Basic Probability Concepts

Tossing a Coin

When a coin is tossed, there are two possible outcomes:

heads (H) or
tails (T)

We say that the probability of the coin landing H is ½.

And the probability of the coin landing T is ½.

Throwing Dice

When a single die is thrown, there are

six possible outcomes: 1, 2, 3, 4, 5, 6.

The probability of any one of them is 1/6.

Probability

In general:

Probability of an event happening = $\frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$

Example: **the chances of rolling odd number with a die**

Number of ways it can happen: 3 (1,3,5)

Total number of outcomes: 6 (6 faces altogether)

So the probability = 3/6 = 1/2

Example: **There are 10 marbles in a bag:**
4 are blue,1 is red, 2 are green and 3 are yellow.
What is the probability that a blue marble will be picked?

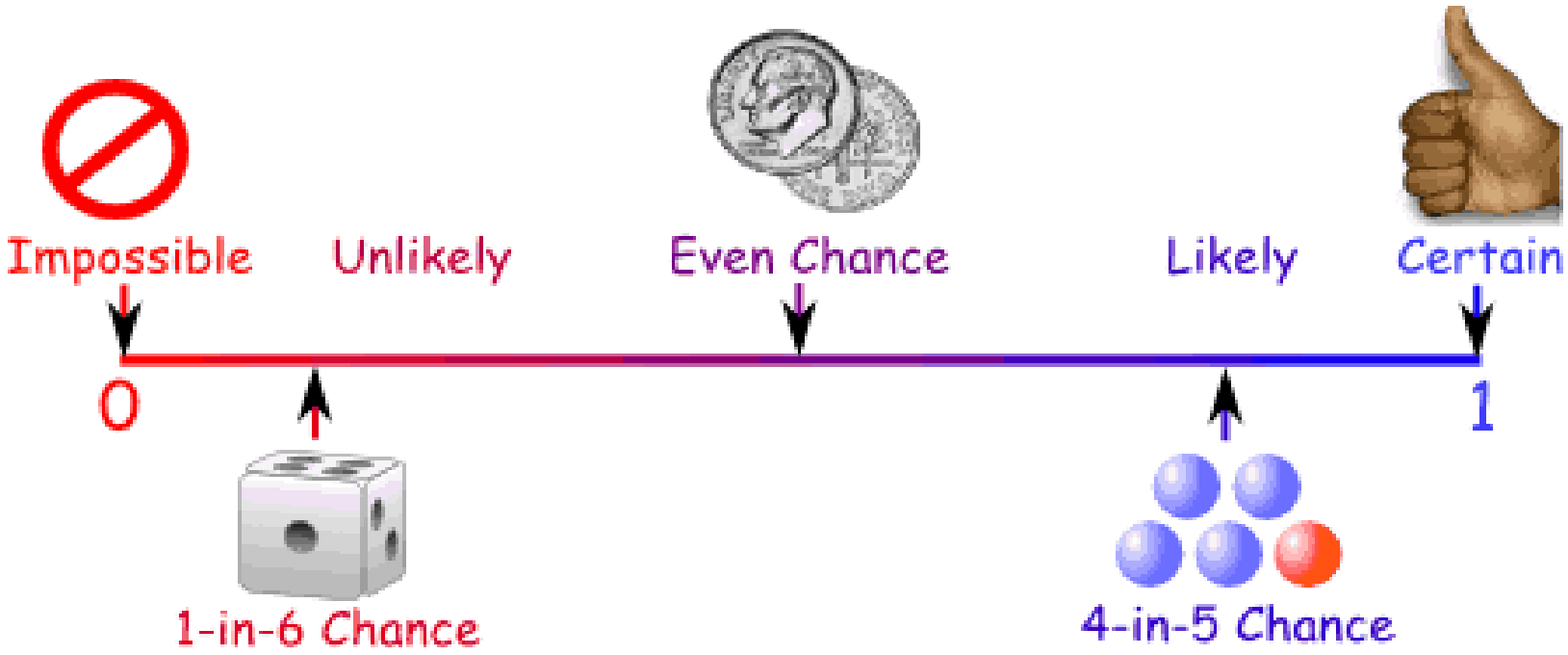
P(blue) = 4/10 = 2/5

What is the probability that a green or a red marble is picked?

P(green or red) = 3/10

Probability Line

You can show probability on a Probability Line:



Probability is always between 0 and 1, inclusive

Terminology

Some words have special meaning in Probability:

Experiment or Trial: an action where the result is uncertain.

Tossing a coin, throwing dice, seeing what pizza people choose are all examples of experiments.

Sample Space: all the possible outcomes of an experiment

Example: choosing a card from a deck

There are 52 cards in a deck (not including Jokers)

The Sample Space is all 52 possible cards:

The Sample Space is made up of Sample Points:

Sample Point: just one of the possible outcomes

<u>Example:</u> Deck of Cards

the 5 of Clubs is a sample point

the King of Hearts is a sample point

"King" is not a sample point. As there are 4 Kings that is 4 different sample points.

Event: a single result of an experiment

Example Events:

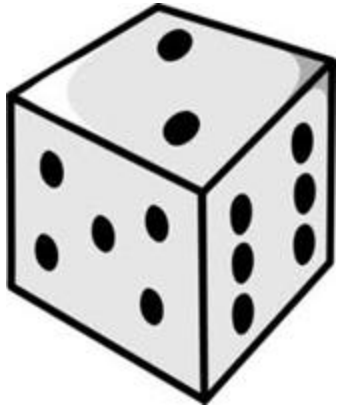
Getting a Tail when tossing a coin is an event

Rolling a "5" is an event.

An event can include one or more possible outcomes:

Choosing a "King" from a deck of cards (any of the 4) is an event

Rolling an "even number" (2, 4 or 6) is also an event



Complement:

Complement of an Event: All outcomes that are **NOT** the event.

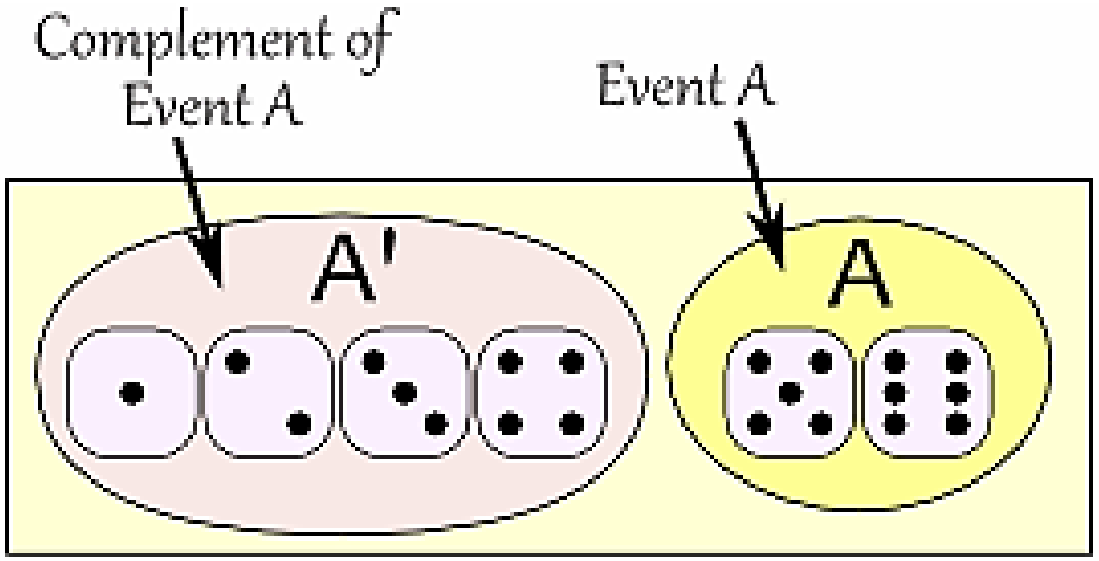
When the event is **Heads**, the complement is **Tails**

When the event is **{Monday, Wednesday}** the complement is **{Tuesday, Thursday, Friday, Saturday, Sunday}**

The probability of an event is shown using "P":
P(A) means "Probability of Event A“

P(A') means "Probability of the complement of Event A"
The two probabilities always add to 1

P(A) + P(A') = 1



Definitions:

- Probability – the chance or likelihood that an uncertain (particular) event will occur
- **Probability is always between 0 and 1, inclusive**
- Sample Space – the collection of all possible events
- An Event – Each possible type of occurrence or outcome from the sample space
- Simple Event – an event that can be described by a single characteristic
- Complement of an event A -- All outcomes that are not part of event A

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If we roll a die, what is the probability of getting a 3?

Event A = Getting a 3 when rolling a die.

Sample space S = {1,2,3,4,5,6}

n(S) = 6

n(A) = 1

Hence, P(A) = 1/6

The complement of A = A'

P(A')= probability of not getting a 3 = 5/6

Definitions:

- There are three approaches to assessing the probability of un uncertain event:

1. **Theoretical or a Priori or Classical Probability**– based of a prior knowledge

$$\text{prob. of occurrence} = \frac{X}{T} = \frac{\text{number of occurance of the event}}{\text{total number of possible outcomes}}$$

2. **Empirical or Experimental or Relative Frequency Probability**– based on observed data

$$\text{prob. of occurrence} = \frac{\text{number of favorable outcomes observed}}{\text{total number of outcomes observed}}$$

3. **Subjective probability** – an individual judgment or opinion about the probability of occurrence

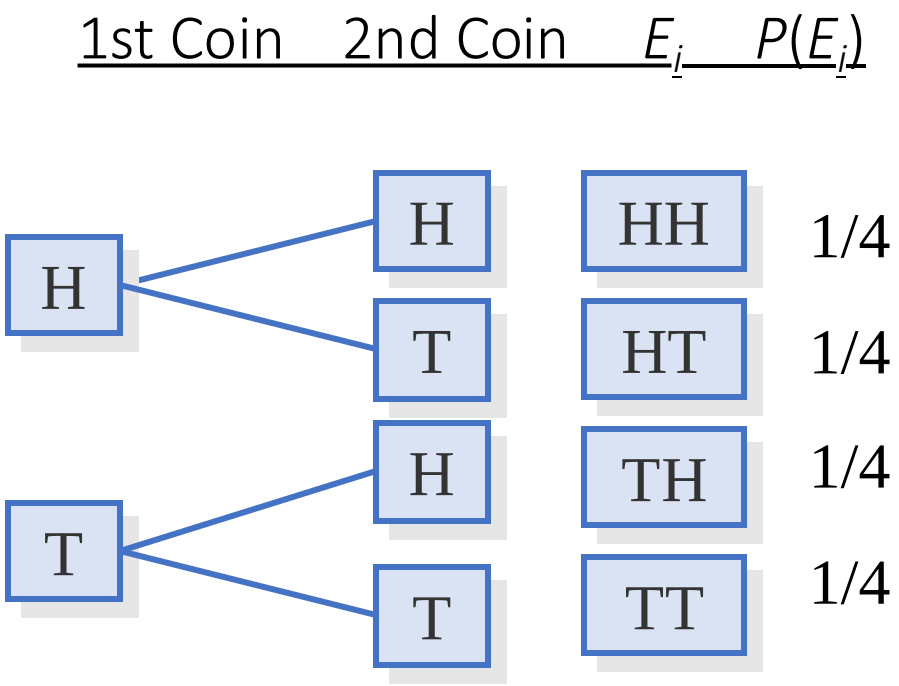
The *mn* Rule

- If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment
  - This rule is easily extended to ***k*** stages, with the number of ways equal to $n_1 n_2 n_3 \ldots n_k$
- **Example**: Toss two coins. The total number of simple events is:

$$2 \times 2 = 4$$

Example

- Toss a fair coin twice. What is the probability of observing at least one head?



P(at least 1 head)

$= P(HH) + P(HT) + P(TH)$

$= 1/4 + 1/4 + 1/4 = 3/4$

Examples

- Example: Toss three coins. The total number of simple events is:

$$2 \times 2 \times 2 = 8$$

- Example: Toss two dice. The total number of simple events is:

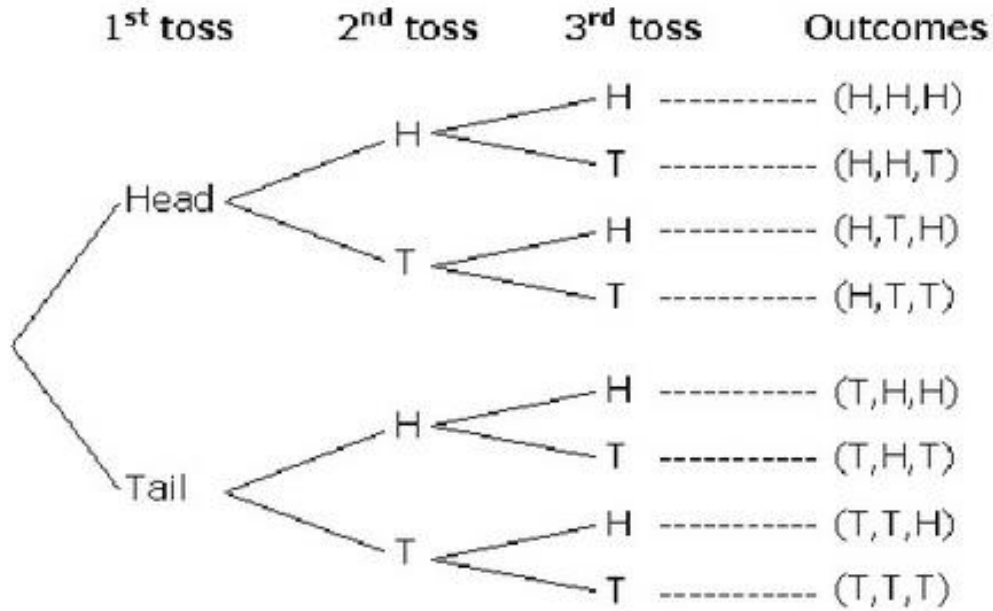
$$6 \times 6 = 36$$

- Example: Two M&Ms are drawn from a dish containing two red and two blue candies. The total number of simple events is:

$$4 \times 3 = 12$$

Tree Diagram:

- A tree diagram can be used to illustrate all possible outcomes
- Example: Tossing of 3 coins



- What is the probability of tossing 3 heads?

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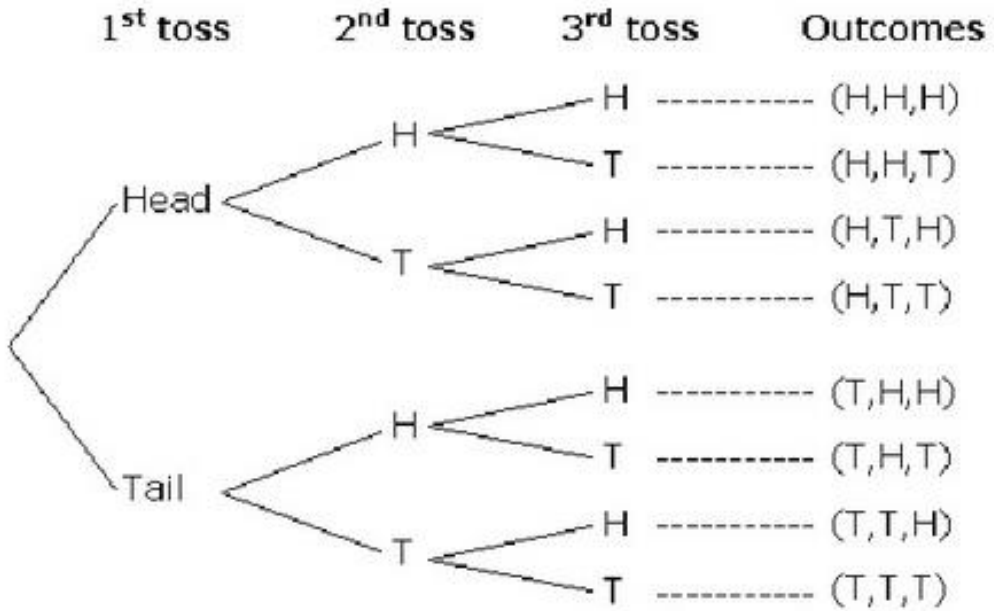

- What is the probability of tossing 3 heads? 1/8 = 0.125

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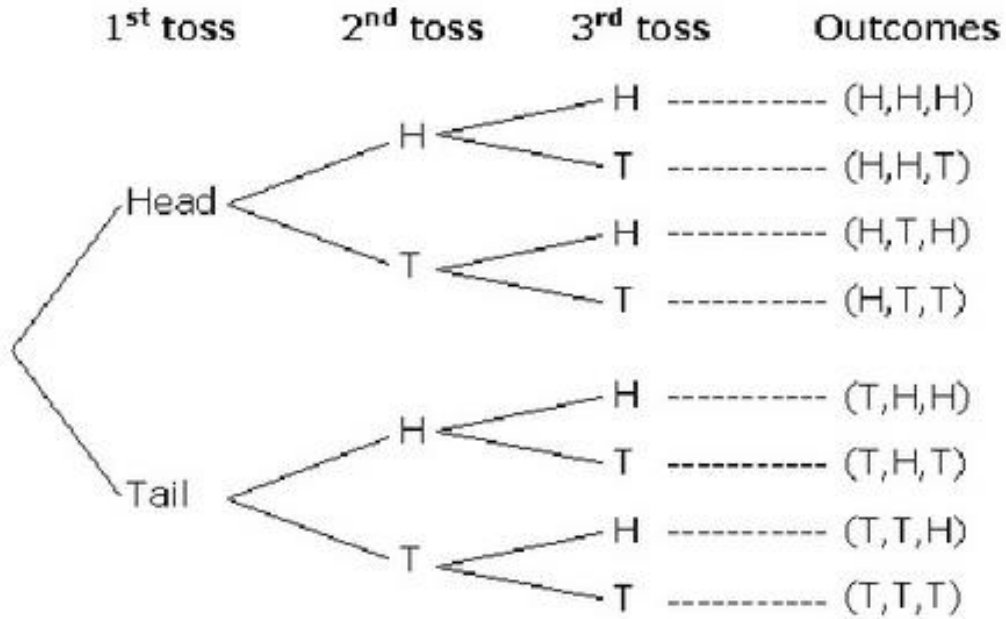

- What is the probability of tossing 2 tails?

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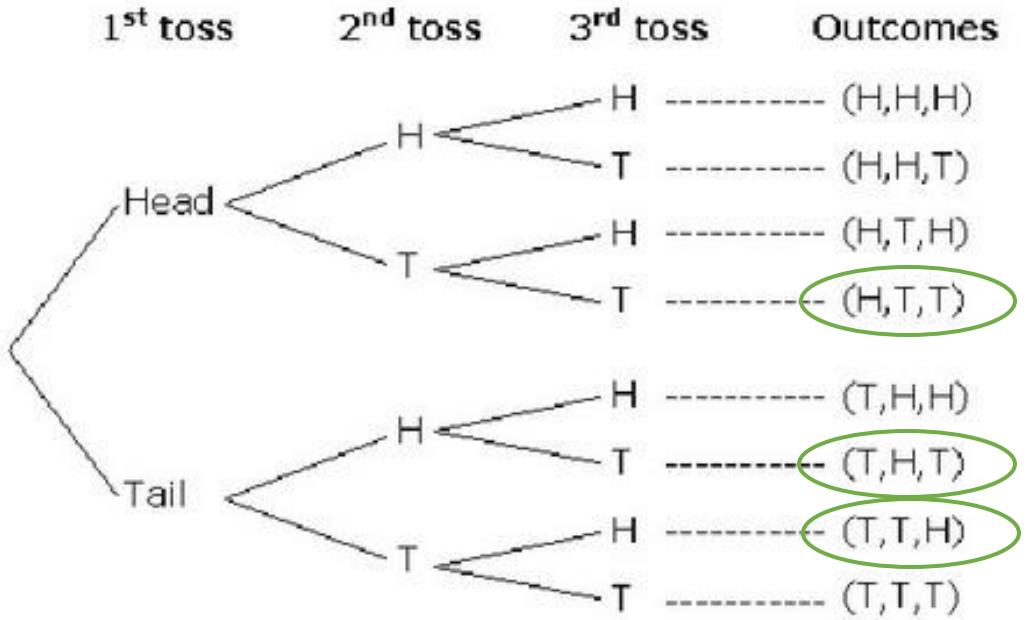

- What is the probability of tossing 2 tails? 3/8 = 0.375

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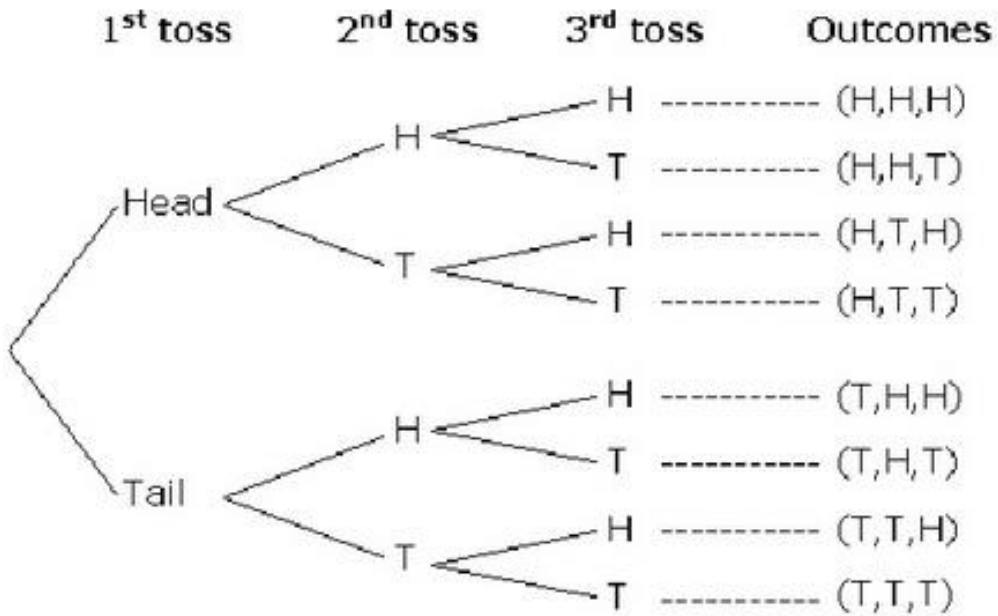

- What is the probability of tossing at least 2 tails?

Tree Diagram:

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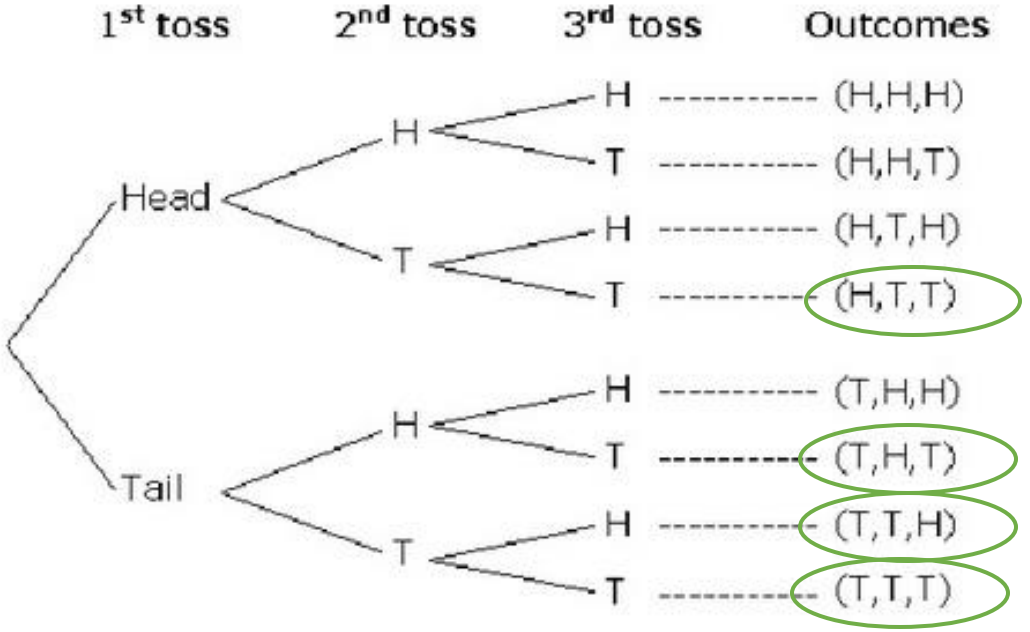
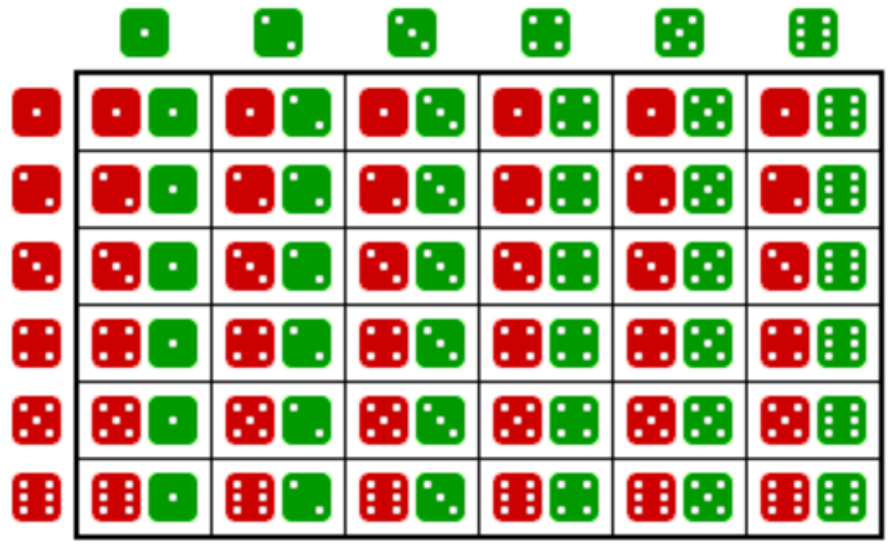
- Example: Tossing of 3 coins



- What is the probability of tossing at least 2 tails? ½ = 0.5

Tossing Two Dice



Tossing Two Dice

- **What is the probability of rolling a sum of 7 with two dice?**
  P(sum of 7) = 6/36 = 1/6 or 0.167