Unsigned Binary Integers

- Given an n-bit number

$$x = x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

- Range: 0 to $+2^n - 1$
- Example
  - $0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 1011_2$
    $= 0 + \ldots + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$
    $= 0 + \ldots + 8 + 0 + 2 + 1 = 11_{10}$
- Using 32 bits
  - 0 to +4,294,967,295

2s-Complement Signed Integers

- Given an n-bit number

$$x = -x_{n-1}2^{n-1} + x_{n-2}2^{n-2} + \cdots + x_1 2^1 + x_0 2^0$$

- Range: -2^{n-1} to $+2^{n-1} - 1$
- Example
  - $1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1100_2$
    $= -1\times2^{31} + 1\times2^{30} + \ldots + 1\times2^2 + 0\times2^1 + 0\times2^0$
    $= -2{,}147{,}483{,}648 + 2{,}147{,}483{,}644 = -4_{10}$
- Using 32 bits
  - –2,147,483,648 to +2,147,483,647

2s-Complement Signed Integers

- Bit 31 is sign bit
  - 1 for negative numbers
  - 0 for non-negative numbers
- $-(-2^{n-1})$ can't be represented
- Non-negative numbers have the same unsigned and 2s-complement representation
- Some specific numbers
  - 0: 0000 0000 … 0000
  - –1: 1111 1111 … 1111
  - Most-negative: 1000 0000 … 0000
  - Most-positive: 0111 1111 … 1111

Signed Negation

$A - B = A + (-B)$

- Complement and add 1
  - Complement means 1 → 0, 0 → 1

$$x + \bar{x} = 1111...111_2 = -1$$

$$\bar{x} + 1 = -x$$

- Example: negate +2
  - +2 = 0000 0000 … 0010_2
  - –2 = 1111 1111 … 1101_2 + 1
    = 1111 1111 … 1110_2

Sign Extension

- Representing a number using more bits
  - Preserve the numeric value
- In MIPS instruction set
  - `addi`: extend immediate value
  - `lb`, `lh`: extend loaded byte/halfword
  - `beq`, `bne`: extend the displacement
- Replicate the sign bit to the left
  - c.f. unsigned values: extend with 0s
- Examples: 8-bit to 16-bit
  - +2: 0000 0010 => 0000 0000 0000 0010
  - –2: 1111 1110 => 1111 1111 1111 1110

Representing Instructions

- Instructions are encoded in binary
  - Called machine code
- MIPS instructions
  - Encoded as 32-bit instruction words
  - Small number of formats encoding operation code (opcode), register numbers, …
  - Regularity!
- Register numbers
  - $t0 – $t7 are reg's 8 – 15
  - $t8 – $t9 are reg's 24 – 25
  - $s0 – $s7 are reg's 16 – 23

Assembler Pseudoinstructions

- Most assembler instructions represent machine instructions one-to-one
- Pseudoinstructions: figments of the assembler's imagination

```
move $t0, $t1      →  add $t0, $zero, $t1
blt $t0, $t1, L    →  slt $at, $t0, $t1
                      bne $at, $zero, L
```

  - $at (register 1): assembler temporary

`li` is a combination of `lui` and `ori`

Conditional Operations

- Branch to a labeled instruction if a condition is true
  - Otherwise, continue sequentially
- `beq rs, rt, L1`
  - if (rs == rt) branch to instruction labeled L1;
- `bne rs, rt, L1`
  - if (rs != rt) branch to instruction labeled L1;
- `j L1`
  - unconditional jump to instruction labeled L1

More Conditional Operations

- Set result to 1 if a condition is true
  - Otherwise, set to 0
- `slt rd, rs, rt`
  - if (rs < rt) rd = 1; else rd = 0;
- `slti rt, rs, constant`
  - if (rs < constant) rt = 1; else rt = 0;
- Use in combination with beq, bne

```
slt $t0, $s1, $s2  # if ($s1 < $s2)
bne $t0, $zero, L  #   branch to L
```

Branch Instruction Design

- Why not `blt`, `bge`, etc?
- Hardware for <, ≥, … slower than =, ≠
  - Combining with branch involves more work per instruction, requiring a slower clock
  - All instructions penalized!
- `beq` and `bne` are the common case
- This is a good design compromise

Signed vs. Unsigned

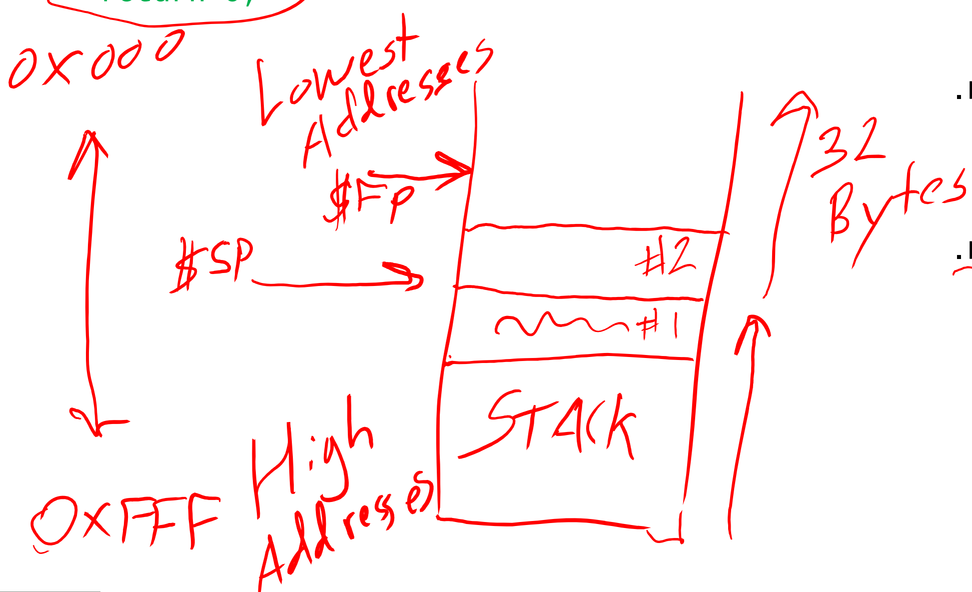
- Signed comparison: `slt`, `slti`
- Unsigned comparison: `sltu`, `sltui`
- Example
  - $s0 = 1111 1111 1111 1111 1111 1111 1111 1111
  - $s1 = 0000 0000 0000 0000 0000 0000 0000 0001
  - `slt  $t0, $s0, $s1  # signed`
    - $-1 < +1 \Rightarrow$ $t0 = 1
  - `sltu $t0, $s0, $s1  # unsigned`
    - $+4{,}294{,}967{,}295 > +1 \Rightarrow$ $t0 = 0

Assembly Review

```c
int main() {
        int num_a = 25;
        int num_b = 75;

        while (num_a != num_b) {
                if (num_a > num_b)
                        num_a = num_a - num_b;
                else
                        num_b = num_b - num_a;
        }

        return 0;
}
```




```
main:
        addiu   $sp,$sp,-32
        sd      $fp,24($sp)
        move    $fp,$sp
        li      $2,25           # 0x19
        sw      $2,0($fp)   25
        li      $2,75           # 0x4b
        sw      $2,4($fp)   75
        j       .L2
        nop
.L4:
        lw      $3,0($fp)
        lw      $2,4($fp)
        slt     $2,$2,$3
        beq     $2,$0,.L3
        nop

        lw      $3,0($fp)
        lw      $2,4($fp)
        subu    $2,$3,$2
        sw      $2,0($fp)
        j       .L2
        nop
.L3:
        lw      $3,4($fp)
        lw      $2,0($fp)
        subu    $2,$3,$2
        sw      $2,4($fp)
.L2:
        lw      $3,0($fp)   25
        lw      $2,4($fp)   75
        bne     $3,$2,.L4
        nop

        move    $2,$0
        move    $sp,$fp
        ld      $fp,24($sp)
        addiu   $sp,$sp,32
        j       $31
        nop
```

Floating Point

$254 = 2.54 \times 10^2$

- Representation for non-integral numbers
  - Including very small and very large numbers
- Like scientific notation
  - -2.34×10^{56} ← normalized
  - $+0.002 \times 10^{-4}$ ← not normalized
  - $+987.02 \times 10^{9}$ ← not normalized
- In binary
  - $\pm 1.xxxxxxx_2 \times 2^{yyyy}$
- Types `float` and `double` in C

Floating Point Standard

- Defined by IEEE Std 754-1985
- Developed in response to divergence of representations
  - Portability issues for scientific code
- Now almost universally adopted
- Multiple representations
  - Single precision (32-bit)
  - Double precision (64-bit)

$$1.5 \rightarrow 1 + \frac{5}{10}$$

$$10^0 . 10^{-1}$$

$$\frac{1}{2^0} . \frac{1}{2^{-1}} \frac{1}{2^{-2}} \rightarrow 1 + \frac{1}{2} + \frac{1}{4}$$

$$1.75$$

decimal

IEEE Floating-Point Format

1.5×2^{5}

Fraction

exponent

	single: 8 bits double: 11 bits	single: 23 bits double: 52 bits
S	Exponent	Fraction/mantissa

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- S: sign bit (0 ⇒ non-negative, 1 ⇒ negative)
- Normalize significand: 1.0 ≤ |significand| < 2.0
  - Always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (hidden bit)
  - Significand is Fraction with the “1.” restored
- Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned
  - Single: Bias = 127; Double: Bias = 1203

Single-Precision Range

- Exponents 00000000 and 11111111 reserved
- Smallest value
  - Exponent: 00000001
    $\Rightarrow$ actual exponent = 1 – 127 = –126
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- Largest value
  - exponent: 11111110
    $\Rightarrow$ actual exponent = 254 – 127 = +127
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

Double-Precision Range

- Exponents 0000…00 and 1111…11 reserved
- Smallest value
  - Exponent: 00000000001
    $\Rightarrow$ actual exponent = 1 – 1023 = –1022
  - Fraction: 000…00 $\Rightarrow$ significand = 1.0
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- Largest value
  - Exponent: 11111111110
    $\Rightarrow$ actual exponent = 2046 – 1023 = +1023
  - Fraction: 111…11 $\Rightarrow$ significand ≈ 2.0
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

Floating-Point Precision

- Relative precision
  - all fraction bits are significant
  - Single: approx 2^{-23}
    - Equivalent to $23 \times \log_{10}2 \approx 23 \times 0.3 \approx 6$ decimal digits of precision
  - Double: approx 2^{-52}
    - Equivalent to $52 \times \log_{10}2 \approx 52 \times 0.3 \approx 16$ decimal digits of precision

Floating-Point Example

- Represent –0.75
  - $-0.75 = (-1)^1 \times 1.1_2 \times 2^{-1}$
  - S = 1
  - Fraction = $1000\ldots00_2$
  - Exponent = –1 + Bias
    - Single: –1 + 127 = 126 = 01111110_2
    - Double: –1 + 1023 = 1022 = 01111111110_2
- Single: 1011111101000…00
- Double: 1011111111101000…00

sign= – → 1

$-0.1\,1\,0\,0\,0$

$1\quad ½\quad ¼$

1.1×2^{-1}

implied

= 32 bits

sign

Exponent

Fraction/mantissa

Floating-Point Example

- What number is represented by the single-precision float

  11000000101000…00

  - S = 1
  - Fraction = $01000\ldots00_2$
  - Fxponent = 10000001_2 = 129
- $x = (-1)^1 \times (1 + 01_2) \times 2^{(129 - 127)}$

  $= (-1) \times 1.25 \times 2^2$

  $= -5.0$