Set 13: Section 5.1, Continuous Distributions

Definition: A rv is *continuous* if it takes on real values in an interval.

Example: Let X be the temperature in degrees Celsius at UVic.

Definition: Let X be a continuous rv. Then the *probability density function* (pdf) $f(x) \geq 0$ of X is such that

$$P(a \leq X \leq b) = \int_a^b f(x)\ dx \qquad \forall a < b$$

$$P(X \leq b) = P(-\infty \leq X \leq b) = \int_{-\infty}^b f(x)\ dx$$

Properties of pdf $f(x)$:

1. $f(x) \geq 0$ **and**
2. $\int_{-\infty}^{\infty} f(x)\ dx = 1$

For a continuous distribution, $P(X = c) = 0$**, for** c **constant.**

Consequences:

$$P(X \leq c) = P(X < c)$$
$$P(a \leq X \leq b) = P(a < X < b)$$

Definition: A rv X has a Uniform(a, b) distribution if it has pdf

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \textbf{otherwise} \end{cases}$$

Exercise: What is $P(X \leq \frac{b+a}{2})$?

Special case: Uniform(0,1)

Definition: The *cumulative distribution function* (cdf) of a continuous rv X is given by

$$F(x) = \mathrm{P}(X \leq x) = \int_{-\infty}^{x} f(y)\ dy \qquad a < x < b$$

Definition: The 100p-th *percentile* of the continuous distribution with cdf $F(x)$ is the value $\eta(p)$ such that

$$p = F[\eta(p)]$$

Definition: The *median* $\tilde{\mu}$ of the continuous distribution with cdf $F(x)$ is the 50-th percentile (i.e. $0.5 = F(\tilde{\mu})$).

Example: Find the median of the $\mathrm{Uniform}(a, b)$ distribution.

Definition: The expected value of a continuous rv X with pdf $f(x)$ is

$$\mu = \mathrm{E}(X) = \int_{-\infty}^{\infty} x f(x)\ dx \qquad a < x < b$$

Proposition: If X is a continuous rv with pdf $f(x)$

$$\mathrm{E}[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\ dx$$

Definition: The variance of a continuous rv X with pdf $f(x)$ is

$$\mathrm{Var}(X) = \mathrm{E}[X - \mathrm{E}(X)]^2 = \int_{-\infty}^{\infty} [x - \mathrm{E}(X)]^2 f(x)\ dx$$

Proposition: If X is a continuous rv, then as in the discrete case,

- $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
- $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$
- $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$