

1. For a 4KiB page, and a 32 bit address space, calculate the amount of memory needed to store a process's page tables. Assume each entry in the page table requires 12 bytes. Show all calculations.

$$\begin{aligned}\text{Number of pages in a table} &= \text{Total possible Logical Address Entries} / \text{Page size} \\ \text{Total size of page table} &= \text{Page Table Entry} * \text{Number of pages}\end{aligned}$$

Page = 4KiB = 4,096 bytes =  $2^{12}$  bytes  
Address space = 32bit  
PTE = 12 bytes  
max # of page =  $2^{(32-12)} = 2^{20} = 1\text{Mi}$  pages  
Total size of a page table = 12 bytes \*  $2^{20} = \mathbf{12\ MiB}$

2. For a 4KiB page, and a 64 bit address space, calculate the amount of memory needed to store a process's page tables. Assume each entry in the page table requires 12 bytes. Show all calculations.

Page = 4KiB = 4,096 bytes =  $2^{12}$  bytes  
Address space = 64bit  
PTE = 12 bytes  
max # of page =  $2^{(64-12)} = 2^{52} = 4\ \text{Pi}$  pages  
Total size of a page table = 12 bytes \*  $2^{52} = \mathbf{48\ PiB}$

3. For a 16KiB page, and a 32 bit address space, calculate the amount of memory needed to store a process's page tables. Assume each entry in the page table requires 12 bytes. Show all calculations.

Page = 16KiB = 16,384 bytes =  $2^{14}$  bytes  
Address space = 32bit  
PTE = 12 bytes  
max # of page =  $2^{(32-14)} = 2^{18} = 256\ \text{Ki}$  pages  
Total size of a page table = 12 bytes \*  $2^{18} = 3,072\text{KiB} = \mathbf{3\ MiB}$

4. For a 16KiB page, and a 64 bit address space, calculate the amount of memory needed to store a process's page tables. Assume each entry in the page table requires 12 bytes. Show all calculations.

Page = 16KiB = 16,384 bytes =  $2^{14}$  bytes  
Address space = 64bit  
PTE = 12 bytes  
max # of page =  $2^{(64-14)} = 2^{50} = 1\text{Pi}$  pages  
Total size of a page table = 12 bytes \*  $2^{50} = \mathbf{12\ PiB}$

5. Suppose the existence of an associative cache design with  $N = 2$ . Assume that the main memory uses a 32-bit address (and that each address maps to a single byte). If the cache scheme uses the following breakdown of the address bits:

Tag	Index	Offset
31-6 (26bits)	5-3 (3bits)	2-0 (3bits)

a) What is the cache block size (in words)?

$$3 \text{ offsets} = \frac{2^3 \text{ bytes}}{\text{block}} * \frac{\text{word}}{4 \text{ bytes}} = \frac{8}{4} \text{ words} = \mathbf{2 \text{ words.}}$$

b) How many block indices does the cache have?

$$3 \text{ bit index} = \frac{2^3}{2 \text{ sets}} = \mathbf{4 \text{ rows of indices with 2 blocks on each indices.}}$$

(4 rows \* 2 sets) \* 8 bytes = 64 possible entries with 32 entries maximum at given point.

c) Including space for the valid bits, tags, and actual block data, how many bits would be required to actually implement this hypothetical cache?

$$\text{Total Cache} = (\text{no. of rows} * \text{no. of sets}) * (\text{valid bit} + \text{tag bits} + \text{data bits})$$

$$\text{Total Cache} = (4 * 2) * (1 + 26 + 8 \text{ bytes} * \frac{8 \text{ bits}}{1 \text{ byte}}) = \mathbf{728 \text{ bits.}}$$

6. Using the cache design from the previous problem, assume the following byte-addressed cache references are recorded. Assume that the cache is initially empty. Also assume that accesses occurred from left to right (e.g. address 0 was requested, then address 4, followed by address 16, etc).

Byte Address											
0	4	16	132	232	160	1024	30	140	3100	180	2180

a) What was the number of cache hits?

Address (binary)	Address	Block #	Index (mod4)	H/M
000 000 000	0	0	0	Miss
000 000 100	4	0	0	Hit
000 010 000	16	2	2	Miss
010 000 100	132	16	0	Miss*
011 101 000	232	29	1	Miss
010 100 000	160	20	0	Hit
10000 000 000	1024	128	0	Miss*
000 011 110	30	3	3	Miss
<b>010 001 100</b>	<b>140</b>	<b>17</b>	<b>1</b>	<b>Miss*</b>
<b>110000 011 100</b>	<b>3100</b>	<b>387</b>	<b>3</b>	<b>Miss*</b>
<b>010 110 100</b>	<b>180</b>	<b>22</b>	<b>2</b>	<b>Miss*</b>
<b>100010 000 100</b>	<b>2180</b>	<b>272</b>	<b>0</b>	<b>Miss*</b>

\* means replaced

**2 Hits**

(Bold indicates the final state of the index)

- b) Create a table to show the final state of the cache including the value of each index, the valid bits, and the tags.

Block Index	Valid Bit	Least Recently Used	Set 0		Set 1	
			Tag (Leading 0's are not shown)	Data	Tag (Leading 0's are not shown)	Data
000 (0)	Y	0	10000	Mem[1024]	<b>100010</b>	<b>Mem[2180]</b>
001 (1)	Y	0	11	Mem[232]	<b>10</b>	<b>Mem[140]</b>
010 (2)	Y	0	0	Mem[16]	<b>10</b>	<b>Mem[180]</b>
011 (3)	Y	0	0	Mem[30]	<b>110000</b>	<b>Mem[3100]</b>