Modeling Blood Alcohol and Breath Alcohol Concentrations

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Math 486

Summary

Many people like to have a drink every once and a while, whether at dinner with friends, at a party, or for some other reason. While drinking can be fun, drunk driving is a very big issue in the United States and police have been cracking down on drinking and driving for many years. Police use breathalyzers that determine how much someone has drank. While it is expected that these devices accurately determine how drunk someone is, many people are doubtful that they are accurate. There have been many stories about people only having 1 drink or not even having any drinks, but they still somehow fail the breathalyzer test and get arrested. Some people will refuse to take a breathalyzer test because they do not trust the device. This paper, is going to explore the science and model behind the breathalyzer in order to see if it really is an accurate way to measure how much someone has drank.

Introduction

Alcohol consumption is very popular and it can be enjoyable; however, it can also be very dangerous. If someone drinks too much alcohol then it could be dangerous to their own health with the possibility of alcohol poisoning. Others could also be in danger if someone drinks too much and decides to drive. According to MADD Statistics, in 2016, 10,497 people died from accidents caused by drunk drivers, and 290,000 were injured. If we look at this per minute, approximately 1 person dies every 50 minutes and 1 person is injured every 2 minutes from drunk drivers. Because of these dangers, it is important to understand how our body absorbs alcohol and gets rid of the alcohol in our system over time. It is also important for police to be able to know how much someone has drank if they are driving, in order to ensure that they are not putting others in harm's way. The way to determine how much someone has drank can be measured by the person's blood alcohol concentration. This is a percentage that tells you how much alcohol is in a person's blood per some volume. The legal limit to drive is 0.08, meaning in order to drive, you must have less than 0.08% alcohol in your blood. Someone's blood alcohol concentration is most accurately measured by a blood test, however police cannot give a blood test on the road, so they need a quick and efficient way to measure a person's blood alcohol concentration. When a police officer suspects someone of drinking and driving, then they can put them through some coordination tests and/or have them take a breathalyzer test, which approximates their blood alcohol level. If the breathalyzer test shows that the person has a blood alcohol concentration greater than 0.08, then the police officer will take them into custody and have them get a blood test to get a more accurate reading of their blood alcohol concentration.

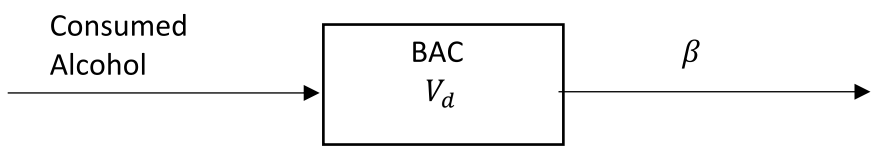
In this paper, I am going to explore different kinds of 1-compartment models to determine a person's blood alcohol concentration over time and analyze how the concentration changes. The differences in the 1-compartment models include different ways of modeling the consumption rate and the removal rate. I am also going to explore a model for breath alcohol concentration and compare it to the different blood alcohol models to see how accurate someone's breath alcohol concentration reflects their blood alcohol concentration. This will help show the accuracy or inaccuracy of breathalyzers.

Blood Alcohol Models

Widmark Model

The first model we are going to explore is a simple 1-compartment model. This model assumes that the alcohol intake goes directly to the blood, and over time the alcohol is eliminated from the blood with zero order. This model may be simple, but it is still very accurate when it comes to real data. Figure 1 shows a drawing of this model.

Figure 1



In this model, V_d is the Volume of distribution, β is the rate of metabolism in g/l/h, and BAC is the Blood Alcohol Concentration. The differential equation for the elimination of alcohol in blood is given as equation (1). The model for the BAC is shown as equation (2).

$$\frac{dBAC}{dt} = -\beta t, \;\; BAC(0) = BAC_0 \tag{1}$$

$$BAC = \frac{D}{r*W} - \beta t \tag{2}$$

In equation 1, the initial BAC is given. This equation can be solved for the initial value, which is how we get equation 2. In the BAC model (equation 2), D is the amount of alcohol in grams consumed, r is the Widmark factor, W is the body weight of the person in kilograms, and t is time in hours. The Widmark factor is different for every person, but for men it is typically 0.68 and for women 0.55 (Andre Heck). The rate of metabolism is also different for each person, but it typically varies from 0.10 to 0.20 g/l/h (Andre Heck). Lastly, the denominator in equation 2, r*W is also equal to the volume of distribution.

An average drink is about 14 grams. My body weight is 185 pounds, which is 83.91 kg. I am male, so my Widmark factor would be about 0.68. I am assuming that my metabolism rate is average at 0.15. If I were to model the BAC of myself in 5 hours after drinking two drinks, then I would get the plot shown in figure 2.

Figure 2

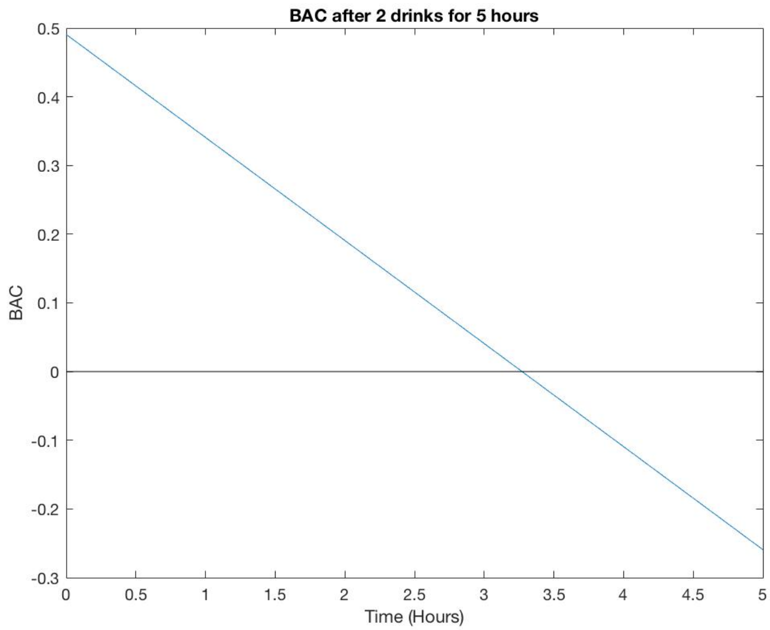


Figure 2 shows that my BAC will decrease at a constant rate, and my BAC will be back to 0 after about three and a half hours. After three and a half hours, the plot shows that my BAC becomes negative, which is not possible This is because the model continues to decrease the values even after the BAC is 0. In reality, the BAC will continue to stay at 0 once the alcohol is eliminated from the blood.

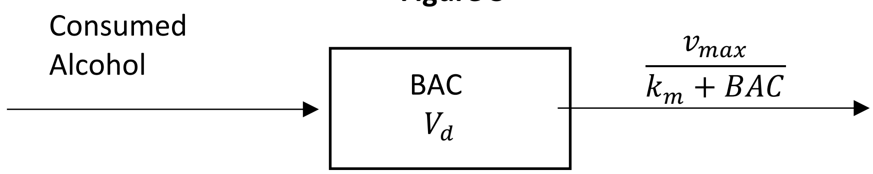
<u>Wagner Model</u>

The Wagner model is very similar to the Widmark model in that it is a one compartment model. The only difference is that the Clearance rate is following Michaelis-Menton kinetics. The clearance rate is given by equation 3.

$$CL = \frac{v_{max}}{k_m + BAC} \tag{3}$$

In the clearance rate, v_{max} is the maximum disappearance rate, and $k_m$is the Michaelis-Menton constant. Figure 3 shows a drawing of the compartment model.

Figure 3



The change of alcohol in the blood can be modeled by a differential equation. This formula is shown in equation 4.

$$V_d \frac{d}{dt} BAC = -\frac{v_{max}}{k_m + BAC} BAC \tag{4}$$

This model looks linear while the BAC is high, but once the BAC is lower, you can start to see the curve, and the BAC goes to 0 over time. This can be seen in figure 4.

Figure 4

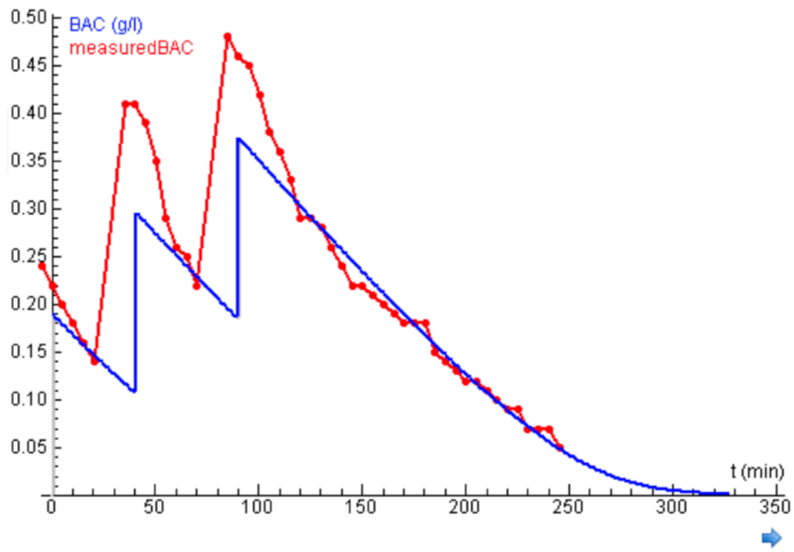


Image taken from Andre Heck

In figure 4, the subject took 3 drinks with 45 minutes in between each drink. This model assumes that the alcohol is ingested and affect the BAC instantaneously. This can also be done in the Widmark model. In order to add the ability to have multiple drinks over time, both models must be adjusted to use heavy side step functions for the times at which the alcohol is consumed.

2 stage differential equations

The last model I am going to explore is a model that has two differential equations to model the two stages of drinking. The first stage is the drinking phase, and the second stage is the recovery phase. The reason there are two stages is so the model is more realistic in the way that alcohol does not affect you instantaneously, but it takes time for your body to absorb it and it will eliminate some alcohol while you are still consuming it. During the drinking phase, there is both an elimination part, along with an alcohol consumption variable. The recovery phase, only has the elimination part that is in the drinking phase. The drinking phase is modeled by equation 5, while the recovery phase is modeled by equation 6.

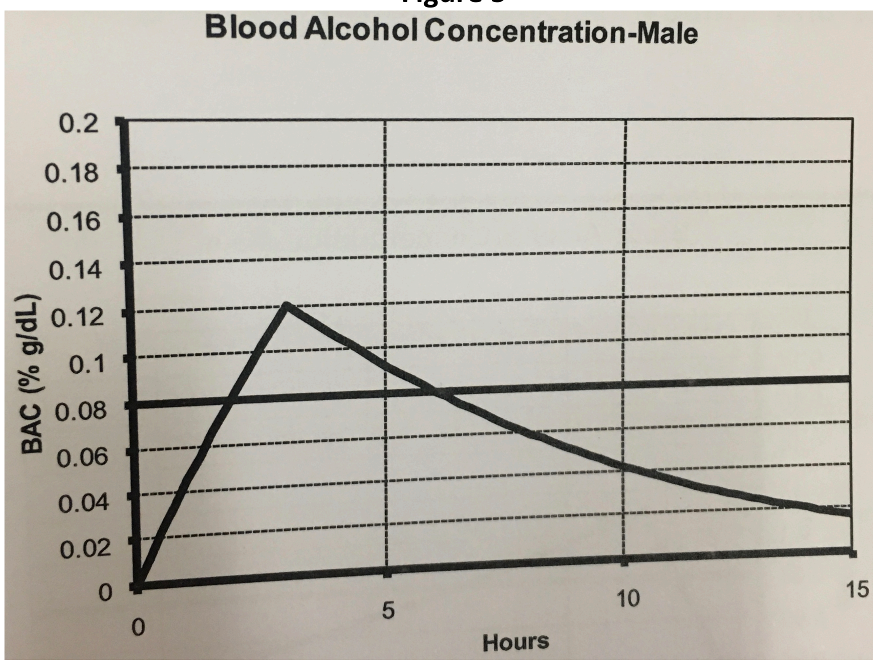
$$\frac{d}{dt} BAC = u - \frac{\alpha * BAC}{BAC + \beta^{v}}, 0 \le t \le s,\ BAC(0) = 0 \tag{5}$$

$$\frac{d}{dt} BAC = -\frac{\alpha * BAC}{BAC + \beta^{v}},\ t > s,\ BAC(s) = M \tag{6}$$

In these equations, u is b* amount of body water per given weight * s, b is the gender ratio, s is the length of the alcohol consumption period, α is the liver function parameter, β is the kidney function parameter, υ is the gender parameter, t is time, and M is the peak BAC at time s. (taken from S.J. Kouba)

This model is much more realistic, because it takes into account the liver and kidney. It is also more realistic in that the BAC increases gradually during the time that the alcohol is being consumed, and then when the alcohol is not being consumed, it gradually decreases, similarly to the Wagner model. This can be seen from figure 5.

Figure 5



This photo was taken from S.J. Kouba

Figure 5 is a graph taken from the paper "Exploring Mathematical Models for Calculating Blood Alcohol Concentration" by S.J. Kouba, M.B.M. Eligindi, R.W. Langer. This plot is showing the BAC of a male that weighs 150 pounds and has had 4 drinks in 3 hours.

This model averages out the drinks over the time, rather than assuming instantaneous drinking at certain times. Because of this, the peak BAC will not be as high as it would be in the Wagner or Widmark model. All of these models have been tested with data, and though they are obviously not perfect, they are all fairly accurate.

Breath Alcohol Model

Breath alcohol concentration is what police officers measure with breathalyzers in order to determine someone's blood alcohol concentration. According to Coste, there are 6 equations that are accurate in determining the BAC when the Breath Alcohol Concentration (BrAC) is less than 0.08. One of these models is the Widmark formula as discussed above. This means that someone's BrAC can accurately determine someone's BAC if less than 0.08. We can model the BrAC similar to the BAC using to the Widmark model. If the BAC is greater than 0.08, then this model may not be as accurate, but for our sake, it is accurate enough. Figure 6 shows a diagram of the BrAC according to the Widmark model.

Figure 6

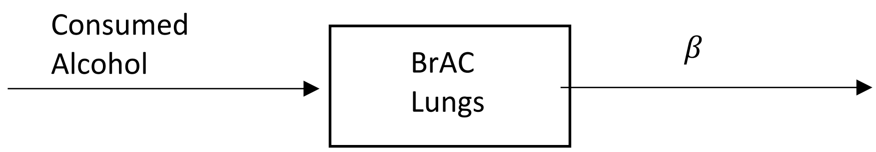


Figure 6 is the same as figure 1, except instead of the alcohol going into the blood, it goes into the lungs. The removal rate will be different in the model for BrAC than it is in BAC, since alcohol may diffuse from the lungs in a different way than it diffuses from the blood. The equation for BrAC is shown as equation 7.

$$BAC = \frac{D}{r*W} - \beta t \tag{7}$$

In equation 7, D is the amount of alcohol consumed in grams, r is the Widmark factor, and W is the weight in kilograms. The Widmark factor according to Coste is 0.73 for men and 0.66 for women. However, the Widmark factor is unique for each individual, and therefore it is the same for both the BAC and BrAC models. There are three cases for comparing the BAC and BrAC. If we let β be the elimination rate of alcohol from the blood and let β' be the elimination rate of alcohol from the lungs.

The first case for comparing the two models is when $\beta = \beta'$. When the two reaction rates are the same, the two differential equations are equal. This is because all variables in the BAC and BrAC models are the same except for the β variables. However, since $\beta = \beta'$, BAC=BrAC.

The second case is when $\beta < \beta'$. When the elimination rate of alcohol from the blood is slower than the elimination rate of alcohol from the lungs, we get that the BAC>BrAC. This means that the alcohol in the lungs is eliminated faster than the alcohol in the blood, so the BrAC which approximates the BAC will be an underestimate.

The last case is when $\beta > \beta'$. Similar to the second case, the elimination of the alcohol from the lungs will be slower than the elimination from the blood. Therefore, the BAC<BrAC, so the BrAC will be an overestimate of the BAC.

I could not find any data on BAC and BrAC together, so I was not able to find β and β' for the two models. However, I am able to graph the different cases to show how much of a difference the β can make compared to each other. Figure 7 shows a plot where β and β' are not equal, but close. Figure 8 shows a plot where β and β' have a large difference.

Figure 7

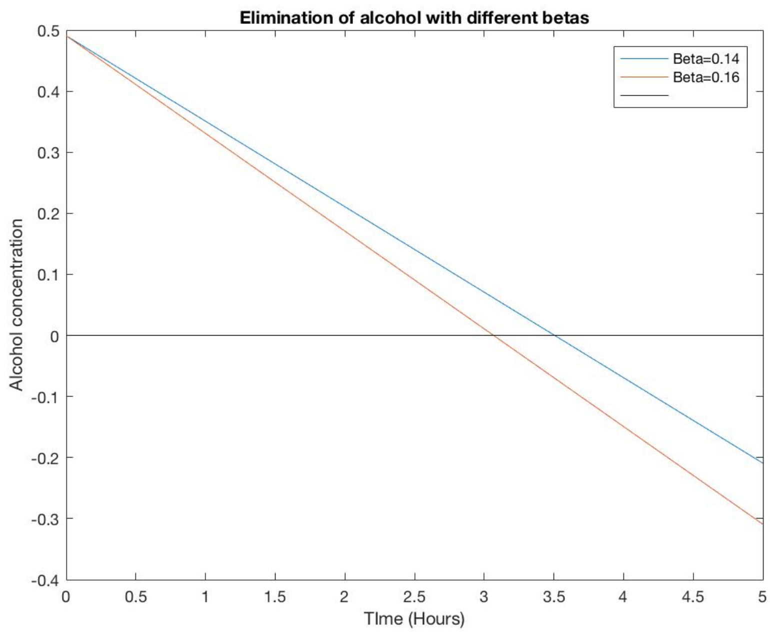
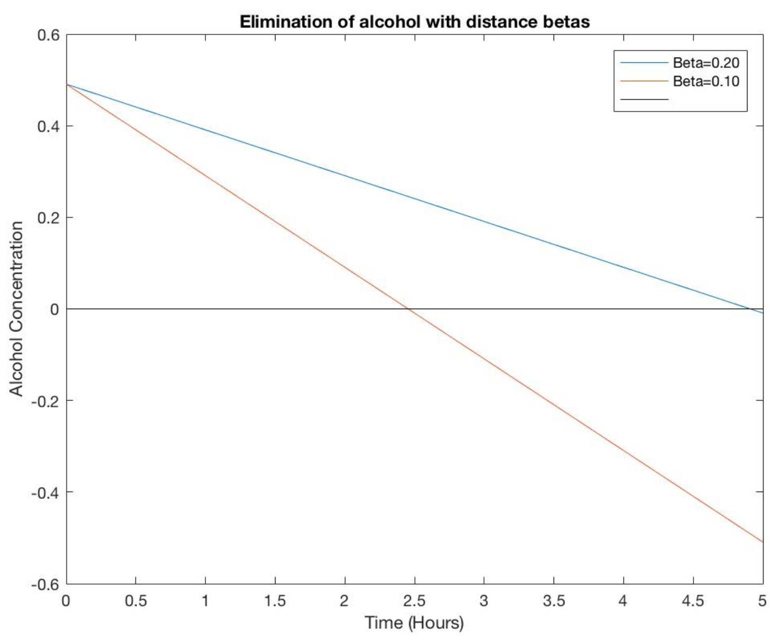


Figure 8



In figure 7, where the β and β' are fairly close, the difference in the change is not much. The model with $\beta = 0.16$ is about half and hour faster, but this is not that much of a difference. If the difference between β and β' was smaller, then the difference in the models and time would be much smaller. In figure 8 where the difference between β and β' is large, the models also have a large difference. The model with $\beta = 0.20$ gets to an alcohol concentration of 0 about twice as fast as the model with $\beta = 0.10$. This makes sense, because 0.20 is 2 times larger than 0.10. Overall, the models are more similar when β and β' have a small difference. If they have a large difference, then the BrAC approximation will not be similar to the BAC. Depending on which elimination rate is faster, this will either be an overestimation or underestimation, either will cause problems when the β's are not close. The two elimination rates are very close in reality because one way the alcohol in the blood Is eliminated is through the breath. This means that they should have similar β's, since elimination through the lungs affects the elimination of the alcohol in the blood.

Since the two β's are similar, it can be concluded that the breath alcohol concentration is typically an accurate way to estimate the blood alcohol concentration. The two concentrations will not be the same, but they will be similar. This means that as long as the breathalyzers work correctly, police should be able to accurately determine someone's BAC.

The next question that needs to be answered is how does a breathalyzer work? According to Wikipedia, when someone exhales into the breathalyzer, any ethanol in the breath is oxidized into acetic acid at the anode. This reaction is shown in the following equation.

$$CH_3CH_2OH(g) + H_2O(l) \rightarrow CH_3CO_2H(l)4H^+(aq) + 4e^- \tag{8}$$

Next, at the cathode, the atmospheric oxygen is reduced. The following equation shows this chemical reaction.

$$O_2(g) + 4H^+(aq) + 4e^- \rightarrow 2H_2O \tag{9}$$

The breathalyzer measures the electric current produced in the reaction in order to determine the approximate BAC. With each molecule of ethanol in the breath, 4 charges will be produced. Based on the charge, the amount of ethanol can be estimated. The amount of ethanol in the lungs can be used in the models to approximate the BAC.

Conclusion

There can also be some problems with breathalyzers that make them inaccurate. First, breathalyzers can get contaminated after being used many times, and they may overestimate the BAC. This is more typical with handheld breathalyzers that are sold to consumers. These also must be calibrated and recalibrated in order to be more accurate. Another source of error is other compounds that may interfere. According to Wikipedia, the National Highway Traffic Safety Administration has found that dieters and those with diabetes may have much higher acetone levels than others. Breathalyzers may falsely determine acetone as ethanol, and therefore these people may have a higher BrAC. There are also some other substances in the environment which may cause the breathalyzer to read higher. The last thing that may cause breathalyzers to be inaccurate is the presence of mouth alcohol. Mouth alcohol is natural and is produced by the mouth or throat. Breathalyzers do not know the difference between ethanol from the lungs and mouth alcohol, so this could end up in a higher reading for the BrAC. Police are aware of these problems, so they are supposed to observe a possible drunk driver before testing them.

In the end, blood alcohol and breath alcohol models are very similar, so the breath alcohol concentration can be used to approximate the blood alcohol concentration. Breathalyzers use this fact to give police, or consumers and idea of what their BAC is. The approximation from the breathalyzers are not completely accurate and can give false readings. Therefore, the best way to determine somebody's true BAC is to get a blood test, but breathalyzers can be useful to give someone an idea of what their BAC is.

References

Breathalyzer. (2017, October 26). Retrieved November 28, 2017, from https://en.wikipedia.org/wiki/Breathalyzer

Coste, Katherine. (2013). Determining Blood Alcohol Concentration From Transdermal Alcohol Data: Calibrating a Mathematical Model Using a Drinking Diary. *Thesis for University of Southern California.*

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Statistics. (n.d.). Retrieved November 28, 2017, from http://www.madd.org/statistics/

Appendix

Matlab Code for Figure 2:

```
D=28;
r=0.68;
B=0.15;
W=83.91;
t=0:5;
BAC=zeros(1,6);
for i=1:6
    BAC(i)=D/(r*W)-B*t(i);
end
plot(t,BAC, t, zeros(1,6),'k')
```

Matlab Code for Figure 7:

```
D=28;
r=0.68;
B1=0.14;
B2=0.16;
W=83.91;
t=0:5;
BAC=zeros(1,6);
BrAC=zeros(1,6);
for i=1:6
    BAC(i)=D/(r*W)-B1*t(i);
    BrAC(i)=D/(r*W)-B2*t(i);
end
plot(t,BAC, t, BrAC, t, zeros(1,6),'k')
```

Matlab Code for Figure 8:

```
D=28;
r=0.68;
B1=0.10;
B2=0.20;
W=83.91;
t=0:5;
BAC=zeros(1,6);
BrAC=zeros(1,6);
for i=1:6
    BAC(i)=D/(r*W)-B1*t(i);
    BrAC(i)=D/(r*W)-B2*t(i);
end
plot(t,BAC, t, BrAC, t, zeros(1,6),'k')
```