Exercise 3.1. Prove by mathematical induction that the following property P holds for all natural numbers.

$$P(n) \iff \Sigma_{i=1}^{n}(2i-1) = n^2$$

(The notation $\Sigma_{i=k}^{l} s_i$ abbreviates $s_k + s_{k+1} + s_{k+2} + ... + s_l$ when k, l are integers with $k < l$).

We proceed by mathematical induction on n.

Base Case $P(1)$.

We chose $P(1)$ as the base case. $P(0)$ is vacuously true; i begins at 1, so $\Sigma_{i=1}^{0}(2i-1)$ is not defined.

$$\begin{aligned} P(1) &\iff \Sigma_{i=1}^{1}(2i-1) = 1^2 \\ &\iff (2(1)-1) = 1^2 \\ &\iff 1 = 1 \end{aligned}$$

Inductive Case

Assume the proposition P holds for an arbitrary natural number m. We need to show that the proposition $P(m+1)$ also holds.

$$\begin{aligned} P(m+1) &\iff \Sigma_{i=1}^{m+1}(2i-1) = (m+1)^2 \\ &\iff \Sigma_{i=1}^{m}(2i-1) + (2(m+1)-1) = (m+1)^2 \\ &\iff m^2 + 2m + 2 - 1 = (m+1)^2 \\ &\iff m^2 + 2m + 1 = m^2 + 2m + 1 \end{aligned}$$

Thus: $\forall m \in \mathbb{N}, P(m) \implies P(m+1)$

We've proven the base case and the inductive case, completing the proof. QED