Exercise 2.3. Write down rules to evaluate boolean expressions of the form $b_0 \vee b_1$, which take advantage of the fact that there is no need to evaluate b in $true \vee b$ as the result will be true independent of the result of evaluating b. The rules written down should describe a method of left-sequential evaluation. Of course, by symmetry, there is a method of right-sequential evaluation.

$$\frac{\langle b_0, \sigma \rangle \Downarrow true}{\langle b_0 \vee b_1, \sigma \rangle \Downarrow true}$$

$$\frac{\langle b_0, \sigma \rangle \Downarrow false \qquad \langle b_1 \sigma \rangle \Downarrow true}{\langle b_0 \vee b_1 \rangle \Downarrow true}$$

$$\frac{\langle b_0, \sigma \rangle \Downarrow false \qquad \langle b_1 \sigma \rangle \Downarrow false}{\langle b_0 \vee b_1 \rangle \Downarrow false}$$

Exercise 2.4. Write down rules which express the "parallel" evaluation of b_0 and b_1 in $b_0 \vee b_1$ so that $b_0 \vee b_1$ evaluates to true if either b_0 evaluates to true, and b_1 is unevaluated, or b_1 evaluates to true, and b_0 is unevaluated.

$$\frac{\langle b_0, \sigma \rangle \Downarrow true}{\langle b_0 \vee b_1, \sigma > \Downarrow true}$$

$$\frac{\langle b_1, \sigma \rangle \Downarrow true}{\langle b_0 \vee b_1, \sigma > \Downarrow true}$$

Exercise 2.7. Let $w \equiv$ *while true do skip*. By considering the form of derivations, explain why, for any state σ, there is no state σ' such that $\langle w, \sigma \rangle \Downarrow \sigma'$.

Based on the structure of w, we know that a derivation tree for w would apply the *while* rule last in order to complete the derivation. Specifically, the tree would apply the rule in which the loop guard b evaluates to true. This derivation tree would have the following shape:

$$\frac{\langle b, \sigma \rangle \Downarrow true \qquad \langle skip, \sigma \rangle \Downarrow \sigma'' \qquad \langle while\ true\ do\ skip, \sigma'' \rangle \Downarrow \sigma'}{\langle while\ true\ do\ skip, \sigma \rangle \Downarrow \sigma'}$$

From this derivation, we can see that our structure w is recursive; every application of the *while* rule to w will just yield w again. Thus, there is no complete, terminating derivation of w, proving that $\langle w, \sigma \rangle$ will never evaluate to state σ'.

Exercise 2.9. Complete the task, begun above, of writing down the rules for $\rightarrow_1$, one step in the evaluation of integer and boolean expressions. What evaluation strategy have you adopted (left-to-right sequential or ...)?

Subtraction

$$\frac{\langle a_0, \sigma\rangle \Downarrow \langle a_0', \sigma\rangle}{\langle a_0 - a_1, \sigma\rangle \Downarrow \langle a_0' - a_1, \sigma\rangle}$$

$$\frac{\langle a_1, \sigma\rangle \Downarrow \langle a_1', \sigma\rangle}{\langle n - a_1, \sigma\rangle \Downarrow \langle n - a_1', \sigma\rangle}$$

$$\langle n - m, \sigma\rangle \Downarrow \langle p, \sigma\rangle$$

Multiplication

$$\frac{\langle a_0, \sigma\rangle \Downarrow \langle a_0', \sigma\rangle}{\langle a_0 \times a_1, \sigma\rangle \Downarrow \langle a_0' \times a_1, \sigma\rangle}$$

$$\frac{\langle a_1, \sigma\rangle \Downarrow \langle a_1', \sigma\rangle}{\langle n \times a_1, \sigma\rangle \Downarrow \langle n \times a_1', \sigma\rangle}$$

$$\langle n \times m, \sigma\rangle \Downarrow \langle p, \sigma\rangle$$

Equal

$$\frac{\langle a_0, \sigma\rangle \Downarrow \langle a_0', \sigma\rangle}{\langle a_0 = a_1, \sigma\rangle \Downarrow \langle a_0' = a_1, \sigma\rangle}$$

$$\frac{\langle a_1, \sigma\rangle \Downarrow \langle a_1', \sigma\rangle}{\langle n = a_1, \sigma\rangle \Downarrow \langle n = a_1', \sigma\rangle}$$

$$\langle n = m, \sigma\rangle \Downarrow \langle true, \sigma\rangle \qquad \text{if n and m are equal}$$
$$\langle n = m, \sigma\rangle \Downarrow \langle false, \sigma\rangle \qquad \text{if n and m are not equal}$$

Leq

$$\frac{\langle a_0, \sigma\rangle \Downarrow \langle a_0', \sigma\rangle}{\langle a_0 \leq a_1, \sigma\rangle \Downarrow \langle a_0' \leq a_1, \sigma\rangle}$$

$$\frac{\langle a_1, \sigma\rangle \Downarrow \langle a_1', \sigma\rangle}{\langle n \leq a_1, \sigma\rangle \Downarrow \langle n \leq a_1', \sigma\rangle}$$

$$\langle n \leq m, \sigma\rangle \Downarrow \langle true, \sigma\rangle \qquad \text{if n is less than or equal to m}$$
$$\langle n \leq m, \sigma\rangle \Downarrow \langle false, \sigma\rangle \qquad \text{if n is greater than m}$$

And

$$\frac{\langle b_0, \sigma\rangle \Downarrow \langle b_0', \sigma\rangle}{\langle b_0 \wedge b_1, \sigma\rangle \Downarrow \langle b_0' \wedge b_1, \sigma\rangle}$$

$$\frac{\langle b_1, \sigma\rangle \Downarrow \langle b_1', \sigma\rangle}{\langle n \wedge b_1, \sigma\rangle \Downarrow \langle n \wedge b_1', \sigma\rangle}$$

$$\langle n \wedge m, \sigma\rangle \Downarrow \langle true, \sigma\rangle \qquad \text{if n and m are both true}$$
$$\langle n \wedge m, \sigma\rangle \Downarrow \langle false, \sigma\rangle \qquad \text{if n or m is false}$$

Or

$$\frac{\langle b_0, \sigma\rangle \Downarrow \langle b_0', \sigma\rangle}{\langle b_0 \vee b_1, \sigma\rangle \Downarrow \langle b_0' \vee b_1, \sigma\rangle}$$

$$\frac{\langle b_1, \sigma\rangle \Downarrow \langle b_1', \sigma\rangle}{\langle n \vee b_1, \sigma\rangle \Downarrow \langle n \vee b_1', \sigma\rangle}$$

$$\langle n \vee m, \sigma\rangle \Downarrow \langle true, \sigma\rangle \qquad \text{if n or m is true}$$
$$\langle n \vee m, \sigma\rangle \Downarrow \langle false, \sigma\rangle \qquad \text{if n and m are both false}$$