Parker deBruyne

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Stat 261 Assignment 0

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Due date: May 12, 2023, 11:59 pm

Answer the questions (handwritten on paper or on a tablet or computer file). Create a PDF file of your answers (scan handwritten notes or save tablet notes to pdf). Upload your PDF file to Brightspace.

NOTE: jpeg files are not acceptable.

For each of the following questions, indicate whether the statement is true or false and justify it. (4 points for each question)

1. $$\frac{\sum_{i=1}^{n} a_i b_i}{\sum_{i=1}^{n} a_i^3} = \sum_{i=1}^{n} \frac{b_i}{a_i^2}$$

False:

$\boxed{n_1=1}$

$$\frac{\sum_{i=1}^{1} a_i b_i}{\sum_{i=1}^{1} a_i^3} = \frac{a_1 b_1}{a_1^3} = \frac{b_1}{a_1^2} \langle = \rangle \sum_{i=1}^{1} \frac{b_i}{a_i^2}$$ Base case true

$\boxed{n_2=n_1+1}$

$$\frac{\sum_{i=1}^{2} a_i b_i}{\sum_{i=1}^{2} a_i^3} = \frac{a_1 b_1 + a_2 b_2}{a_1^3 + a_2^3} = \frac{a_1 b_1}{a_1^3+a_2^3} + \frac{a_2 b_2}{a_1^3+a_2^3} \langle \neq \rangle \sum_{i=1}^{2} \frac{b_i}{a_i^2} = \frac{b_1}{a_1^2} + \frac{b_2}{a_2^2}$$ Induction is false

2. $$\prod_{i=1}^{n} e^{2y_i} = e^{2\sum_{i=1}^{n} y_i}$$

False:

$\boxed{n_1=1}$

$$\prod_{i=1}^{1} e^{2y_i} = e^{2y} = e^{2\sum_{i=1}^{1} y_i}$$ Base case is true

$\boxed{n_2=n_1+1}$

$$\prod_{i=1}^{2} e^{2y_i} = e^{2y} \cdot e^{2y_2} = e^{4yy_2} (\neq) e^{2\sum_{i=1}^{2} y_i} = e^{2(y+y_2)}$$ Induction step false

3. $$\ln\left(\prod_{i=1}^{n} \lambda e^{\lambda x_i}\right) = n \ln\lambda + \lambda \sum_{i=1}^{n} x_i$$

True:

$\boxed{n_1=1}$

$$\ln\left(\prod_{i=1}^{1} \lambda e^{\lambda x_i}\right) = \ln\left(\lambda e^{\lambda x}\right) = \ln\lambda + \lambda x = n\ln\lambda + \lambda\sum_{i=1}^{1} x_i$$ Base case True

$\boxed{n_2=n_1+1}$

$$\ln\left(\prod_{i=1}^{2} \lambda e^{\lambda x_i}\right) = \ln\left((\lambda e^{\lambda x_1})(\lambda e^{\lambda x_2})\right) = 2\ln\lambda + \lambda x_1 + \lambda x_2 \langle = \rangle 2\ln\lambda + \lambda\sum_{i=1}^{2} x_i = 2\ln\lambda + \lambda x_i + \lambda x_2$$ Induction step True!!

4. $$\sum_{i=1}^{n} 2(x_i+1) = 2\left(\sum_{i=1}^{n} x_i\right) + n$$

False:

$$\sum_{i=1}^{n} 2(x_i+1) = 2\sum_{i=1}^{n}(x_i+1) = 2\sum_{i=1}^{n} x_i + 2\sum_{i=1}^{n} 1 = 2\sum_{i=1}^{n} x_i + 2n \langle \neq \rangle 2\left(\sum_{i=1}^{n} x_i\right) + n$$

5. $$\prod_{i=1}^{n} \rho^{x_i}(1-\rho)^{k-x_i} = \rho^{\sum_{i=1}^{n} x_i}(1-\rho)^{k-\sum_{i=1}^{n} x_i}$$

False:

$\boxed{n_1=1}$

$$\prod_{i=1}^{1} p^{x_i}(1-p)^{k-x_i} = p^{x}(1-p)^{k-x} \langle = \rangle p^{\sum_{i=1}^{1} x_i}(1-p)^{k-\sum_{i=1}^{1} x_i} = p^{x}(1-p)^{k-x}$$ Base case True

$\boxed{n_2=n_1+1}$

$$\prod_{i=1}^{2} p^{x_i}(1-p)^{k-x_i} = \left(p^{x_1}(1-p)^{k-x_1}\right)\left(p^{x_2}(1-p)^{k-x_2}\right) = p^{x_1+x_2}(1-p)^{2k-x_1-x_2} \langle \neq \rangle p^{\sum_{i=1}^{2} x_i}(1-p)^{k-\sum_{i=1}^{2} x_i} = p^{x_1+x_2}(1-p)^{k-x_1-x_2}$$ Induction False