IEEE Floating-Point Format

	single: 8 bits double: 11 bits	single: 23 bits double: 52 bits
S	Exponent	Fraction/mantissa

1.5×2^{5}

Fraction

exponent

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- S: sign bit (0 ⇒ non-negative, 1 ⇒ negative)
- Normalize significand: 1.0 ≤ |significand| < 2.0
  - Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
  - Significand is Fraction with the "1." restored
- Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned
  - Single: Bias = 127; Double: Bias = 1023

Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
  - Exponent: 00000001
    $\Rightarrow$ actual exponent = 1 – 127 = –126
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
  - exponent: 11111110
    $\Rightarrow$ actual exponent = 254 – 127 = +127
  - Fraction: 111…11 $\Rightarrow$ significand $\approx$ 2.0
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

Double-Precision Range

- Exponents 0000…00 and 1111…11 reserved
- Smallest value
  - Exponent: 00000000001
    $\Rightarrow$ actual exponent = 1 – 1023 = –1022
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
  - Exponent: 11111111110
    $\Rightarrow$ actual exponent = 2046 – 1023 = +1023
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

Floating-Point Precision

- Relative precision
  - all fraction bits are significant
  - Single: approx 2^{-23}
    - Equivalent to $23 \times \log_{10}2 \approx 23 \times 0.3 \approx 6$ decimal digits of precision
  - Double: approx 2^{-52}
    - Equivalent to $52 \times \log_{10}2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

Floating-Point Example

- Represent –0.75
  - $-0.75 = (-1)^1 \times 1.1_2 \times 2^{-1}$
  - S = 1
  - Fraction = $1000\ldots00_2$
  - Exponent = –1 + Bias
    - Single: $-1 + 127 = 126 = 01111110_2$
    - Double: $-1 + 1023 = 1022 = 01111111110_2$
- Single: 1011111101000…00 = 32 bits
- Double: 1011111111101000…00

sign = − → 1

-0.11000

$1 \quad \tfrac{1}{2} \quad \tfrac{1}{4}$

1.1×2^{-1}

implied

sign

Exponent

Fraction/mantissa

Floating-Point Example

- What number is represented by the single-precision float

  11000000101000…00

  - S = 1
  - Fraction = $01000\ldots00_2$
  - Exponent = $10000001_2 = 129$
- $x = (-1)^1 \times (1 + 01_2) \times 2^{(129 - 127)}$

  $= (-1) \times 1.25 \times 2^2$

  $= -5.0$

Floating-Point Operations

- Consider decimal floating point operations
  - Addition/subtraction

b could be base 10, base 2, etc

$$a \times b^e + d \times b^e = (a+d) \times b^e$$

$$1.5 \times 10^5 + -2.6 \times 10^3 = 1.5 \times 10^5 + -0.026 \times 10^5 = 1.474 \times 10^5$$

  - Multiplication/division

$$(a \times b^c) \div (d \times b^e) = (a \div d) \times b^{(c-e)}$$

$$1.5 \times 10^5 \div -2.6 \times 10^3 = \frac{1.5}{-2.6} \times 10^{(5-3)}$$

for division, subtract exponents.
For multiplication, add exponents.

- How would this apply in binary?

Same exact idea, except the math is performed in base 2.

See next slide for example of addition.

Floating-Point Operations

Dec: $0.5 + (-)0.4375$

Binary:

$1.000 \times 2^{-1} + (-)\ 0.0111 \times 2^{0}$ $\quad -1.110 \times 2^{-2}$

Shift to same exponent $\quad -0.1110 \times 2^{-1}$

$$\begin{array}{r} 1.0000 \\ -\ 0.1110 \\ \hline 0.0010 \times 2^{-1} \end{array}$$

1.0×2^{-4}

$-4 + 127 = 123$

IEEE Representation:

0 01111011 0000

Drop the leading "1" $\quad$ 23 zeros →

$1.0 \rightarrow .00000$

Using 2's complement:

001110 → 110001 → 110010

Notice I added an extra sign bit

$$\begin{array}{rl} 01.0000 & \times 2^{-1} \\ +\ 11.0010 & \times 2^{-1} \\ \hline 00.0010 & \times 2^{-1} \end{array}$$

Ignore carry bit $\quad = 1.0 \times 2^{-4}$

MIPS R-format Instructions

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

- Instruction fields
  - op: operation code (opcode)
  - rs: first source register number
  - rt: second source register number
  - rd: destination register number
  - shamt: shift amount (00000 for now)
  - funct: function code (extends opcode)

R-format Example

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

```
add $t0, $s1, $s2
```

special	$s1	$s2	$t0	0	add
0	17	18	8	0	32
000000	10001	10010	01000	00000	100000

$00000010001100100100000000100000_2 = 02324020_{16}$

MIPS I-format Instructions

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

- Immediate arithmetic and load/store instructions
  - rt: destination or source register number
  - Constant: $–2^{15}$ to $+2^{15} – 1$
  - Address: offset added to base address in rs
- *Design Principle 4:* Good design demands good compromises
  - Different formats complicate decoding, but allow 32-bit instructions uniformly
  - Keep formats as similar as possible

Logical Operations

- Instructions for bitwise manipulation

Operation	C	Java	MIPS
Shift left	<<	<<	`sll`
Shift right	>>	>>>	`srl`
Bitwise AND	&	&	`and, andi`
Bitwise OR	|	|	`or, ori`
Bitwise NOT	~	~	`nor`

- Useful for extracting and inserting groups of bits in a word

Shift Operations

op	rs	rt	rd	shamt	funct
6 bits	5 bits	5 bits	5 bits	5 bits	6 bits

- shamt: how many positions to shift
- Shift left logical
  - Shift left and fill with 0 bits
  - `sll` by *i* bits multiplies by 2^i
- Shift right logical
  - Shift right and fill with 0 bits
  - `srl` by *i* bits divides by 2^i (unsigned only)

AND Operations

- Useful to mask bits in a word
  - Select some bits, clear others to 0

```
and $t0, $t1, $t2
```

$t2	0000 0000 0000 0000 0000 1101 1100 0000
$t1	0000 0000 0000 0000 0011 1100 0000 0000
$t0	0000 0000 0000 0000 0000 1100 0000 0000

OR Operations

- Useful to include bits in a word
  - Set some bits to 1, leave others unchanged

```
or $t0, $t1, $t2
```

\$t2	0000 0000 0000 0000 0000 1101 1100 0000
\$t1	0000 0000 0000 0000 0011 1100 0000 0000
\$t0	0000 0000 0000 0000 0011 1101 1100 0000

NOT Operations

- Useful to invert bits in a word
  - Change 0 to 1, and 1 to 0
- MIPS has NOR 3-operand instruction
  - a NOR b == NOT (a OR b)

```
nor $t0, $t1, $zero
```

Register 0: always read as zero

\$t1	0000 0000 0000 0000 0011 1100 0000 0000
\$t0	1111 1111 1111 1111 1100 0011 1111 1111

32-bit Constants

- Most constants are small
  - 16-bit immediate is sufficient
- For the occasional 32-bit constant

  `lui rt, constant`

  - Copies 16-bit constant to left 16 bits of rt
  - Clears right 16 bits of rt to 0

`lhi $s0, 61`	0000 0000 0111 1101	0000 0000 0000 0000
`ori $s0, $s0, 2304`	0000 0000 0111 1101	0000 1001 0000 0000

§2.10 MIPS Addressing for 32-Bit Immediates and Addresses

Branch Addressing

- Branch instructions specify
  - Opcode, two registers, target address
- Most branch targets are near branch
  - Forward or backward

op	rs	rt	constant or address
6 bits	5 bits	5 bits	16 bits

- PC-relative addressing
  - Target address = PC + offset × 4
  - PC already incremented by 4 by this time

Jump Addressing

Recall: × 4 = left shift of 2

- Jump (`j` and `jal`) targets could be anywhere in text segment
  - Encode full address in instruction

op	address
6 bits	26 bits

- (Pseudo)Direct jump addressing
  - Target address = $PC_{31\ldots28}$: (address × 4)

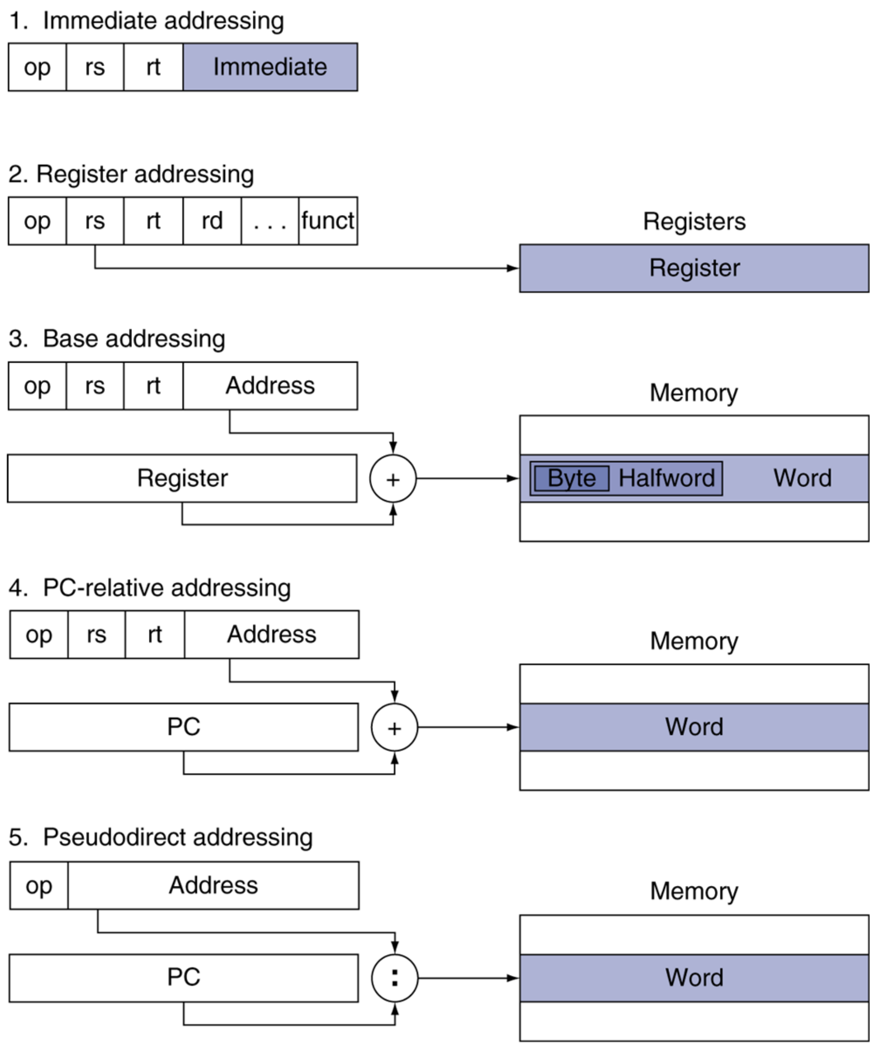
31 28 27 2 00

Branching Far Away

- If branch target is too far to encode with 16-bit offset, assembler rewrites the code
- Example

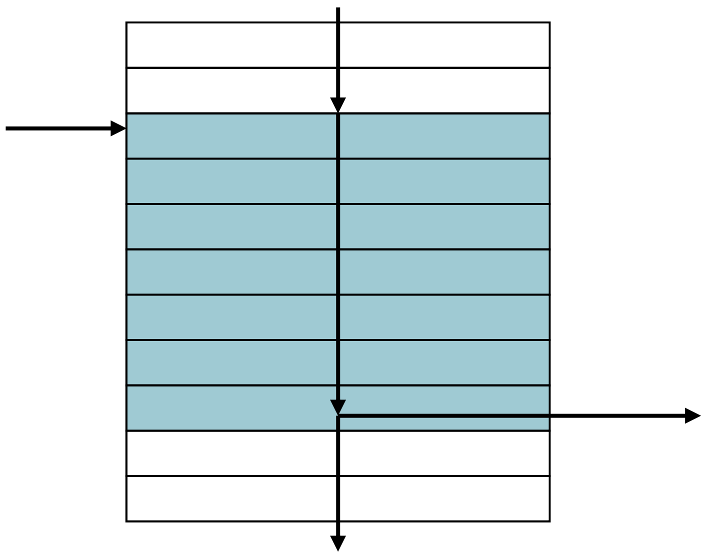
```
        beq $s0,$s1, L1
              ↓
        bne $s0,$s1, L2
        j L1
    L2: …
```

Addressing Mode Summary



Basic Blocks

- A basic block is a sequence of instructions with
  - No embedded branches (except at end)
  - No branch targets (except at beginning)



- A compiler identifies basic blocks for optimization
- An advanced processor can accelerate execution of basic blocks

Procedure Calling

- Steps required
  1. Place parameters in registers
  2. Transfer control to procedure
  3. Acquire storage for procedure
  4. Perform procedure's operations
  5. Place result in register for caller
  6. Return to place of call

Register Usage

- $a0 – $a3: arguments (reg's 4 – 7)
- $v0, $v1: result values (reg's 2 and 3)
- $t0 – $t9: temporaries
  - Can be overwritten by callee
- $s0 – $s7: saved
  - Must be saved/restored by callee
- $gp: global pointer for static data (reg 28)
- $sp: stack pointer (reg 29)
- $fp: frame pointer (reg 30)
- $ra: return address (reg 31)

Procedure Call Instructions

- Procedure call: jump and link

  ```
  jal ProcedureLabel
  ```

  - Address of following instruction put in $ra
  - Jumps to target address
- Procedure return: jump register

  ```
  jr $ra
  ```

  - Copies $ra to program counter
  - Can also be used for computed jumps
    - e.g., for case/switch statements

Leaf Procedure Example

- C code:

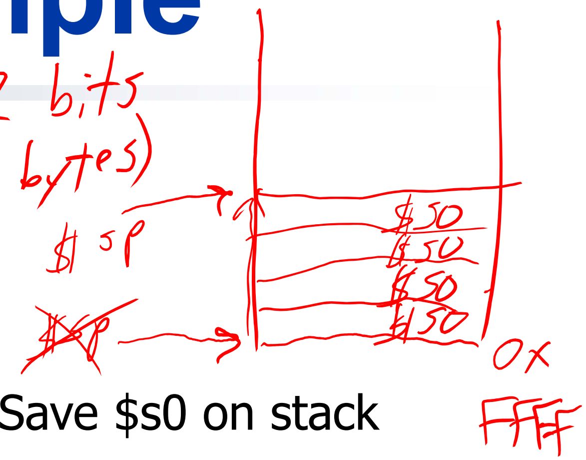
```
int leaf_example (int g, h, i, j)
{ int f;
  f = (g + h) - (i + j);
  return f;
}
```

  - Arguments g, …, j in $a0, …, $a3
  - f in $s0 (hence, need to save $s0 on stack)
  - Result in $v0

Leaf Procedure Example

$s0 is 32 bits
(4 bytes)

- MIPS code:



`leaf_example:`	
`addi $sp, $sp, -4` `sw   $s0, 0($sp)`	Save $s0 on stack

`add  $t0, $a0, $a1`
`add  $t1, $a2, $a3`
`sub  $s0, $t0, $t1`
	Procedure body

`add  $v0, $s0, $zero`	Result
`lw   $s0, 0($sp)` `addi $sp, $sp, 4`	Restore $s0
`jr   $ra`	Return