

Week 1 Tuesday Lecture Notes

Prep for Class: read DPV Chapter 0. You can find the prep as well as Lecture notes on Canvas under Pages.

Pictures: at the end of these notes are pictures of key proofs and concepts covered in class.

Lecture 1: Introduction, Asymptotic run time, Big-Oh Notations:

Information about Instructor:

Prof. Xiaoli Fern (Office: Kelley 3073)

Email: xfern@eecs.oregonstate.edu

Office Hours: Tuesday/Thursday after class (3:20-4:00pm)

4 TA's for the class: Juneki, Hamed, Souti, Juan.

Grade Breakdown:

5 Quizzes – 25%

3 group (up to three people per group) implementation Assignments – 25%

1 Midterm (Thursday of 6th week) – 25%

1 Final – 25%

Information about the course:

- Very large class size: 150+ people. Stay focused and ask questions if you need help.
- In Class Quizzes will be at the end of classes on Thursday. From 3:05-3:20.
- You can switch to different group members between assignments.
- Communication is key with group members.
- **Problem sets practice questions:** Will be provided as written homework. However, they **will NOT be graded**. Exam questions are based on problem sets practice questions. These practice questions will be handed out a week in advance before the quiz.
- There will be recitations with TA's the Wednesday before the quiz. (times not yet
- *(See *Lecture 1: Page 4 for complete overview list):*

Algorithm: a term coined to Honor Al Khwarizmi

- He Introduced a general way of solving problems.

Why Study Algorithms?

- They are important for all branches of CS. For example:
- Bioinformatics: comparing two gene sequences and how similar/different they are.
- Algorithms are very satisfying to accomplish.
- Knowledge of Algorithm Design are great for job interviews.

Two Fundamental questions of algorithm:

1: Is it Correct

2: Is it Efficient

Run time of algorithm depends allot on external factors: Processor speed, language, implementation... ect.

- We will abstract away from these fine details.
- Analytical, the run time at the Asymptotic Level.

In Class Example: Largest Power of n Example:

- n is a Number
- c is a Constant

Example: $c + nc + n(n+1)/2 * 2c$.

Conclusion: We don't care about constants, summation and multiplication of n.

All that matters is the largest power of n. The largest power will dominate for large numbers of N and thus creates an estimate for computation time.

Insertion Sort Example: (See Insertion Sort Picture**)**

- Give array a with n numbers. The insertion sort builds the assorted array one element at a time. The runtime of **Insertion Sort** is **not deterministic** (every run can have different runtime speeds)

- **The best case scenario:** the array is already sorted: takes 1 run through.
Big-Oh run time: cn . It only runs n times
- **The Worst Case Scenario:** The list is completely out of order. Takes $n * n$ times through. Big-Oh time: n^2 .

Insertion Sort Example: Sort this array in order from lowest to highest:

- Fastest: Best case example: [1, 2, 3, 4, 5]
- Slowest: Worst case example: [5, 4, 3, 2, 1]

Runtime Depends on input: Use Worst-Case to determine runtime.

We focus on the **“Worst-Case Scenario”**.

- Why not the **“Average-case”**? It is much harder to compute, requires probabilistic analysis, out of the scope of this class.
- Why not the **“Best-case”**? Rarely encountered and so is not relevant to runtime speed for real life simulations.
- Constants and lower order terms **don't matter**.
- Constant factors: $2n$ vs $3n$. **n** is what matters.
- In contrast **n vs n^2** is a big difference.
- You can simplify constant and lower order terms:

For example:

$2n^2 + 2n$ vs $3n^2$. Becomes **n^2** .

$2n + 5n + 4$. Becomes **n** .

Why don't we care? Because we focus on very large n values. (*Big-Oh Notation*)

Asymptotic Growth Example: (See Significant n Picture**)**

Shows that on small numbers the constants have an effect, but the larger the number gets the less important the constants become. This is a main reason we care about Big-Oh Notation.

Big-Oh Notation:

Intuitive meaning: Even though $f(n)$ starts above $g(n)$ (** See Significant n Picture**) if the two functions cross that point $n(0)$. The upper bound is what is important, since $g(n)$ grows faster than $f(n)$.

Simplified: Even though one function may start at a higher value than another it is what happens at very high values and how functions grow that determines significance.

Proof by Substitution Example: (See Proof 1 Picture**)**

Claim: $T(n) = O(n^k)$

Solution: Explains why the lower terms can be ignored; the upper bounded power is all that matters when determining Big-Oh Complexity.

Proof by contradiction example. (See Proof 2 Picture**)**

If this is true then $n^k \leq C * n^{k-1}$ for $n \geq n_0$.

$n \leq c$. which is a contradiction to $n \geq n_0$.

(covered pages 1-16 of Lecture 1 Powerpoint)

Pictures from Lecture 1:

- Significant n.jpg

$f(n) = \log n$ $g(n) = n^2$

Asymptotic Analysis: Big-Oh notation

English definition:
Let $f(n)$ and $g(n)$ be two functions

We say that $f(n) = O(g(n))$ if eventually (for all sufficiently large n), $f(n)$ is bounded above by a constant multiple of $g(n)$

Intuitive Meaning: $f(n)$ grows no faster (asymptotically no worse) than $g(n)$

$f = O(g)$

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- Insertion Sort.jpg

Runtime of insertion sort

```

1. for i ← 1 to n
2.   x ← A[i]
3.   j ← i - 1
4.   while j > 0 and A[j] > x
5.     A[j + 1] ← A[j]
6.     j ← j - 1
7.   end while
8.   A[j + 1] ← x
9. end for
    
```

Line 4-7

- Best case (already sorted): 1
- Worst case (reverse order): $i - 1$ for $A[i]$

$n \cdot c$

$(1 + 2 + \dots + n) \cdot c$
 $\approx n^2 \cdot c$

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Proof 1.jpg

for $n > 1$

Example #1

- $T(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_0 \leq C n^k$
- Claim: $T(n) = O(n^k)$
- Proof:

$a_k \cdot n^k + \dots$

$n \geq n_0$

$C = a_k + 1$

$C = |a_k| + |a_{k-1}| + |a_{k-2}| + \dots + |a_0|$

$a_k n^k + |a_{k-1}| n^k + |a_{k-2}| n^k + \dots + |a_0|$

$a_k n^k + a_{k-1} n^{k-1} + a_{k-2} n^{k-2} + \dots + a_0$

Proof 2.jpg

Big-Oh: formal definition

$f(n) \leq_0 O(g(n))$ if and only if

there exist constants c, n_0 such that

$f(n) \leq c g(n)$

for all $n \geq n_0$

Note: c and n_0 cannot depend on n

$T(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_0$

$\leq |a_k| n^k + |a_{k-1}| n^k + \dots + |a_0| n^k$

$\leq (|a_k| + |a_{k-1}| + |a_{k-2}| + \dots + |a_0|) \cdot n^k$

C

for $n > 1$

End of Week 1 Tuesday Lecture

~Information composed by Notetaker Scott Russell for CS 325 DAS students