} 3 & -2 & 10 \\ 5 & 3 & 15 \end{matrix}$

$\begin{matrix} 1 & \frac{-2}{3} & \frac{10}{3} \\ 5 & 3 & 15 \end{matrix}$

$\begin{matrix} 1 & \frac{-2}{3} & \frac{10}{3} \\ 0 & 19/3 & -5/3 \end{matrix}$

$\begin{matrix} 1 & -2/3 & 10/3 \\ 0 & 1 & -5/19 \end{matrix}$

$\begin{matrix} 1 & 0 & 60/19 \\ 0 & 1 & -\frac{5}{19} \end{matrix}$

$L_1: x = \frac{60}{19} - 8t, \ y = \frac{-5}{19} + 26t, \ z = -t$

**Answer:**

$L_1$: $x = \frac{60}{19} - 8t$, $y = \frac{-5}{19} + 26t$, $z = -t$, for $-\infty < t < \infty$.

(b) Prove that $L_1$ and $L_2$ are (i) not parallel and (ii) not intersecting (and hence they are skew).

$\left.\begin{matrix} \vec{n} \text{ of } L_1 = \langle -8, 26, -1 \rangle \\ \vec{n} \text{ of } L_2 = \langle 2, -5, 6 \rangle \end{matrix}\right\}$ Not scalar multiples!

Not parallel

Out of time...

thank you for the extension!

(c) Determine the (minimum) distance between $L_1$ and $L_2$.

**Answer:**

| |
|---|
| **Distance:** |