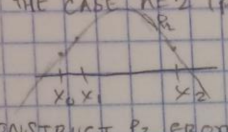
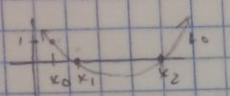
MIDTERM EXAM FEB 13 4FEB2019

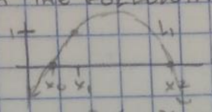
SECTION 3.4 POLYNOMIAL INTERPOLATION, CONTINUED

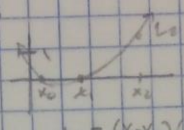
THE CASE $n=2$ (p120-122), CONTINUED



CONSTRUCT P_2 FROM THE FOLLOWING:







$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \begin{cases} 0 & \text{IF } x = x_1 \text{ OR } x_2 \\ 1 & \text{IF } x = x_0 \end{cases}$

$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \begin{cases} 0 & \text{IF } x = x_0 \text{ OR } x_2 \\ 1 & \text{IF } x = x_1 \end{cases}$

$L_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \begin{cases} 0 & \text{IF } x = x_0 \text{ OR } x_1 \\ 1 & \text{IF } x = x_2 \end{cases}$

NOW FIND P_2 WANT $P_2(x_0)=y_0$, $P_2(x_1)=y_1$ AND $P_2(x_2)=y_2$

LET $P_2 = y_0L_0(x) + y_1L_1(x) + y_2L_2(x)$ $\forall x \in \mathbb{R}$

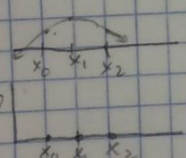
CHECK: $P_2(x_0) = y_0(1) + 0 + 0 = y_0$
$P_2(x_1) = 0 + y_1(1) + 0 = y_1$
$P_2(x_2) = 0 + 0 + y_2(1) = y_2$
} SIMILAR TO LINEAR ALGEBRA

L_i HAS DEGREE 2, $\forall i$ $\therefore$ P_2 HAS DEGREE 2 OR LESS

EXAMPLE: READ EXAMPLE 4.1.3 ON PAGE 121

UNIQUENESS OF THE INTERPOLATION POLYNOMIAL (121-122 IN TEXTBOOK)

CLAIM: ASSUME p AND q ARE POLYNOMIALS OF DEGREE ≤ 2 SUCH THAT $p(x_i)=y_i$ AND $q(x_i)=y_i$ FOR $i=0,1,2$ WHERE x_0, x_1, x_2 ARE DISTINCT. THEN $p(x)=q(x)$ $\forall x \in \mathbb{R}$



PROOF: LET $R(x) = p(x) - q(x)$ $\forall x \in \mathbb{R}$ (GOAL: SHOW $R(x)=0$ $\forall x \in \mathbb{R}$)

$R(x_i) = p(x_i) - q(x_i) = y_i - y_i = 0$ FOR 0,1,2

p AND q EACH HAVE DEGREE ≤ 2 $\therefore$ R HAS DEGREE ≤ 2

BUT R HAS AT LEAST 3 ROOTS.

A POLYNOMIAL OF DEGREE 2 HAS AT MOST 2 REAL ROOTS.

A POLYNOMIAL OF DEGREE 1 HAS AT MOST 1 REAL ROOT

A POLYNOMIAL OF DEGREE 0 HAS NO REAL ROOTS, EXCEPT IF IT IS ZERO EVERYWHERE

$\therefore R(x) = 0$ $\forall x \in \mathbb{R}$

$\therefore p(x) - q(x) = 0$, ie $p(x) = q(x)$ $\forall x \in \mathbb{R}$ qed "ws"

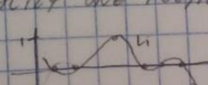
qed: LATIN ENDING FOR A PROOF

ws: WHICH WAS WHAT WE WANTED

THE CASE OF GENERAL n (SECTION 4.1.3 p123-124)

THEOREM 4.1.5 p123

GIVEN $n+1$ POINTS $(x_0,y_0), (x_1,y_1), \ldots, (x_n,y_n)$ WITH THE x_i's DISTINCT THEN AMONG ALL POLYNOMIALS OF DEGREE ≤ 2, $\exists$ EXACTLY ONE POLYNOMIAL P_n SUCH THAT $P_n(x_i) = y_i$ FOR $0 \leq i \leq n$



CALCULATION OF P_n USING THE LAGRANGE FORM

CHOOSE ANY i WITH $0 \leq i \leq n$

WANT POLYNOMIAL L_i OF DEGREE $\leq n$ SUCH THAT $L_i(x) = \begin{cases} 0 & \text{IF } x = x_j \;\forall j \neq i \\ 1 & \text{IF } x = x_i \end{cases}$

LET $L_i(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)} = \prod_{\substack{j=0 \\ j\neq i}}^{n} \left(\frac{x-x_j}{x_i-x_j}\right)$