

1. Suppose the existence of a direct-mapped cache design with a 32-bit address. Assume that each address maps to a single byte. Assume the cache scheme uses the following breakdown of the address bits:

Tag	Index	Offset
31-9 (23bits)	8-5 (4bits)	4-0 (5bits)

a) What is the cache block size (in words)?

$$5 \text{ offsets} = \frac{2^5 \text{ bytes}}{\text{block}} * \frac{\text{words}}{4 \text{ bytes}} = \frac{32}{4} \text{ words} = \mathbf{8 \text{ words / block}}$$

b) How many entries does the cache have?

$$4 \text{ Index} = 2^4 \text{ rows} = 16 \text{ rows of index}$$

$$16 \text{ rows} * 32 \text{ bytes} = \mathbf{512 \text{ bytes of entries}}$$

c) Including space for the valid bits, tags, and actual block data, how many bits would be required to actually implement this hypothetical cache?

$$\mathbf{\text{Total Cache size} = (\# \text{ of Rows}) * (\text{valid bit} + \text{tag bits} + \text{data bits})}$$

$$\text{Cache} = 16 * (1 + 23 + 32 \text{ bytes} * (\frac{8 \text{ bits}}{1 \text{ byte}})) = \mathbf{4,480 \text{ bits}}$$

2. Using the cache design from problem 1, assume the following byte-addressed cache references are recorded. Assume that the cache is initially empty.

Also assume that accesses occurred from left to right (e.g. address 0 was requested, then address 4, followed by address 16, etc).

Byte Address											
0	4	16	132	232	160	1024	30	140	3100	180	2180

a) What was the number of cache hits?

Address (binary)	Address	Block address	Index (mod16)		Hit/Miss
0000 00000	0	0	0	0	Miss
<b>0000 00100</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>Hit</b>
<b>0000 10000</b>	<b>16</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>Hit</b>
0100 00100	132	4	4	4	Miss
0111 01000	232	7	7	7	Miss
0101 00000	160	5	5	5	Miss
10 0000 00000	1024	32	0	0	Miss*
0000 11110	30	0	0	0	Miss*
<b>0100 01100</b>	<b>140</b>	<b>4</b>	<b>4</b>	<b>4</b>	<b>Hit</b>
110 0000 11100	3100	96	0	0	Miss*
<b>0101 10100</b>	<b>180</b>	<b>5</b>	<b>5</b>	<b>5</b>	<b>Hit</b>
100 0100 00100	2180	68	4	4	Miss*

**4 Hits.**

(Bold indicates final state of the index)

\* means replaced

- b) Create a table to show the final state of the cache including the value of each Index, Valid bit, and Tag.

<b>Index</b>	<b>Valid</b>	<b>Tag</b> (leading 0's not shown)	<b>Data</b> (address in decimal)
0000 (0)	<b>Y</b>	110	Mem[96]
0001 (1)	N		
0010 (2)	N		
0011 (3)	N		
0100 (4)	<b>Y</b>	100	Mem[68]
0101 (5)	<b>Y</b>	0	Mem[180]
0110 (6)	N		
0111 (7)	<b>Y</b>	0	Mem[232]
1000 (8)	N		
1001 (9)	N		
1010 (10)	N		
1011 (11)	N		
1100 (12)	N		
1101 (13)	N		
1110 (14)	N		
1111 (15)	N		

3. Suppose the existence of a different direct-mapped cache design with a 32-bit address. Assume that each address maps to a single byte. Assume this cache scheme uses the following breakdown of the address bits:

<b>Tag</b>	<b>Index</b>	<b>Offset</b>
31-6 (26bits)	5-3 (3bits)	2-0 (3bits)

- a) What is the cache block size (in words)?

$$3 \text{ offsets} = \frac{2^3 \text{ bytes}}{\text{block}} * \frac{\text{words}}{4 \text{ bytes}} = \frac{8}{4} \text{ words} = \mathbf{2 \text{ words}}$$

- b) How many entries does this cache have?

$$3 \text{ Index} = 2^3 \text{ rows} = 8 \text{ rows of index}$$

$$8 \text{ rows} * 8 \text{ bytes} = \mathbf{64 \text{ bytes}}$$

- c) Including space for the valid bits, tags, and actual block data, how many bits would be required to actually implement this hypothetical cache?

**Total Cache size = ( # of Rows ) \*( valid bit + tag bits + data bits )**

$$Cache = 8 * \left( 1 + 26 + \frac{8 \text{ bytes per block}}{8 \text{ bits}} \right) = 728 \text{ bits}$$

4. Again assuming an initially empty cache, repeat the steps described in problem 2.

- a) What was the number of cache hits?

Address (binary)	Address	Block #	Index (mod8)	H/M	
000 000 000	0	0	0	Miss	
000 000 100	4	0	0	Hit	<b>1 Hit.</b>
<b>000 010 000</b>	<b>16</b>	<b>2</b>	<b>2</b>	<b>Miss</b>	
010 000 100	132	16	0	Miss*	
<b>011 101 000</b>	<b>232</b>	<b>29</b>	<b>5</b>	<b>Miss</b>	
<b>010 100 000</b>	<b>160</b>	<b>20</b>	<b>4</b>	<b>Miss</b>	
10000 000 000	1024	128	0	Miss*	
000 011 110	30	3	3	Miss	
<b>010 001 100</b>	<b>140</b>	<b>17</b>	<b>1</b>	<b>Miss</b>	
<b>110000 011 100</b>	<b>3100</b>	<b>387</b>	<b>3</b>	<b>Miss*</b>	
<b>010 110 100</b>	<b>180</b>	<b>22</b>	<b>6</b>	<b>Miss</b>	(Bold indicates final state of the index)
<b>100010 000 100</b>	<b>2180</b>	<b>272</b>	<b>0</b>	<b>Miss*</b>	

\* means replaced

- b) Create a table to show the final state of the cache including the value of each Index, Valid bit, and Tag.

Index	Valid	Tag (leading 0's not shown)	Data (address in decimal)
0000 (0)	<b>Y</b>	100010	Mem[272]
0001 (1)	N		
0010 (2)	<b>Y</b>	0	Mem[16]
0011 (3)	<b>Y</b>	110000	Mem[387]
0100 (4)	<b>Y</b>	10	Mem[160]
0101 (5)	<b>Y</b>	11	Mem[232]
0110 (6)	<b>Y</b>	10	Mem[180]
0111 (7)	N		
1000 (8)	N		