4. Pricing Forwards and Futures

Instructor: Byungjin Kang

숭덕경상관 **418**호**, Tel. 828-7392**

E-mail. bjkang@ssu.ac.kr

I. Forward Pricing

Overview

- Forward Price
  - The delivery price that makes the forward contract have zero value to both parties
  - How do we identify this zero – value price?
- Combine
  - (1) Key assumption: No arbitrage
  - (2) Guiding principle: Replication

No Arbitrage

- Maintained Assumption
  - Market does not permit arbitrage!

- What is <u>Arbitrage</u>?
  - A profit opportunity which guarantees net cash inflows with no net cash outflows
  - Such an opportunity represents an extreme form of market inefficiency where two identical securities (or baskets of securities) trade at different prices

- Keep in Mind
  - Assumption is <u>not</u> that arbitrage opportunities can <u>never</u> arise, but that the cannot <u>persist</u>
  - This is a minimal market rationality condition – it is impossible to say anything sensible about a market where such opportunities can persist

Replication

- Replication
  - Fundamental idea underlying the pricing of all derivative securities

- The Argument:
  - Derivative's payoff is determined by price of the underlying asset
  - So, it "should" be possible to recreate (or replicate) the derivative's payoffs by directly using the spot asset and, perhaps, cash
  - By definition, the derivative and its replicating portfolio (should one exist) are equivalent
  - So, by no – arbitrage, they must have the same cost
  - Thus, the cost of the derivative (its so – called "fair price") is just the cost of its replicating portfolio, i.e., the cost of manufacturing its outcomes synthetically

- Key Step
  - Identifying the replicating portfolio

Replicating Forwards

Illustration

- Forward contracts are relatively easy to price by replication
- Consider an investor who wants to take a long forward position
- Notation
  - S, T, F
- At maturity of the contract, the investor pays \$$F$ and receives one unit of the underlying
- To replicate this final holding:
  - Buy one unit of the asset today and hold it to date T
- Both strategies result in the same final holding of one unit of the underlying at T
- So, viewed from today, they must have the same cost
- Wheat are these costs?

Replicating Forwards

❒ Cost of Forward Strategy

- The forward strategy involves a single cash outflow of the delivery price F at time T
- So, cost of forward strategy: PV(F)

❒ Cost of Replicating Strategy

- To replicate, we must
- (1) Buy the asset today at its current spot price S
- (2) "Carry" the asset to date T
  - This involves,
  - (a) Possible holding/carrying costs (storage, insurance)
  - (b) Possible holding benefits (dividends, convenience yield)
- Let
  - M=PV(Holding Costs) – PV(Holding Benefits)
- Net Cost of replicating strategy = $S + M$

The Forward Pricing Condition

- By No – Arbitrage,
  - $PV(F) = S + M$
  - Solving this condition for F, we obtain the unique forward price consistent with no – arbitrage

- Violation of This Condition ➔ Arbitrage
  - If $PV(F) > S + M$, the forward is overvalued relative to spot
    - Arbitrage profits may be made by selling forward and buying spot ➔ "Cash and carry" arbitrage
    - Forward contract has positive value to the short, and negative value to the long
  - If $PV(F) < S + M$, the forward is undervalued relative to spot
    - Arbitrage profits can be made by buying forward and selling spot ➔ ("Reverse cash and carry" arbitrage
    - The contract has positive value to the long, and negative value to the short

Determinants of the Forward Price

- Three Inputs
  - Current price S of the spot asset
  - The cost M of "carrying" the spot asset to date T
  - The level of interest rates which determine present values
  - This is commonly referred to as the "cost – of – carry" model of pricing forwards

- Two Comments
  - Forward and spot prices are tied together by arbitrage
    - They must move in "lockstep"
  - To what extent then do (or can) forward prices embody expectations of future spot prices?

Pricing Formula

- I. Continuous Compounding

$$PV(F) = S + M, \quad PV(F) = e^{-rT} F \quad \Rightarrow \quad F = e^{rT}(S + M)$$

When there are no holding costs or benefits $(M = 0) \quad \Rightarrow \quad F = e^{rT} S$

- 2. Money Market Conventions

$$PV(F) = S + M, \quad PV(F) = \frac{F}{1 + r \times \frac{d}{360}} \quad \Rightarrow \quad F = (S + M)\left(1 + r \times \frac{d}{360}\right)$$

When there are no holding costs or benefits $(M = 0) \quad \Rightarrow \quad F = S\left(1 + r \times \frac{d}{360}\right)$

Numerical Example 1

- Consider a Forward Contract on Gold
  - Spot price: S = \$1,140/oz
  - Contract length: T = 1 month = 1/12 years
  - Interest rate: r = 2.80% (continuously compounded)
  - No holding costs or benefits
  - Then, what is the (theoretical) forward price, F?

- Arbitrage with an Over – or Under – valued Forward
  - (1) Market forward price is \$1,160/oz
  - (2) Market forward price is \$1,125/oz

Holding Costs and/or Benefits

- Holding Costs are Often Non – Zero
  - With equities or bonds, there are often holding benefits such as dividends or coupons
  - With commodities, there are often holding costs such as storage and insurance

- Then,
  - Such interim cash flows affect the total cost of the replication strategy and should be taken into account in pricing

Numerical Example 2

- Consider a Forward Contract on a Bond
  - Spot price of bond: $S = 95$
  - Contract length: $T = 6$ months
  - Interest rate: $r = 10\%$ (continuously compounded) for all maturities
  - Coupon of $5 will be paid to bond holders in 3 months

- What is the Forward Price of the Bond?

Numerical Example 2

- Arbitrage Trading Strategies
  - For example, suppose F (market price) = 98
  - Then, the forward is overvalued relative to spot, so we want to sell forward, buy spot, and borrow
  - Buying and holding the spot asset leads to a cash outflow of 95 today, but we receive a coupon of 5 in 3 months
  - There are many ways to structure the arbitrage strategy. Here is one. We split the initial borrowing of 95 into two parts, with
    - One part repaid in 3 months with the $5 coupon, and
    - The balance repaid in six months with the delivery price received on the forward contract

Numerical Example 2

- Arbitrage Trading Strategies
  - So the full arbitrage strategy is:
    - Enter into short forward with the delivery price of 98
    - Buy the bond for 95 and hold for 6 months
    - Finance spot purchase by
      - (1) Borrowing 4.877 for 3 months at 10%
      - (2) Borrowing 90.123 for 6 months at 10%
  - In 3 months:
    - Receive coupon $5
    - Repay the 3 month borrowing
  - In 6 months:
    - Deliver bond on forward contract and receive $98
    - Repay 6 month borrowing

Numerical Example 2

- Cash Flows from the Arbitrage

CF Source	CF Today	CF in 3 mo	CF in 6 mo
Short Forward			+98.00
Long bond	−95		
Coupon		+5.00	
3-month borrowing	+4.88	−5.00	
6-month borrowing	+90.12		−94.74
Net CF	**0**	**0**	**+3.26**

- Question: What is the Arbitrage Strategy If $F = 91.50$?

Currency Forward

❒ Forwards on Currencies & Related Assets

- ❖ Forwards on currencies need a slightly modified argument
- ❖ For example, suppose you want to be long £1 on date T
- ❖ Two strategies:
  - ➢ Forward contract: Pay $\$F$ at time T, receive £1
  - ➢ Replicating strategy: Buy £x today and invest it to T, where x = PV(£1)
- ❖ PV(£1) is the amount that when invested at the sterling interest rate will grow to £1 by time T
  - ➢ The "£" inside the PV expression is to emphasize that present values are being taken with respect to the £ interest rate

❒ Replication Costs

- ❖ Cost of the forward strategy in USD: PV($\$F$) = F * PV($1)
- ❖ Cost of the spot (or replicating s) strategy in USD: S * PV(£1)
- ❖ Here, S is the spot exchange rate ($ per £)

Currency Forward

❑ General Pricing Expression

- By no arbitrage, we must have: S * PV(£1) = F * PV($1)

$$F = S \times \frac{PV(\text{GBP } 1)}{PV(\text{USD } 1)}$$

❑ Compounding Frequency and Pricing Formula

- (1) Continuous compounding

$$F = e^{(r-r_f)T} S$$

- (2) Money market conventions

$$F = \frac{1 + r \times \frac{d}{360}}{1 + r_f \times \frac{d}{360}} \times S, \quad \text{where } r(\text{ACT/360}), \ \ r_f(\text{ACT/360})$$

Numerical Example 3

- Data
  - Foreign currency: GBP
  - Spot exchange rate S (USD per GBP): 1.63146
  - Contract length T: 3 months = 90 days
  - 3 month USD Libor rate: 0.251% (ACT/360)
  - 3 month GBP Libor rate: 0.610% (ACT/365)

- In This Case,

$$F = \frac{1 + 0.00251 \times \frac{90}{360}}{1 + 0.0061 \times \frac{90}{365}} \times 1.63146 = 1.63003$$

Numerical Example 3

- Arbitrage Trading
  - Suppose we had F(market) = \$1.60/£
  - Then, the forward is undervalued relative to spot, so we want to buy forward, sell spot, and invest
    - Long forward contract to buy £1 for \$1.60 in 3 months
    - Short PV(£1) ➔ What does it mean?
      - (1) Borrow PV(£1) = £0.9985 for 3 months at 0.61%
      - (2) Sell £0.9985 for \$ at the spot rate of \$1.63146/ £
    - Invest the proceeds for 3 months at 0.251%
    - In 3 months:
      - Pay USD 1.6000 and receive GBP 1 from the forward
      - Repay GBP borrowing

Numerical Example 3

- At Inception,

CF Source	CF in GBP	CF in USD
Borrowing GBP	+0.9985	
Selling GBP for USD	−0.9985	+1.62901
Investing proceeds		−1.62901
Net CF	**0**	**0**

- At Maturity,

CF Source	CF in GBP	CF in USD
Long Forward	+1	−1.60
Due on GBP borrowing	−1	
From USD investment		+1.63003
Net CF	**0**	**+0.03003**

Stock Index Forward

❒ Pricing Stock Index Forward

- ❖ We can also price forwards on stock indices using this approach
  - ➢ A stock index is essentially a basket of a number of stocks
  - ➢ If the stocks pay dividends at different times, we can approximate the dividend payments well by assuming they are continuously paid
  - ➢ Dividend yield on the index plays the role of the variable r_f in the formula
- ❖ Literally speaking, the idea of continuous dividends is an unrealistic one, but in general, the approximation works very well
- ❖ Computationally, much simpler than calculating cash value of dividend payments expected over contract life and using the known – cash – payouts formula

Numerical Example 4

- Data
  - Current level of S&P 500 index: 1,343
  - One month interest rate (continuously compounded): 2.80%
  - Dividend yield on the S&P 500: 1.30%
  - Contract length is one month

- Then, $F = 1343 \times e^{(0.0280 - 0.0130) \times (1/12)} = 1344.68$

- S&P 500 Futures Prices: Jan 15, 2010

Expiry	Futures Price
March 2010	1132.00
June 2010	1127.50
September 2010	1118.50
December 2010	1116.00

Spot: 1136.03 (S&P on Jan 15, 2010)

II. Valuing Forwards and Futures Pricing

Valuing Existing Forwards

Illustrations

- Consider a forward contract with delivery price K that was entered into earlier and now has T years left to maturity
- What is the current value of such a contract?
- Suppose we are long the existing contract
- Suppose also that the current forward price for the same contract (same underlying, same maturity date) is F

Offsetting the Existing Forward

- Consider offsetting the existing long forward position with a short forward position in a new forward contract
  - Original portfolio: long forward contract with delivery price K and maturity T
  - New portfolio: (1) long forward contract with delivery price K and maturity T + (2) short forward contract with delivery price F and maturity T
- Value of original portfolio = Value of new portfolio (Why?)

Valuing Existing Forwards

Valuation by Offset

- What happens to the new portfolio at maturity?
  - Physical obligations in the underlying offset
  - Net cash flow: $F - K$
    - So new portfolio – certainty cash flow of $F - K$ at time T
  - This means that the value of new portfolio is $\text{PV}(F - K)$
- Therefore,
- (1) Value of long forward = $\text{PV}(F - K)$
- (2) Value of corresponding short forward = $\text{PV}(K - F)$

Intuition?

Numerical Example 5

- Data
  - You enter into a forward contract to sell 10,000 shares of Dell stock in 3 months' time at a delivery price of \$25.25
  - A month later:
    - The price of Dell is \$25.40
    - The two month rate of interest at this point is 4.80% (money market convention)
    - There are 61 days in the two month period
    - Dell is not expected to pay any dividends over the next two months
  - What is the value of the contract you hold?
- Solution

Futures Pricing

- Considerations
  - Analytical valuation of futures contracts difficult for two reasons:
    - 1. Delivery options provided to the short position
    - 2. Margining which creates uncertain interim cash flows
  - These features will have an impact on futures prices compared to another wise identical forward contract
  - The question is
    - How much of an effect?
    - Is it quantitatively significant?

Futures Pricing

- Delivery Options
  - Consider delivery options
  - Such options make the futures contract
    - More attractive to the seller (the short position)
    - Less attractive to the buyer (the long position)
  - Thus, other things being equal the futures price must be lower than the forward price on this account
  - How much lower? That is, how economically valuable is the delivery option?

  - The delivery option is provided primarily to guard against squeezes
  - However, provision of the delivery option degrades the quality of the hedge
  - Intuitively, the more valuation this option, the grater this uncertainty, and the more the hedge is degraded
  - Thus, we would expect that in a successful futures contract, the delivery option des not have much economic value

Futures Pricing

❐ Margin Accounts

- What about margin accounts?
- These create interim cash flows which earn interest at possibly uncertain rates
- Thus, the quantitative impact will depend on
  - How cash flows into the margin account occur
  - How interest rates change when futures price change
- Once again, in a successful futures contract, our expectation would be that this impact would be quantitatively small
- Best laboratory for testing the effect: currency contracts, where no delivery options exist
- One study reported that difference in a forward and futures prices were smaller than the bid – ask spread in the currency market

Futures Pricing

- Forward Price = Futures Price
  - When the risk free interest rate is constant and the same for all maturities
  - It can be extended to the situation, where the interest rate is a known function of time

- Forward Price ≠ Futures Price
  - When interest rates vary unpredictably
  - (Case 1) A strong positive correlation between interest rates and the underlying asset price implies that the futures price is slightly _______ than the forward price
  - (Case 2) A strong negative correlation implies the reverse

- Difference between Forward and Futures Prices
  - The longer the maturity, ___________________
  - The more the difference in tax, transaction cost and margins, ___________________

III. From Theory to Reality

The Empirical Performance

- Issues
  - How well does the "cost of carry" model of pricing forwards hold up empirically?
  - The theory is based on <u>replication</u>
  - If replication is not possible, then of course, the theory is inapplicable
    - Non – traded underlying (e.g., catastrophe futures)
    - Non – storable underlying (e.g., electricity forwards)
  - But these are exceptional cases
    - In most cases, the underlying is traded and storable
    - How well does the theory do here?

The Empirical Performance

- Financial Forwards
  - The vast majority of forward contracts traded are on financial assets
  - For such assets, in most developed markets, the assumptions underlying the theory are a very good first approximation
    - Transaction costs are generally low for institutional participants
    - There are generally no impediments to taking long and short positions in both the underlying and the forward contract
  - So carrying out the theoretical arbitrage strategies is feasible, and this means the divergence between the theory and practice is minimal
  - <u>Caveat</u>
    - Even for financial forwards, the theoretical "spot – forward relationship" can break down in times of market stress
    - In 2008, during the financial crisis, <u>covered interest rate parity (CIRP)</u> was violated in currency forward markets because traders could not obtain funding at close to the risk–free rate to implement the short forward – long spot arbitrage strategy

The Empirical Performance

- Commodity Forwards
  - Commodities differ from financial assets in that they get consumed in the process o fuse (e.g., production)
  - Holding inventories of commodities gives producers and other the option to consume the commodity out of storage
    - The commodity's convenience yield measures the value of this option
  - The tighter are supplies of the commodity relative to consumption demand, the more valuable is this option and so the higher the convenience yield
    - Oil is the preeminent example of a high convenience yield commodity
  - Commodities that are in plentiful supply relative to consumption demand (e.g., gold) generally command a lower convenience yield

Commodity Forward Price Behavior

- Suppose We Ignore Convenience Yields
  - Since commodities typically have positive storage costs and the size of the storage costs increases as the horizon increases, the cost of carry model predicts that
    - 1. Forward prices will be higher than spot prices
    - 2. Forward prices will be higher for longer maturities
  - The first condition is called "<u>contango</u>"
    - <u>Backwardation</u>: Futures/forward prices are lower than spot
    - <u>Contango</u>: Futures/forward prices are greater than spot
  - The second condition is called a "normal" market
    - <u>Normal</u> market: Futures/forward prices increase with maturity
    - <u>Inverted</u> market: Futures/forward prices decrease with maturity

The Effect of Convenience Yield

❒ How Does the Convenience Yield Affect the Theory?

- To short sell a commodity, one must borrow it from someone who has surplus inventories
- Inventories of commodities are held because of the convenience yield that holding the commodity provides
- So the short seller must compensate the holder for the lost convenience yield
- This results in a band within which forward prices may lie without violating no – arbitrage
- Letting "*r*" denote the risk – free interest rate, and "*c*" the convenience yield, both in continuously compounded terms, we have

$$S_0 e^{(r-c)T} \le F_0 \le S_0 e^{rT}$$

- Intuition?

The Effect of Convenience Yield

- Implications
  - In the presence of high convenience yields, futures/forward prices can exhibit both backwardation and an inverted market
  - So what looks like an arbitrage opportunity – or a gross violation of the theory – when we ignore convenience yields may not be an arbitrage opportunity at all
  - Unfortunately, the practical implication of these observations is limited since the convenience yield is not observable
  - But we can use these ideas to understand departures in the data from the theory
    - For example, commodities with a higher convenience yield will have a greater rage of possible departures from the theoretical price (calculated ignoring the convenience yield)

The Effect of Convenience Yield

- An Example: Oil
  - Oil is the most prominent examine of a commodity that has exhibited backwardation and inverted market prices over long periods of time
  - However, oil has also been in contango and exhibited normal market patterns at other times
  - Oil futures settlement prices (per barrel, Light Sweet Crude) on NYMEX, 15 – Dec – 2003 and 14 – May – 2010:

Delivery Month	Settlement Price	Delivery Month	Settlement Price
Jan-04	33.04	Jun-10	71.61
Feb-04	32.95	Jul-10	75.43
Mar-04	32.36	Aug-10	77.67
Apr-04	31.78	Sep-10	78.98
May-04	31.23	Oct-10	79.80
Jun-04	30.70	Nov-10	80.47

IV. Index Arbitrage

Index Arbitrage

- Basic Concepts
  - Stock index: basket of stocks constructed according to specified rules
  - Many active futures contracts on stock indices:
    - US: Dow Jones Industrial Average, S&P500, Nasdaq100
    - Asia: Nikkei225, KOSPI200
    - Europe: FTSE100, Dow Jones STOXX 50, DAX, SMI
  - <u>Index arbitrage</u> looks to take advantage of mispricing in the futures – spot relationship, i.e., between
    - The spot price of the basket of stocks composing the index, and
    - The theoretical price of the futures contract

Index Arbitrage

- Pricing Index Forwards / Futures
  - Stock index forwards / futures may be priced in one of two ways:
  - 1. Using the cash holding cost formula
    - If M is the PV of cash dividends received on the index over the contract's life,

$$F = e^{rT}(S + M), \qquad M: negative\ value\ (for\ benefit)$$

  - 2. Using the dividend yield formula
    - Letting d denote the dividend yield on the index, then

$$F = e^{(r-d)T} S$$

  - Both formulations are really approximations, since the dividends to be paid are unknown and cannot be locked in at the outset
    - Formulae should be regarded as offering the correct futures/forward price given the assumed size of dividends

Index Arbitrage

An Example

- Consider the following data on a given index
  - $S = 1020$, $d = 0.014$, $r = 0.03$, $T = 3$ months, $F = 1027.40$
- The theoretical futures price given this data is: ______________
- Thus, the given futures contract is overpriced by ______, and there is an arbitrage opportunity

The Arbitrage Strategy

- Sell futures at 1027.40
- Buy ______ units of the spot index: the cost is ___________
- Reinvest all dividends received in purchasing more units of spot
- Borrow _______ for three months at 3%
- Given the reinvestment strategy, the holding of the index will grow to one unit in 3 months
- In 3 months: Deliver unit of index and receive 1027.40 + Repay borrowing

Index Arbitrage and Program Trading

- Definition of Program Trading
  - A type of trading in securities, usually consisting of baskets of fifteen stocks or more that are executed by a computer program simultaneously based on predetermined conditions

- Long Arbitrage
  - Index basket _________ + Index futures ___________
  - When basis is ___________
  - Increase in long arbitrage positions ➔ in the future, ___________

- Short Arbitrage
  - Index basket _________ + Index futures ___________
  - When basis is ___________
  - Increase in short arbitrage positions ➔ in the future, ___________

Real World Example

[주식 ABC] 차익거래와 비차익거래, 유한빛 기자 hanvit@chosun.com – 2011. 11 월

18 일 유가증권시장은 전날보다 2% 급락해 장을 마쳤다. 전문가들은 프로그램매매, 그 중에서도 차익거래가 코스피지수를 하락시킨 요인이라고 보고 있다. 이날 프로그램매매는 6600 억원 매도우위였고, 그 중 5400 억원이 차익거래 물량이었다.

프로그램매매는 말 그대로 프로그램을 이용해 원하는 주식을 원하는 자동으로 사거나 파는 거래방식이다. 주로 자금운용 규모가 큰 외국인이나 기관투자자들이 사용한다. 매번 종목을 일일이 고르는 게 아니라 거래할 종목과 구성을 미리 설정해두고 원하는 금액만큼 주식을 사고 파는 것이다.

프로그램매매는 차익거래와 비차익거래로 나뉜다. 차익거래는 현물과 선물의 가격차이가 나는 경우 둘 중 높게 평가돼 있는 상품을 팔고 낮게 평가된 상품을 사 차익을 노리는 투자방법이다. 차익거래의 기준은 선물가격에서 현물가격을 뺀 값인 베이시스(basis)다. 이 값이 0 이면 선물가격과 현물가격이 같다는 뜻이다. 베이시스가 클수록 거래차익도 커 프로그램매매가 활발해진다.

우리투자증권의 최창규 애널리스트는 "한국 증시에 대한 기대심리가 약해진 외국인들이 선물시장에서 물량을 대거 내놓으면서 선물가격이 떨어졌다"며 "투신 등 기관계에서 차익거래에 나섰는데, 보유한 주식을 팔아 상대적으로 저평가돼 있는 선물을 사들인 것"이라고 말했다. 투신과 보험 등 기관계의 차익거래가 코스피지수를 떨어뜨리는데 영향을 미쳤다는 뜻이다.

Real World Example

선물가격이 더 비싼 상태는 콘탱고(contango), 현물가격이 더 비싼 경우는 백워데이션(backwardation) 또는 역조시장이라고 부른다. 투자자들은 콘탱고 상태에선 선물을 팔고 현물을 사는 매수차익거래를 하고, 백워데이션 상태에선 현물을 팔고 선물을 사는 매도차익거래를 한다. 18 일엔 베이시스가 장중 한때 마이너스 1 을 기록할 정도로 선물가격이 낮게 형성된 백워데이션 상태였다.

이승재 대신증권 애널리스트는 “선물은 시장에 대한 기대감이 있어야 사는 것”이라며 “외국인들이 선물을 5000 억원 넘게 판 것은 우리나라 증시를 부정적으로 전망했다는 의미”라고 설명했다. 증권사 관계자들은 프랑스로까지 재정위기가 전이될 것이라는 우려가 나오는 국제경제 상황도 외국인들이 투자전략을 보수적으로 세우는 데 영향을 줬다고 말했다. 이 애널리스트는 “외국인과 국내기관이 갖고 있는 주식이 2 조원에 이를 것이라고 추정하는데, 이들이 차익거래를 위해 물량 내놓으면 코스피지수가 더 떨어질 수 도 있다”고 덧붙였다.

비차익거래는 차익을 얻는 게 목적이 아니라 프로그램을 이용해 한꺼번에 주식을 사들이거나 파는 거래방식이다. 기준으로 삼은 지수에 해당되는 종목을 여러 개 묶어 거래하는데, 이 주식 묶음을 '바스켓'이라고 부른다. 일반적인 인덱스펀드 상품을 운용하는 증권사나 자산운용사가 사용하는 방식이다.

예를 들어 규모가 1 조원이고 코스피 200 을 기준으로 하는 펀드상품이 있다. 상품가입자들이 펀드계좌를 해지해 2000 억원을 찾으려고 할 때 펀드운용사는 보유한 주식을 팔아 현금으로 바꾼다. 이때 운용사가 투자종목을 모아놓은 바스켓을 사고파는 것은 비차익거래이다. 보유하고 있는 종목에 대한 투자금액을 늘리거나 줄이기 위한 목적으로 주식을 매매하기 때문이다.

Real World Example

<증시 브리핑> 일장춘몽 프로그램

[이데일리 2008년 8월 29일, 손희동기자] 코스피가 또 한번 연중 최저치를 경신했다. 전날 코스피는 밤사이 강세로 마감한 뉴욕증시의 훈풍덕분에 개장초 소폭 오르기도 했지만, 이 같은 상승흐름을 끝까지 지켜지는 못했다. 지금의 지수대는 지난해 대세 상승장이 시작되기 전 수준이다. 꼬일대로 꼬인 수급이 문제였다. 여드레 연속 계속된 외국인 순매도도 부담스러웠지만 뚜렷한 방향성없이 차익에만 의존하는 프로그램 매매가 결국은 어제 지수의 발목을 잡았다. 프로그램 매매는 전일을 제외한 최근 나흘 동안만 보면 대규모 물량 유입을 통해 지수를 방어해 준 고마운 존재였다. 21일 연속 순매수를 기록한 비차익 매매도 그렇거니와 현선물 차익거래로도 지난 22일부터 27일까지 1조원 넘는 자금이 주식시장에 들어와 투자심리 안정에 적지 않은 기여를 했다. 가뜩이나 수급여건이 불안한 상황에 이 같은 프로그램 매수세는 시장의 불안을 덜어내 준 안정제 역할을 맡았다. 그러나 프로그램 매매로 들어온 물량은 결국 다시 빠져나갈 수 밖에 없다는 점을 뇌리에 되새긴다면 어제와 같은 일은 언제든지 반복될 수 있다는 점 역시 염두에 둬야 한다. 특히 프로그램 차익매매의 경우 현선물 가격차인 베이시스가 벌어질 때 들어오고 좁아질 때 나가는 성질을 가진 만큼, 붉은 색으로 표시된다 해서 마냥 반가워 할 수 만은 없다. 실제 그 동안 꾸준히 물량이 쌓이면서 잔고도 8조원을 넘어서고 있다.

Real World Example

선물시장 외국인이 바로 이 베이시스의 주도권을 가지고 있고, 이들이 매도 포지션 설정에 방점을 찍는다면 결국 프로그램 차익거래는 썰물처럼 빠져나갈 수 밖에 없다. 어제가 바로 그랬다. 전일 선물시장 외국인은 미결제약정의 증가를 동반한 5700계약의 순매도를 감행했는데, 시장에서는 이들 선물 외국인이 향후 하락장을 염두에 두고 신규 매도 포지션을 설정했을거라 진단하고 있다. 결국 베이시스의 하락은 불을 보듯 뻔한 일이었고, 이는 프로그램 매물 출회와 지수 하락으로 연결됐다. 주식을 좀 한다 싶은 사람이라면 한 번쯤 들어봤을 법한 이른바 `왝더독` 장세다. 특히 요즘 들어 주식시장이 프로그램 매매에 휘둘리는 일이 잦아지고 있다. 프로그램 매수세가 강한 날은 보합으로, 매도세가 강한 날은 하락장으로 마감하는 그런 식이다. 그만큼 국내 주식시장에 별다른 모멘텀이 없다는 뜻이기도 하다. 그나마 간밤 뉴욕증시가 크게 올라 오늘은 프로그램 매매를 걱정해야 하는 시름은 잠시 덜었다. 하지만 물량이 터져 나오지 않는다고 좋아할 만한 일도 아니다. 프로그램 매수주문이 들어오지 않는 상황에서 지수가 반짝 올라주기도 힘든 요즘이기 때문이다. 프로그램 매매에 일희일비하지 않는 그런 시장 분위기를 되찾아야 투자자들도 안심하고 투자할 수 있는 여건이 마련될 것으로 보인다.

** “Wag the Dog” :Underlying asset market vs. Derivatives market

Real World Example

기관 1.1조원 순매수 뒤엔 '콘탱고' 있었다 (2020.6.4., 한국경제)

코스피200 선물시장이 지난 3월 공매도 금지 조치 이후 처음으로 현물 가격을 앞지르기 시작했다. '콘탱고'라고 부른다. 코로나19로 인한 폭락장 이후 최근까지 코스피200 선물 가격이 현물 가격보다 낮게 형성되는 이례적인 현상이 나타났지만, 외국인이 선물 매수에 나서며 분위기 변화가 감지되고 있다는 분석이다.

지난 3일 코스피200 선물지수는 286.25로 코스피200 현물지수(285.91)를 0.34포인트 앞섰다. 일반적으로는 선물 가격이 현물 가격보다 비싼 콘탱고 현상이 '정상시장'으로 간주된다. 하지만 3월 급락장 이후 국내 증시에선 이 반대 현상인 '백워데이션'이 나타났다. 금융당국이 공매도를 금지하자 기관 및 외국인투자자들의 '현물 매도-선물 매수'를 통한 차익거래가 불가능해지자 헤지(위험 회피)와 숏(매도 베팅) 수요가 모두 코스피200 선물시장으로 쏠렸다. 이 때문에 2015년 8월 이후 처음으로 현물지수보다 선물지수가 더 낮은 현상이 발생했다.

그러나 2개월여 만에 코스피200 선물을 외국인이 사들이면서 다시 선물 가격이 현물 가격보다 높아졌다. 이는 기관 매수로 이어졌다. 선물 가격이 오르자 기관(금융투자)이 싼 현물을 대거 매수하고, 비싼 선물을 매도하는 차익 거래에 나선 것. 3일 금융투자가 코스피200 현물을 1조원 넘게 순매수한 이유다. 주가는 크게 올랐다. 김동완 유진투자증권 연구원은 "금융투자가 사상 최대 규모로 현물을 순매수한 이유는 현물 잔액이 귀해졌기 때문"이라며 "콘탱고 폭은 크지 않았지만 최근 선물 저평가 상태가 잦아지면서 기관들로선 현물 잔액을 쌓아놓고 백워데이션이 발생하면 청산하려는 의도"라고 해석했다. 즉 현물을 사고 선물을 파는 매수 차익 거래에 진입한 뒤 선물이 다시 싸지면 반대 거래(현물 팔고, 선물 매수)를 통해 차익 청산을 하려는 의도란 설명이다.

외국인이 코스피200 선물시장에 유입된 것은 신흥시장에 대한 투자 심리가 긍정적으로 돌아섰다는 신호로 볼 수 있다는 관측이 나온다. 다만 거시경제 여건과 경기가 회복된 상황이 아니기 때문에 단기적 현상에 그칠 수 있다는 지적도 제기된다.

V. Forward Price and Future Spot Price

Economic Meaning of Forward Price

❒ Question

- ❖ Is the forward price a predictor of the spot price of the underlying at maturity of the contract?
  - ➢ It is a commonly – held belief that this is the case
  - ➢ Called the <u>unbiased expectations hypothesis</u>

❒ Cost of Carry Model

- ❖ Consider a stock forward with no dividends

$$F_0 = S_0 e^{rT} = E(S_T)?$$

- ❖ As a risky asset, the stock will typically command a non – zero risk premium related to is risk characteristics

$$E(S_T) = S_0 e^{\mu T} > S_0 e^{rT} = F$$

- ❖ In this case, the forward price <u>underpredicts</u> the expected future spot price

Economic Meaning of Forward Price

- Example
  - Buy a forward contract maturing in $T-$ years with the price of F_0
  - Make a deposit of __________ to meet the obligations at maturity
  - Cash flow
    - Today: ____________
    - Maturity: ____________
  - NPV of the investment

$$-\frac{F_0}{(1+r)^T}+\frac{E(S_T)}{(1+k)^T}$$

Where, k is the required rate of investors

Economic Meaning of Forward Price

- Example (Continued)
  - If the market is efficient, and thus the NPV of the investment is zero,

$$\Rightarrow F_0 = E(S_T)(1 + r - k)^T$$

- In Conclusion,
  - (1) The underlying asset of a futures/forward contract has no systematic risk, then
    - k _____ r ➔ __________
  - (2) The underlying asset of a futures/forward contract has positive systematic risk, then
    - k _____ r ➔ __________
  - (3) The underlying asset of a futures/forward contract has negative systematic risk, then
    - k _____ r ➔ __________