FLUIDS 2009

Problem Set #6 *Handed out 12/1/2007, due 12/7/2007 at the start of class*

Consider the potential flow problem of 2D flow around a cylinder.

A[5]. Derive the expression for the streamfunction, ψ (work in cylindrical polar coordinates). You can check your answer in Kundu and Cohen. It is useful to have the relations

$$u_R = \frac{\partial \varphi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$
$$u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

B[10]. Sketch (or plot) contours of ψ and φ, indicating the direction in which each grows larger.

C[10]. Use Bernoulli to find the general expression for the pressure anomaly, $p' \equiv p - p_\infty$. Sketch this as a contour map for the region outside of the cylinder. Indicate regions of relatively high and low pressure.

D[10]. Narrate how the change in parcel velocity (both magnitude and direction) along a streamline relates to the pressure gradient.

E[10]. Argue (without resorting to detailed math) why the integrated x-momentum inside the cylinder should be zero. What does this tell you about the *momentum* of the cylinder in the case where we are in a frame of reference moving with the free-stream flow U (and so the ambient far field flow is stationary and the cylinder is moving in the negative x-direction)?