Set 12: Section 4.4, Poisson Distribution

Poisson Process: X **counts the number of events in space or time. Process properties are:**

1. **Number of events in any interval (of space or time) is independent of the number of events in any other non-overlapping interval.**
2. **Probability of an event in an interval is the same for all intervals of equal size.**
3. **As interval size becomes small, the probability of more than one event approaches zero.**
4. **The probability of an event in an interval is proportional to the size of the interval.**

X **has a Poisson distribution,** $X \sim Poisson(\lambda)$, **where** λ**=mean number of events per space/time interval.**

Example: A typist makes on average 10 errors while typing 300 pages. Let X**=the number of errors on a page. Then** $X \sim Poisson(\lambda = 10/300)$ **errors *per page*.**

Example: There are on average 200 brine shrimp per litre of sea water. Let X=the number of brine shrimp in a litre of sea water. Then $X \sim Poisson(\lambda = 200$ shrimp per litre$)$.

Example: At a busy intersection, an average 5 cars pass through the intersection per minute. Let X=the number of cars which pass through the intersection in an *hour*. Then $X \sim Poisson(\lambda = 5x60 = 300$ cars per hour$)$.

Poisson Probability Distribution:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, 3, \ldots$$

Example: A machine makes an average of 5 defective items per hour. What is the probability that the machine will make exactly 4 defective items in an hour?

For $X \sim \text{Poisson}(\lambda)$

- $\text{E}(X) = \lambda$
- $\text{Var}(X) = \lambda$

Cumulative Distribution Tables: These tables give $P(X \leq x)$ **for "nice" values of** λ

Example: The machine which makes on average 5 defectives per hour is watched for three hours. What is the probability that it will make no more than 12 **defective items?**

Let X **denote the number of defective items made in a three hour period. Then** $X \sim Poisson(\lambda =$ 15 **defective items per three hour period**).

We wish to find $P(X \leq 12)$. **Using the table, we find:**

$$P(X \leq 12) = 0.268$$

Example: What is the probability that at least 6 defective items will be made?

Example: What is the probability that exactly 13 defective items will be made?

Example: At a particular intersection, there are an average of 2 car accidents per week. If there is at least one accident this week, what is the probability that there will be no more than 3 accidents in total this week?

Let X denote the number accidents in a week. Then $X \sim Poisson(2)$.

Poisson approximation to Binomial:
If $X \sim binomial(n, p)$ where n is very large and p is very small then X can be approximated with a Poisson distribution with $\lambda = np$. Provided $n \geq 100$ and $p \leq 0.01$, the approximation will be quite good.

Example: Brugada syndrome is a rare disease which afflicts 0.02% of the population. 10,000 people are selected at random and tested for Brugada syndrome. What is the probability that no more than 3 of the tested people will have Brugada syndrome?

Example: A typist makes on average of 2 errors per page. The typist is creating a ten-page document. What is the probability that exactly three of the pages do not contain any errors?