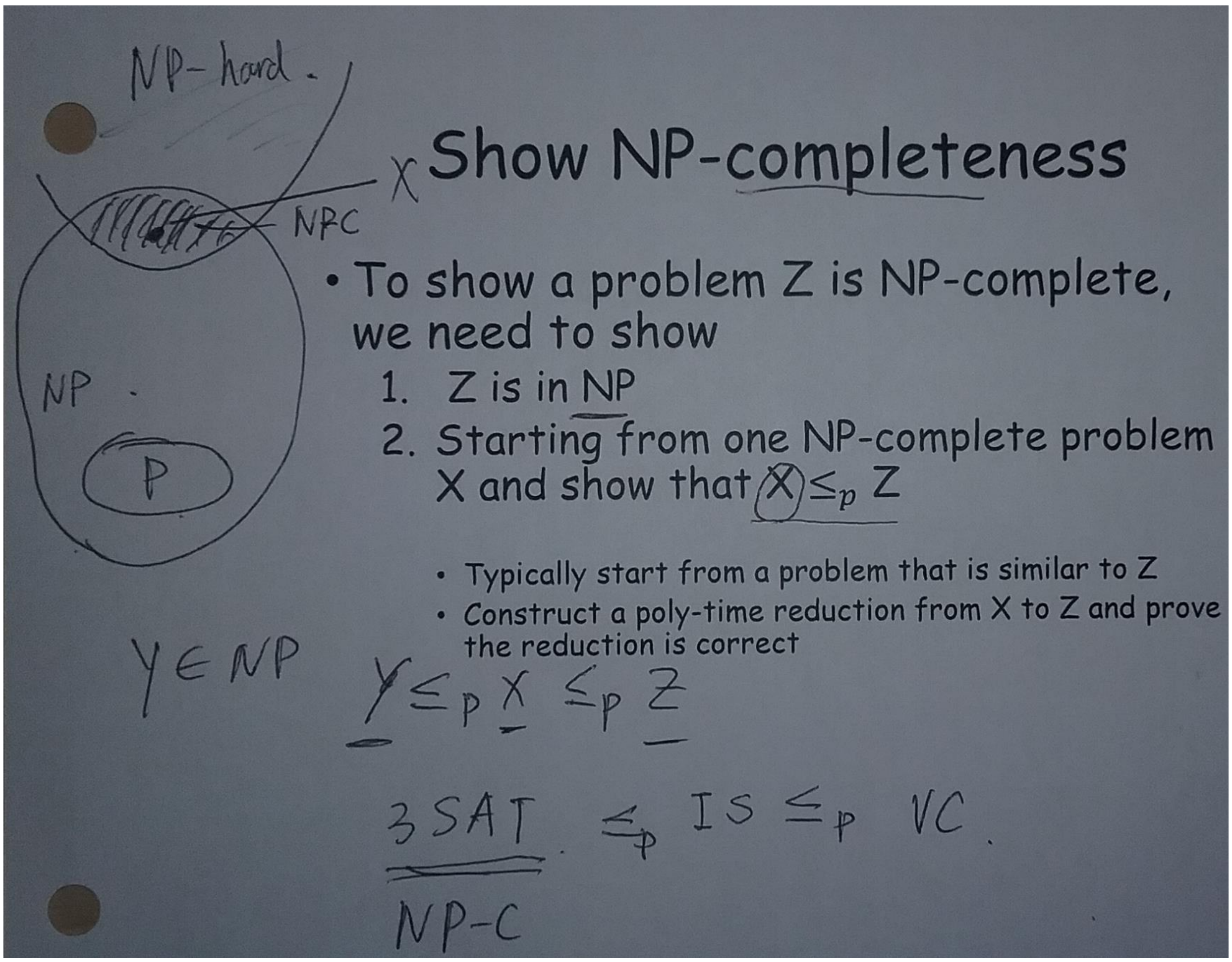
Week 10 Day 2 LECTURE NOTES

Prep:

- Review NP-Completeness (no Lecture Tuesday)
- Implementation 3 Due tonight.
- Reduction and interactive Questions for NP review.
- Practice Q's for Final.

Show NP Completeness:



- To show NP complete it must be in NP and in NP-Hard (both)

- If you can verify that solution is in polynomial time than it is NP.

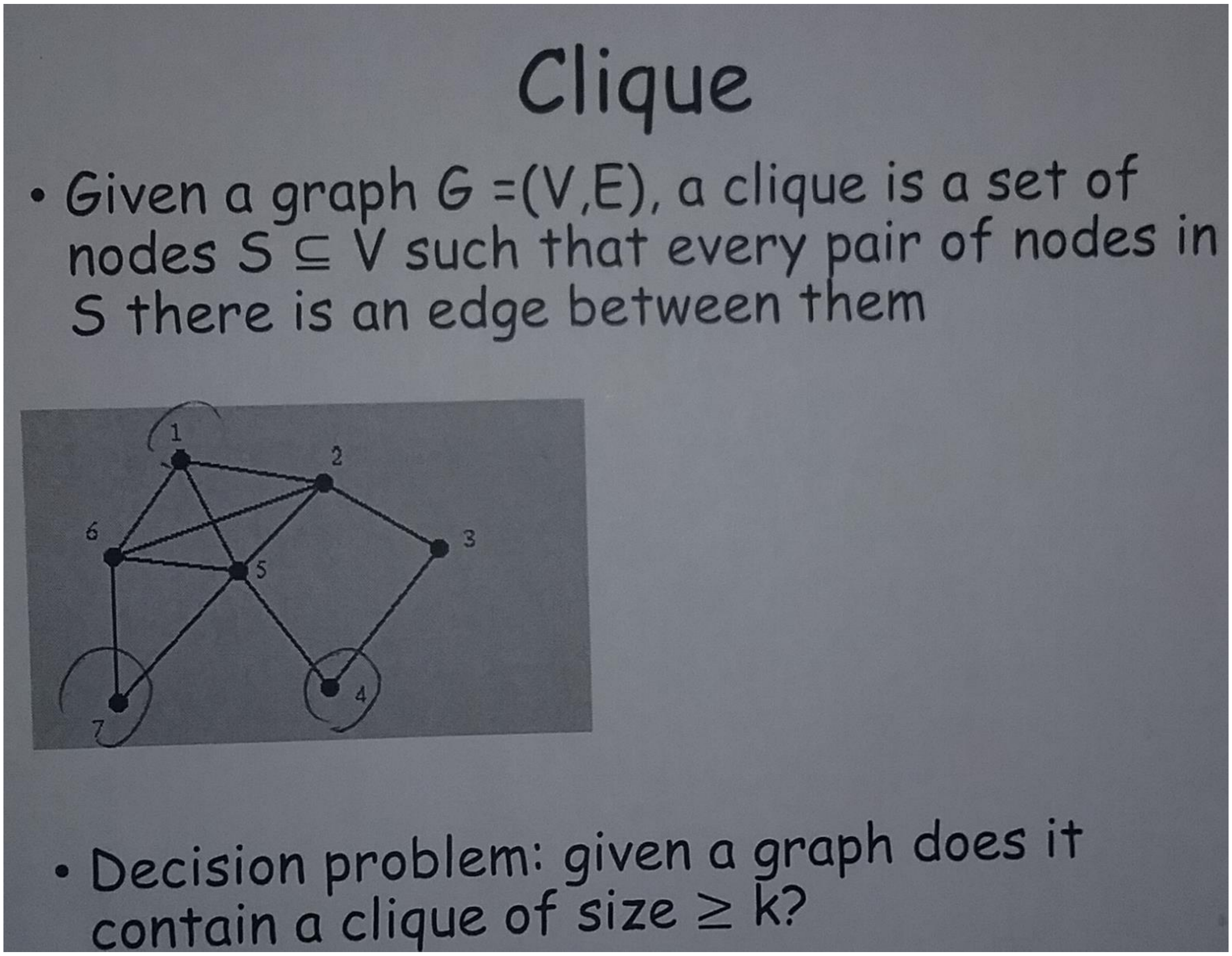
- NP-Hard every problem in NP-Hard is at least as hard as NP.

- If you can find problem x that is in NP complete than you can reduce

Y <= X <= Z

1. Show that Z is in NP
2. Show that one NP-complete problem X reduces to Z.
- If asked if P = NP we don't know.

Clique:



3SAT Reduces to Independent Set Reduces to Vertex Cover.

- And since 3SAT is NP-complete than IS and VC are also NP-Complete.
- The right side is at least as hard as the left.
- **Clique**: every node in a set is connected to each other with an edge.
- Problem: is there a clique of size k or larger?

Step 1: check if the clique is NP

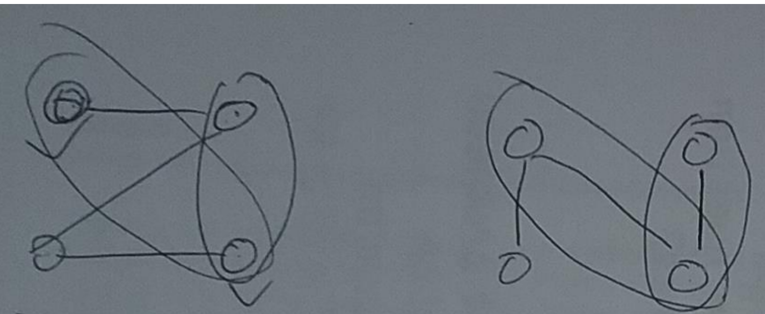
Step 2: Find problem X and show that X reduces to the clique.

- Start from a problem that is as hard and reduce it to clique.

IS → [(Solver for Clique)]

To convert from IS to clique. For every edge that exists delete, for every 2 nodes that don't have an edge. Add an edge

See ***picture*** below on conversion from Independent Set to Clique (and vice versa)



I: G, 2

G': 2.

Y

S is an independent set of G $=(V, E)$ iff

S is a clique in $G' = (V, E')$

$\Rightarrow$ If S is an independent set in G, then for every pair $u, v \in S$, there is no edge between u, v. By construction of G', we must have edge (u, v) in G'. $\Rightarrow$ S is a clique in G'.

$\Leftarrow$ if S is a clique in G', then every $u, v \in S$ $(u, v) \in E'$, then $(u, v) \notin E$.

So S must an independent set in G.

S is an independent set of G iff S is a clique in G.

You can prove from [IS → Clique] or from [Clique → IS]

Thus S is a clique in G^1.

Hamiltonian Cycle: known NP-complete problem

Specifically: Traveling Salesman Problem:

- Reduce Hamiltonian to TSP.

Proof of Hamiltonian Cycle:

$d(u,v) = 1+a$ TSP

G has a Hamiltonian cycle iff G' has a tour of distance $\leq n$. n is # of nodes in G.

$\Rightarrow$ if G has a HC, then that cycle will give us a tour of length $= n$.

$\Leftarrow$ if TSP has a tour of distance $\leq n$, this tour must only use original edges in G, which form a HC.

Reduction can be done in $O(n^2)$.

Notes of Importance:

- HC reduces to TSP.
- TSP is NP-Hard
- TSP is NP
- TSP is NP-Complete (since it is NP-Hard AND NP)

Quick review of Final Review Question 9:

- Thoroughly read through all information.

Next time:

- Final Time: Thursday March 23rd @2:30-3:50 PM
- Will be at usual lecture location.
- Imp 3 Due tonight (March 16th)
- **Final is non-cumulative.**
- Final Grade: 25% of total grade.

Week 10 Day 2 LECTURE NOTES

~Information composed by Notetaker Scott Russell for CS 325 **DAS** student