MIDTERM EXAM FEBRUARY 13 6 FEB 2018

EXERCISES (DO NOT TURN IN) (REPEATED): p88-89/3, 4, 9 p92/5 p54/1ab, 5e, 6af
p132-133/2b, 4, 8b, 10, 11, 12, 13

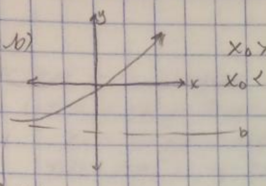
REMARKS ON THE LAST ASSIGNMENT

1) a) $e^x - b = 0$

LET $f(x) = e^x - b \ \forall x$

NEWTON'S METHOD: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ so $x_{n+1} = x_n - \left(\frac{e^{x_n} - b}{e^{x_n}}\right)$

b)

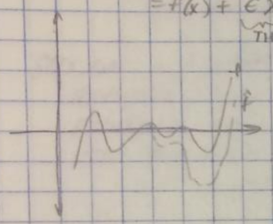


$\left.\begin{array}{l} x_0 > \text{ROOT CONVERGES MONOTONICALLY} \\ x_0 < \text{ROOT CONVERGES} \end{array}\right\}$ CONVERGES $\forall x_0 \in \mathbb{R}$

2) ILL CONDITIONING

c) $\hat{f}(x) = x^7 + (a_6 + \epsilon)x^6 + a_5 x^5 + \dots + a_1 x + a_0$

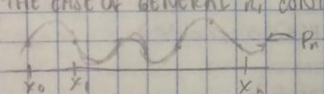
$= f(x) + \underbrace{\epsilon x^6}_{\text{THIS CAN BE LARGE EVEN IF } \epsilon \text{ IS SMALL}}$



THREE INTERSECTION POINTS DISAPPEAR
TWO ROOTS BECOME COMPLEX (WITH NONZERO βi WHERE $\beta \in \mathbb{R}$)

SECTION 4.1 POLYNOMIAL INTERPOLATION, CONTINUED

THE CASE OF GENERAL n, CONTINUED



CONSTRUCTION OF P_n USING THE LAGRANGE FORM, CONT'D

LET $L_i(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}$

$= \prod_{\substack{j=0 \\ i \neq j}}^{n} \left(\frac{x-x_j}{x_i-x_j}\right)$ FOR $0 \leq i \leq n$

THEN $L_i(x) = \begin{cases} 1 & \text{IF } x = x_i \\ 0 & \text{IF } x = x_j, j \neq i \end{cases}$

L_i HAS n FACTORS SO L_i HAS DEGREE n. LET $P_n(x) = y_0 L_0(x) + y_1 L_1(x) + \dots + y_n L_n(x) = \sum_{i=0}^{n} y_i L_i(x) \ \forall x \in \mathbb{R}$

$P_n(x_j) = \sum_{i=0}^{n} y_i L_i(x_j) = y_j \cdot 1 = y_j$ FOR $0 \leq j \leq n$ $= \begin{cases} 1 & \text{IF } j = i \\ 0 & \text{IF } j \neq i \end{cases} = \delta_{ij}$ (KRONEKER DELTA)

L_i HAS DEGREE n, $\forall i$ SO P_n HAS DEGREE $\leq n$

PROOF OF UNIQUENESS

ASSUME p, q ARE POLYNOMIALS OF DEGREE $\leq n$ AND $P(x_i) = y_i = q(x_i)$ FOR $0 \leq i \leq n$
WHERE $x_0, x_1, \dots, x_n$ ARE DISTINCT. SHOW $P(x) = q(x) \ \forall x \in \mathbb{R}$
LET $R(x) = p(x) - q(x) \ \forall x \in \mathbb{R}$ SHOW $R(x) = 0 \ \forall x \in \mathbb{R}$
$R(x_i) = P(x_i) - q(x_i) = y_i - y_i = 0$ FOR $0 \leq i \leq n$
SO R HAS AT LEAST $n+1$ REAL ROOTS. BUT R
HAS DEGREE $\leq n$ AND A NONZERO POLYNOMIAL OF DEGREE $\leq n$ HAS AT MOST n
REAL ROOTS. THEREFORE $R(x) = 0$ SO $P(x) - q(x) = 0 \ \forall x \in \mathbb{R}$ QED