3rd April '2024

Recap: Straight Line Models, Outliers, Influential points, Q-Q plots and Box plots

Section 6.5 :→ Analysis of Paired Measurements.

Let's Look at Example 6.5.1 of ch.6 (Pg-49).

↳ $H_0 : \mu_1 = \mu_2$ or, $H_0 : \mu_1 - \mu_2 = 0$

Where, μ_1 = Mean grade of Test 1.
μ_2 = Mean grade of Test 2.

Because the two groups of measurements are not independent, this is not an appropriate set up for a 2-Sample test.

Define, d_i as the i^{th} difference in test score (Test 1 − Test 2) for $i = 1, 2, \cdots, 12$.

$d_1, d_2, \cdots, d_{12} \overset{iid}{\sim} N(\mu_d, \delta_d^2)$ is our assumption.

↳ to do a 1-Sample t-test (δ^2 not assumed known)

In 2-Samples : dof $= n_1 + n_2 - 2$.

In paired Sample : dof $= n - 1$

Subscript d denotes it's from the differenced data

$H_0 : \mu_d = 0$, δ_d^2 unknown.

Test-Statistic : $T = \dfrac{\bar{x}_d - \mu_0}{s_d/\sqrt{n}} \sim t_{(n-1)}$

In this example, $\mu_0 = 0$, $n = 12$, $s_d^2 = 474.96^2$,

$\bar{x}_d = -16.6$

$$p\text{-value} = 2\,\mathbb{P}(T \geq |t_{obs}|)$$
$$= 2 \cdot \mathbb{P}(t_{11} \geq 2.65)$$

$$2(0.01) \leq p\text{-value} \leq 2(0.025)$$

or, $p\text{-value} \in (0.02, 0.05)$

We have evidence against H_0, given p-value < 0.05

$100(1-\alpha)\%$ C.I for μ_d :-

$$\bar{X}_d \pm t_{1-\alpha/2,(n-1)} \, s_d/\sqrt{n}$$

95% C.I, we can be obtained as,

$$[-30.5, -2.82]$$

↳ C.I does not contain Zero!

p-value < 0.05

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① CES complete.

② Practice Problems — Answer.

END of CHAPTER 6 ☺

CHAPTER 1 : Review

- Experiment, Sample Space, random Variables } Defination
- Various dist^n's and their applications
  - ↳ recognize dist^{n}s from experiment set-up.
  - ↳ familiar with dist^n assumptions.
- Defining pdf's and pmf's and CDF's

CHAPTER 2 : Likelihood Methods.

- Construct a (log)-likelihood function given n observations from a dist^n of interest.
- Derive the MLE from the Score function.
- 2nd derivative test using the information function.
- How to test hypothesis using frequency tables
- Likelihood intervals using Relative likelihood function
  - * Using roots of RLF - p in R.
- Invariance property of MLE's
  - ↪ How to integrate a pdf.
  - ↪ check function is monotone.
  - * Computing Medians/percentiles
    - ↑ Monotone, Can you find the MLE's ?
- Knowing when Calculus fails and, finding MLE's "Heuristically". (unusual Example) =.