

Week 10 Day 2 LECTURE NOTES

Prep:

- Review NP-Completeness (no Lecture Tuesday)
- Implementation 3 Due tonight.
- Reduction and interactive Questions for NP review.
- Practice Q's for Final.

Show NP Completeness:

The image shows handwritten notes on a chalkboard. At the top left, 'NP-hard' is written with a dot next to it. Below it, a Venn diagram shows a large circle labeled 'NP' containing a smaller circle labeled 'P'. A shaded region at the top of the 'NP' circle is labeled 'NPC' (NP-complete). To the right of the diagram, the title 'Show NP-completeness' is written. Below the title, a bullet point states: 'To show a problem Z is NP-complete, we need to show' followed by two numbered steps: '1. Z is in NP' and '2. Starting from one NP-complete problem X and show that $X \leq_p Z$ '. Below these steps, two more bullet points are listed: 'Typically start from a problem that is similar to Z' and 'Construct a poly-time reduction from X to Z and prove the reduction is correct'. At the bottom left, 'Y ∈ NP' is written. In the center, the reduction notation $Y \leq_p X \leq_p Z$ is shown. At the bottom, the reduction $\text{3 SAT} \leq_p \text{IS} \leq_p \text{VC}$ is written, with '3 SAT' underlined and 'NP-C' written below it.

NP-hard.

Show NP-completeness

- To show a problem Z is NP-complete, we need to show
 1. Z is in NP
 2. Starting from one NP-complete problem X and show that $X \leq_p Z$
- Typically start from a problem that is similar to Z
- Construct a poly-time reduction from X to Z and prove the reduction is correct

$Y \in NP$

$Y \leq_p X \leq_p Z$

3 SAT \leq_p IS \leq_p VC.

NP-C

- To show NP complete it must be in NP and in NP-Hard (both)
- If you can verify that solution is in polynomial time then it is NP.
- NP-Hard every problem in NP-Hard is at least as hard as NP.
- If you can find problem x that is in NP complete then you can reduce

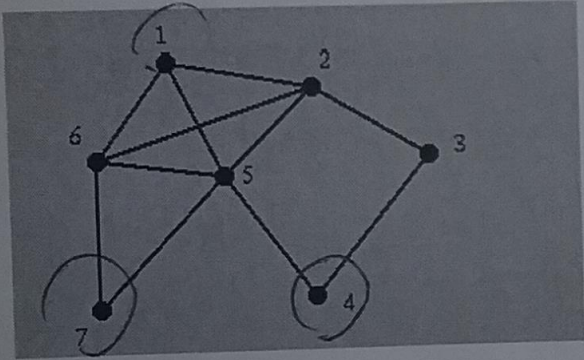
$$Y \leq X \leq Z$$

1. Show that Z is in NP
2. Show that one NP-complete problem X reduces to Z.
 - If asked if $P = NP$ we don't know.

Clique:

Clique

- Given a graph $G = (V, E)$, a clique is a set of nodes $S \subseteq V$ such that every pair of nodes in S there is an edge between them



- Decision problem: given a graph does it contain a clique of size $\geq k$?

3SAT Reduces to Independent Set Reduces to Vertex Cover.

- And since 3SAT is NP-complete then IS and VC are also NP-Complete.
- The right side is at least as hard as the left.
- **Clique:** every node in a set is connected to each other with an edge.
- Problem: is there a clique of size k or larger?

Step 1: check if the clique is NP

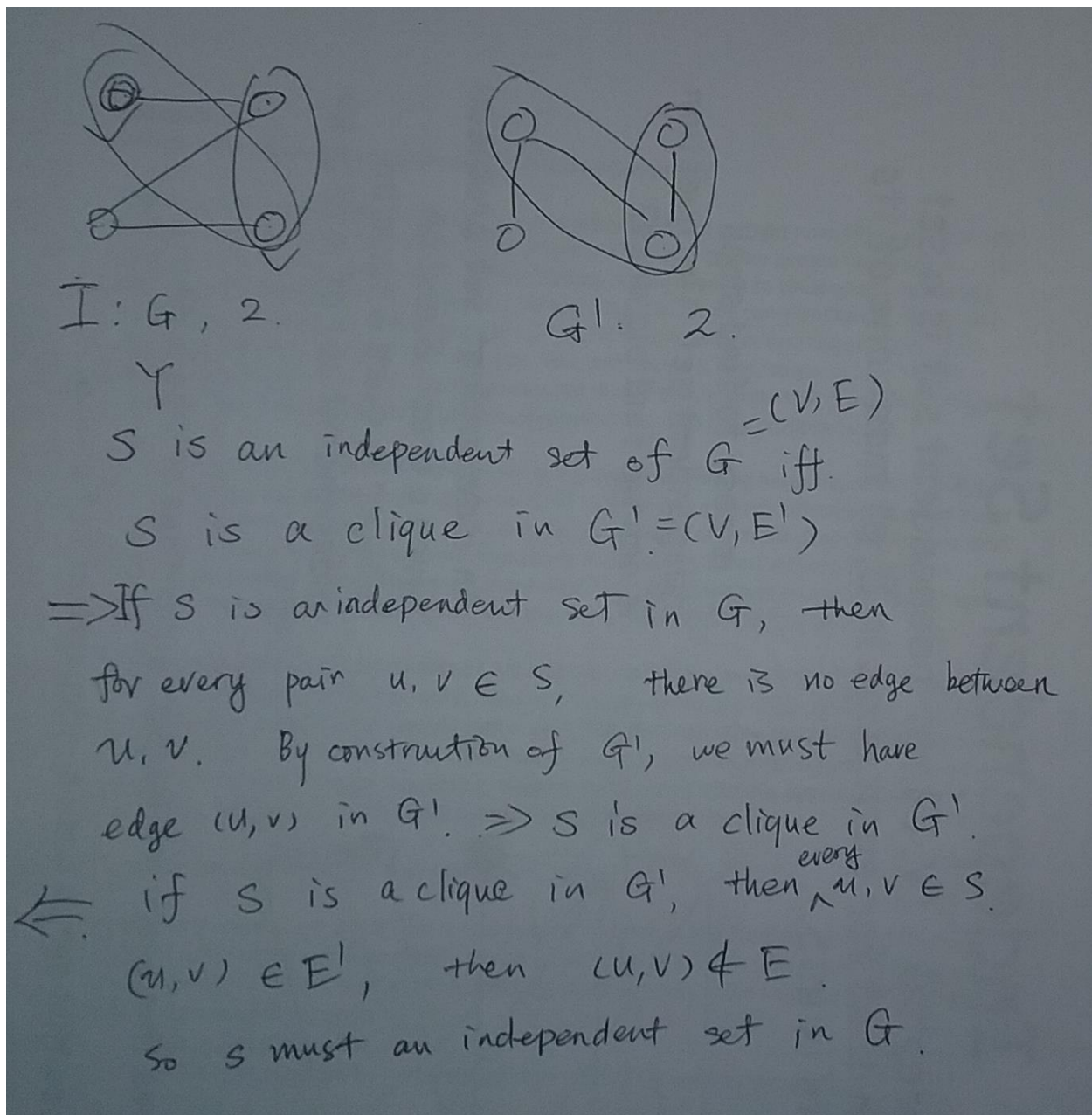
Step 2: Find problem X and show that X reduces to the clique.

- Start from a problem that is as hard and reduce it to clique.

IS \rightarrow [(Solver for Clique)]

To convert from IS to clique. For every edge that exists delete, for every 2 nodes that don't have an edge. Add an edge

See **picture** below on conversion from Independent Set to Clique (and vice versa)



S is an independent set of G iff S is a clique in G .

You can prove from $[IS \rightarrow \text{Clique}]$ or from $[\text{Clique} \rightarrow IS]$

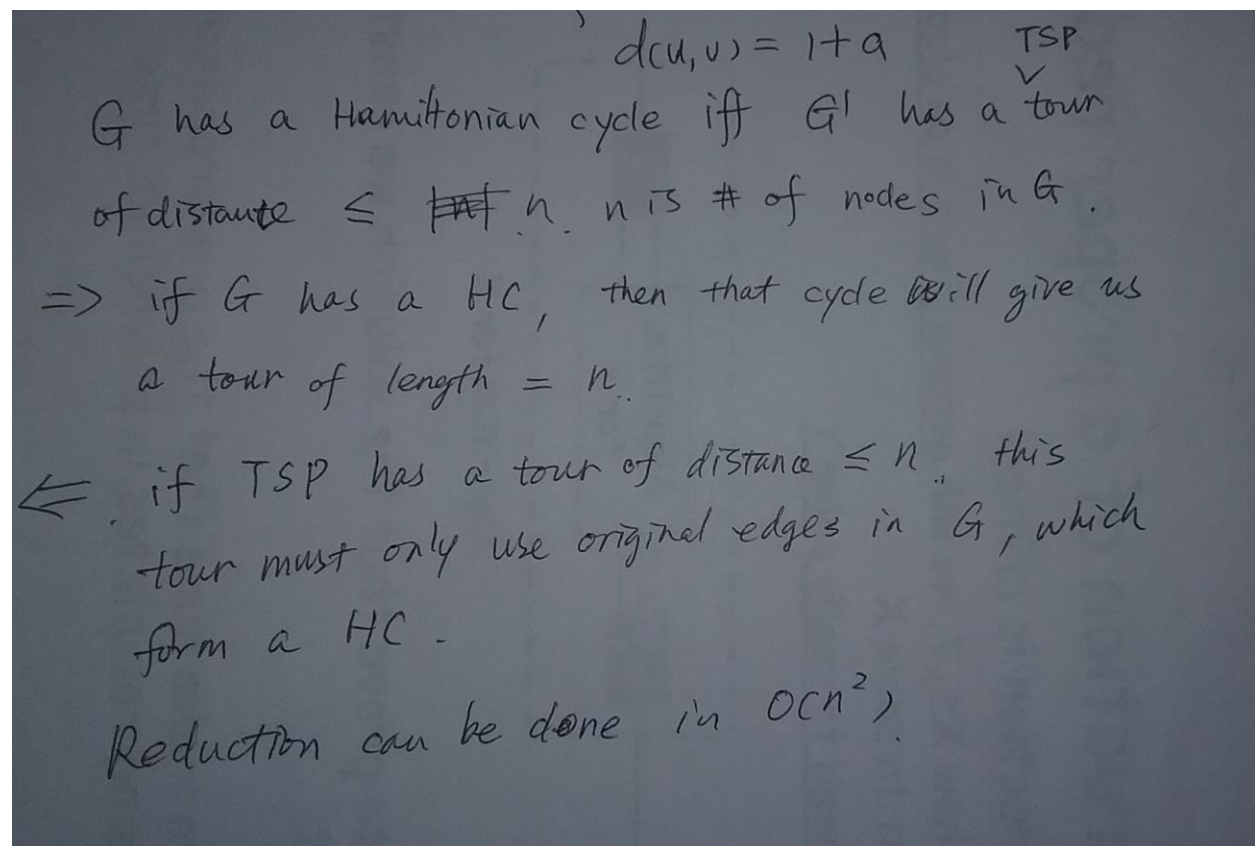
Thus S is a clique in G^1 .

Hamiltonian Cycle: known NP-complete problem

Specifically: Traveling Salesman Problem:

- Reduce Hamiltonian to TSP.

Proof of Hamiltonian Cycle:



Notes of Importance:

- HC reduces to TSP.
- TSP is NP-Hard
- TSP is NP
- TSP is NP-Complete (since it is NP-Hard AND NP)

Quick review of Final Review Question 9:

- Thoroughly read through all information.

Next time:

- Final Time: Thursday March 23rd @2:30-3:50 PM
- Will be at usual lecture location.
- Imp 3 Due tonight (March 16th)
- **Final is non-cumulative.**
- Final Grade: 25% of total grade.

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~Information composed by Notetaker Scott Russell for CS 325 **DAS** student