Set 5: Section 3.3

Kolmogorov (1933) provided the following definition of probability:

A probability measure P satisfies three axioms

1. For any event A, $P(A) \geq 0$
2. $P(S) = 1$ where S is the sample space
3. If $A_1, A_2, \ldots,$ are disjoint, $P(\bigcup A_i) = \sum P(A_i)$

Useful derivations from the Kolmogorov defn:

Example: $P(\bar{A}) = 1 - P(A)$

Example: $P(\phi) = 0$

Example: If $A \subseteq B$**,** $P(A) \leq P(B)$

Example: $P(A \cup B) = P(A) + P(B) - P(AB)$

Example:

$$\begin{aligned} P(A \cup B \cup C) = \quad & P(A) + P(B) + P(C) \\ & -P(AB) - P(AC) - P(BC) \\ & +P(ABC) \end{aligned}$$

Example: Suppose a quantity of red, green, purple, and yellow marbles are placed in a bag. You are twice as likely to select a red marble as a green marble. You are four times as likely to select a purple marble as a red marble. You are three times as likely to select a yellow marble as a red marble.

What is the probability that a randomly selected marble will be red? Green? Purple? Yellow?

Symmetry definition of probability:

In the case of a finite number of equally likely outcomes in an experiment,

$$P(A) = \frac{\textbf{number of outcomes leading to } A}{\textbf{number of outcomes in the experiment}}$$

Example: Suppose for the coin-flipping experiment, we wish to find $P(E)$ where $E = \{HHH, HHT, HTH\}$.

First, we note that E is made up of three sample points. That is, we can think of E as the union of three simple events:

$$E = E_1 \cup E_2 \cup E_3$$

This means that $P(E) = P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$.

Since the sample space consists of eight sample points, then each simple event has probability $\frac{1}{8}$. We have found that:

$$P(E) = P(E_1) + P(E_2) + P(E_3) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

To simplify our work: If A is an event in a space of equally likely events, then A can be written as the union of $n(A)$ simple events, each with probability $1/n(S)$. The probability that A will occur is $n(A)$ times $1/n(S)$,

$$P(A) = \frac{n(A)}{n(S)}$$

Example: The 2001 Census found that in Tofino, there were 790 residents who traveled to work. Here are the results of this census question:

Mode of Transportation	Total Number
Car, truck, van, as driver	425
Car, truck, van, as passenger	10
Public transit	15
Walked or bicycled	250
Other method	90

Suppose a Tofino resident who travels to work is selected at random. What is the probability that this resident walks or bicycles to work?

Problem: If 85% of Canadians like either baseball or hockey, 63% like hockey and 52% like baseball, what is the probability that a randomly chosen Canadian likes both hockey and baseball?