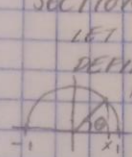
20 FEB 2019

SECTION 4.5 CHEBYSHEV POLYNOMIALS, CONTINUED

LET n BE AN INTEGER WITH $n \geq 0$

DEFINED $T_n(x) = \cos(n \arccos x)$ $\forall x \in [-1,1]$



FOR ANY $x \in [-1,1]$ $\exists$ UNIQUE θ WITH $0 \leq \theta \leq \pi$ AND $x = \cos\theta$

THEN $\theta = \arccos x$ AND $T_n(x) = \cos n\theta$

OBTAINED $T_0(x) = 1$, $T_1(x) = x$, $T_2(x) = 2x^2 - 1$

RECURSION RELATION: $T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x)$ $\forall n \geq 1$

IF T_{n-1} AND T_n ARE POLYNOMIALS, THEN T_{n+1} IS A POLYNOMIAL.

T_0 AND T_1 ARE BASE CASES SO BY INDUCTION T_n IS A POLYNOMIAL $\forall n \geq 0$

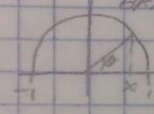
EXAMPLES:

$T_3(x) = 4x^3 - 3x$

$T_4(x) = 8x^4 - 8x^2 + 1$

IN GENERAL, $T_n(x) = 2^{n-1}x^n +$ (LOWER DEGREE TERMS)

GRAPH OF T_n



$T_n(x) = \cos(n \arccos x) = \cos n\theta$ $\forall x \in [-1,1]$

AS x VARIES FROM -1 TO 1, θ VARIES FROM 0 TO π

AS x VARIES FROM -1 TO 1, $n\theta$ VARIES FROM 0 TO $n\pi$

ie. $n\theta$ VARIES OVER n HALF CIRCLES

SO $\cos n\theta$ MAKES n CHANGES BETWEEN 1 AND -1.

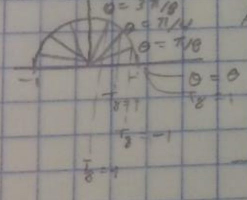
$n\theta = 0 \rightarrow \cos n\theta = 1$

$n\theta = \pi \rightarrow \theta = \frac{\pi}{n}$ AND $\cos n\theta = \cos \pi = -1$

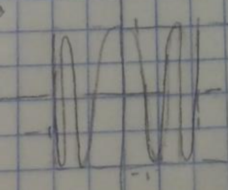
$n\theta = 2\pi \rightarrow \theta = 2\left(\frac{\pi}{n}\right)$ AND $\cos n\theta = \cos 2\pi = 1$

$n\theta = n\pi \rightarrow \theta = n\left(\frac{\pi}{n}\right)$ AND $\cos n\theta = \cos n\pi = (-1)^n$

THE CASE $n = 8$



ALTERNATING SIGN



SIDE VIEW OF A CYLINDER w/ $\cos 8\theta$

POINTED ON THE SIDE

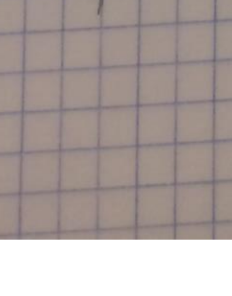
ALSO SEE FIGURES 4.15 AND 4.16 p166-167

FOR PLOTS OF T_0, T_1, T_2, T_3, T_4

MINIMUM SIZE PROPERTY (p 168-169)

HAD $T_n(x) = 2^{n-1}x^n +$ (LOWER DEGREE TERM)

THEN $\frac{1}{2^{n-1}} T_n(x) = x^n +$ LOWER DEGREE TERM



READ: THEOREM 4.5.3 p169