

Week 9 Day 2 Lecture Notes

Prep:

- Quiz 5 Today. (Linear Programming and Reduction)
- Implementation 3 due in a week.
- Read DPV 8.1-8.3

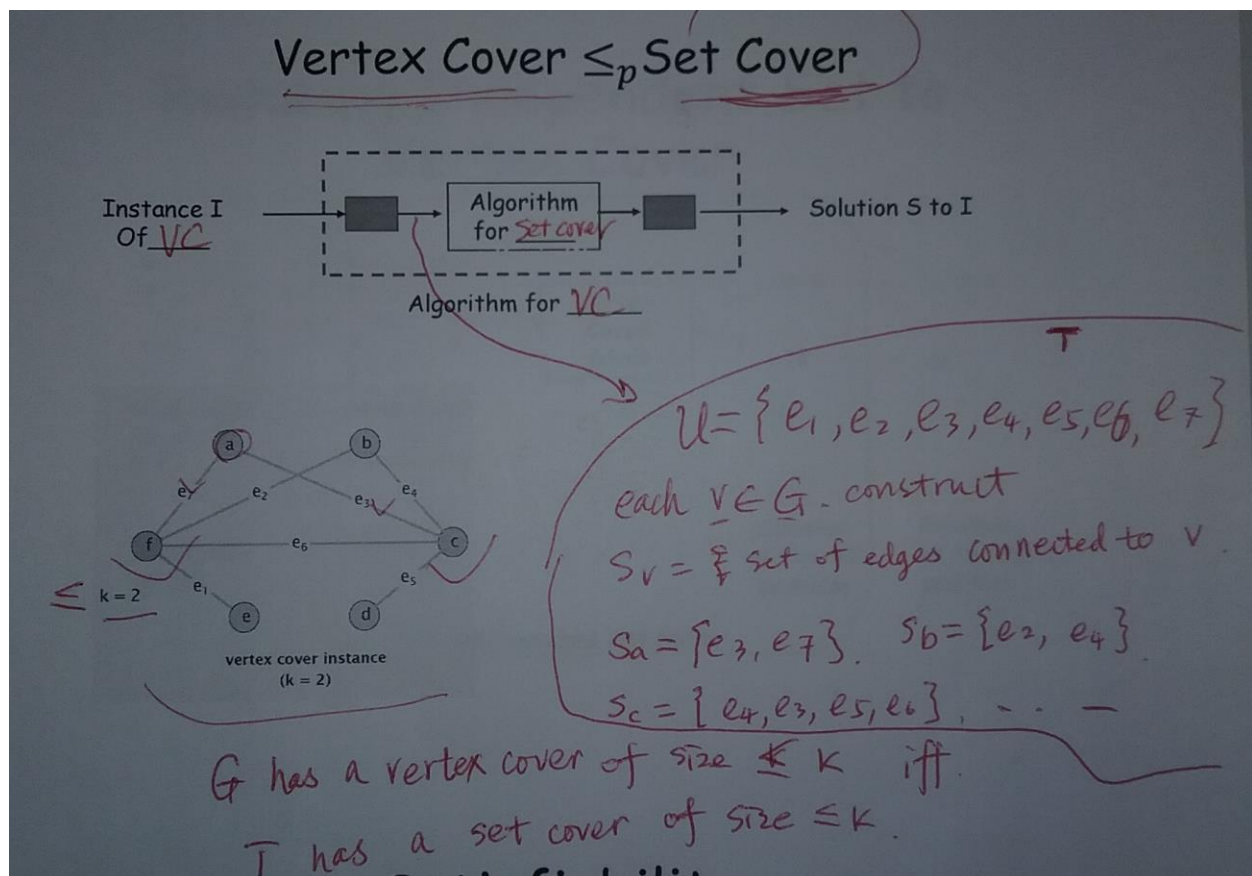
Set Cover:

Given a Degree and a set of classes that provide pre-requisites for the Degree what is the fewest # of classes you can take to complete the degree?

Vertex Cover: Every Edge has at least one end in the selected set. (Find **Minimum** # of nodes)

Independent Set: Each edge, at most has only one end point in the set. (find **Maximum** # of nodes)

Vertex Cover \leq Set Cover



- Vertex Cover Reduces to Set Cover
- For each node n in the graph. Construct a set of edges e that are connected to that node n .
- Write out a set of connecting edges for every node.
- If there's a Vertex Cover then there's also a Set Cover.
- G has a Vertex cover of size $\leq K$ if T has a set cover of size $\leq k$
- Can reduce Vertex Cover to Set Cover but not the other way.

Satisfiability: (SAT.)

Satisfiability

- Literal: A Boolean variable or its negation x_i or $\neg x_i$
- Clause: A disjunction of literals $C_j = x_1 \vee \neg x_2 \vee x_3$
- Conjunctive normal form (CNF): A Boolean formula that is a conjunction of clauses $\Phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$
- SAT. Given a CNF formula Φ , does it have a satisfying truth assignment?
- 3-SAT: SAT where each clause contains exactly 3 literals (and each literal corresponds to a different variable).

$$\Phi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$$

$x_1 = x_2 = x_3 = x_4 = \text{True}$

Literal: Boolean variable or its negative x or $\neg x$

Clause: A disjunction of Literals:

Conjunctive Normal Form (CNF): a formula that consists of clauses:

3-SAT: meaning that each clause has 3 Literals.

Reducing 3-SAT into Independent Set:

3-Sat \leq_p Independent Set

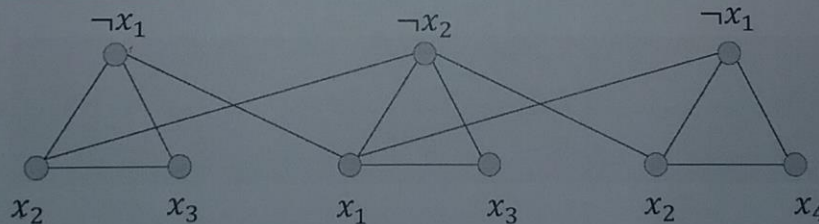
Construction from ϕ to (G, k) :

For each clause, we construct 3 nodes, one for each literal

Connect the 3 nodes in a clause into a triangle

Connect literal to its negations

$k = \#$ of clauses in ϕ



$$\phi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_2 \vee \neg x_4 \vee x_1)$$

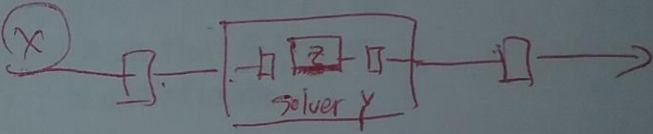
3 literals:

- Drawing the Literals of the 3-SAT (Picture)
- Connect all elements within a Clause.
- Connect all Literals with their Negations across Clauses.
- Every clause contributes one Literal from Each Triangle (Clause)
- You can only pick one Literal for each Independent Set.
- If you pick the same Literal for different Clauses they have to both be true or both false.
- We can construct this graph in $(3n)^2$. Polynomial time.

Basic Reduction Strategies:

Basic reduction strategies

- Simple equivalence
 - $\text{Independent Set} \equiv_p \text{Vertex cover}$
- Special case to a more general case
 - $\text{Vertex cover} \leq_p \text{set cover}$
- Encoding with gadgets
 - $3\text{SAT} \leq_p \text{Independent Set}$
- Transitivity.
 - If $X \leq_p Y$ and $Y \leq_p Z$, then $X \leq_p Z$
 - $3\text{SAT} \leq_p \text{Independent Set} \leq_p \text{Vertex Cover} \leq_p \text{Set Cover}$



- **Transitivity:**
- If X reduces to Y and Y Reduces to Z. Than X reduces to Z.
- Order of reduction: $3\text{SAT} \leq \text{Independent Set} \leq \text{Vertex Cover} \leq \text{Set Cover}$

Search Vs. Optimization:

Optimization: Find the smallest Vertex Cover.

Search: Find a Vertex Cover of Size $\leq k$

Decision Problem: Is there a vertex Cover of Size $\leq k$.

Next Time:

- No Lecture Tuesday (There will be an online lecture uploaded)
- Next Thursday possible Final review if we get through all material.
- Implementation 2 Grades have been posted.
- Implementation 3 Due next Thursday.

- End of Week 9 Day 2 Notes

- ~Information composed by Notetaker Scott Russell for CS 325 **DAS** students