

Set 21: Section 5.6, The Central Limit Theorem

Proposition - The Central Limit Theorem (CLT): Let X_1, \dots, X_n be iid (independent and identically distributed) rvs from a distribution with mean μ and variance σ^2 . Then as $n \rightarrow \infty$,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow \text{Normal}(0, 1)$$

Notes:

- the most important result in Statistics
- weaker versions of the CLT are available
- the CLT is important for inference
- assuming little, the CLT tells us a lot
- applies when the sample size is large ($n \geq 30$)

We motivate the CLT using an applet.

Example: The number of bacteria per mL sample of water has a Poisson distribution, with a mean of 50 bacteria per sample. $n = 100$ samples are tested. What is the probability that the average number of bacteria per sample is at least 52?

Example: It is known that for a particular brand of motherboard, the lifespan in months has exponential distribution with $\lambda = 25$. A campus computer lab has 40 new computers, all with this type of motherboard. What is the probability that the average lifespan of the motherboards in this lab is between 23 and 26 months?

Example: You order 500 apples and you know from previous orders that the mean weight of an apple is 0.2 kg with std dev 0.1 kg. What is the probability that the total weight of the 500 apples is less than 98 kg?

Example: The mean tensile strength of type-A steel is 105 ksi with standard deviation 8 ksi. For type-B steel, the mean tensile strength is 100 ksi and standard deviation 6 ksi. Let \bar{X} be the sample average of 40 type-A specimens and let \bar{Y} be the sample average of 35 type-B specimens.

- (a) What are the approx distbns of \bar{X} and \bar{Y} ?
- (b) What is the approx distbn of $\bar{X} - \bar{Y}$?
- (c) Calculate approximately $P(-1 \leq \bar{X} - \bar{Y} \leq 1)$.

Example: The mean weight of luggage for an economy passenger is 40 lb with std dev 10 lb. The mean weight of luggage for a business class passenger is 30 lb with std dev 6 lb. Suppose that there are 12 business class and 50 economy passengers on a given flight.

- (a) What is the expected total luggage weight and standard deviation?
- (b) What is the prob that the total luggage weight is at most 2500 lb if luggage weights are independent and normally distributed?