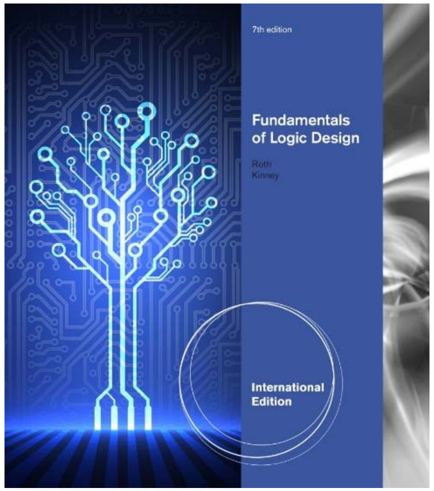
CHAPTER 5

KARNAUGH MAPS



This chapter in the book includes:

Objectives

1. Given a function (completely or in completely specified) of three to five variable, plot it on a Karnaugh map.
   The function may be given in minterm, maxterm, or algebraic form.

2. Determine the essential prime implicants of a function from a map.

3. Obtain the minimum sum-of-products or minimum product-of-sums form of a function from the map.

4. Determine all of the prime implicants of a function from a map.

5. Understand the relation between operations performed using the map and the corresponding algebraic operation.

5.1 Minimum Forms of Switching Functions

1. Combine terms by using $XY' + XY = X$

Do this repeatedly to eliminates as many literals as possible.

A given term may be used more than once because $X + X = X$

2. Eliminate redundant terms by using the consensus theorems.

5.1 Minimum Forms of Switching Functions

Example: Find a minimum sum-of-products

$$F(a,b,c) = \sum m(0,1,2,5,6,7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + b'c + bc' + ab$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= a'b' + bc' + ac$$

5.1 Minimum Forms of Switching Functions

Example: Find a minimum product-of-sums

$$(A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$$

$$= (A+B'+D')\quad(A+B'+C')\quad(B'+C'+D)\quad(B+C'+D)$$

$$= (A+B'+D')\qquad\underbrace{(A+B'+C')}\qquad(C'+D)$$

$$= (A+B'+D')(C'+D)$$

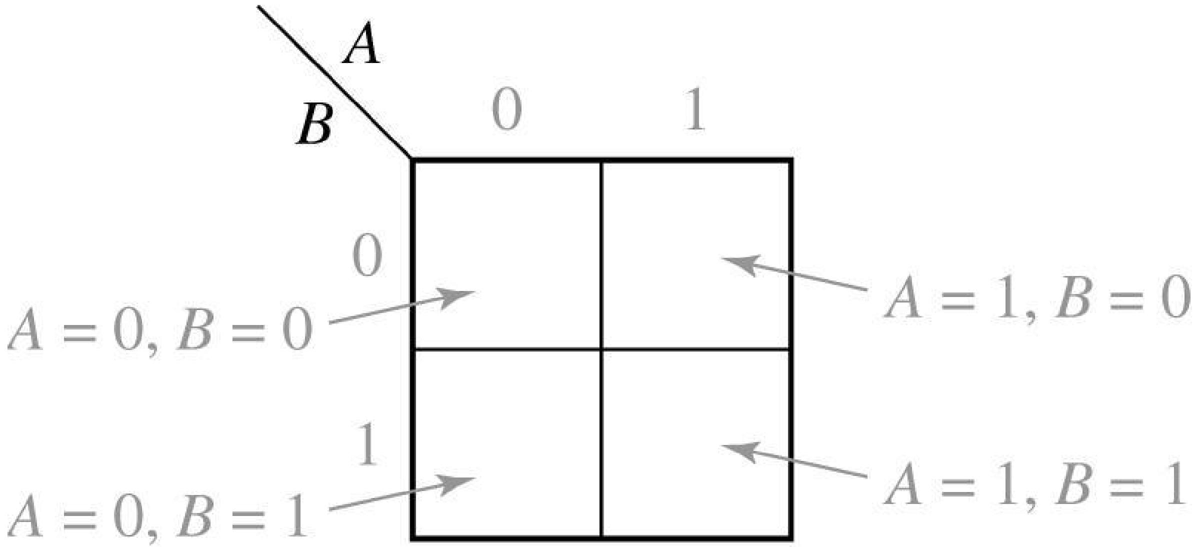
Eliminate by consensus

5.2 Karnaugh Maps

- **Simplification using algebraic rules can be impossible, difficult, tedious**
- **Two-dimensional truth-table.**
- **Karnaugh maps can be used up to 6 variables**

5.2 Two- and Three-Variable Karnaugh Maps

2-variable Karnaugh Map

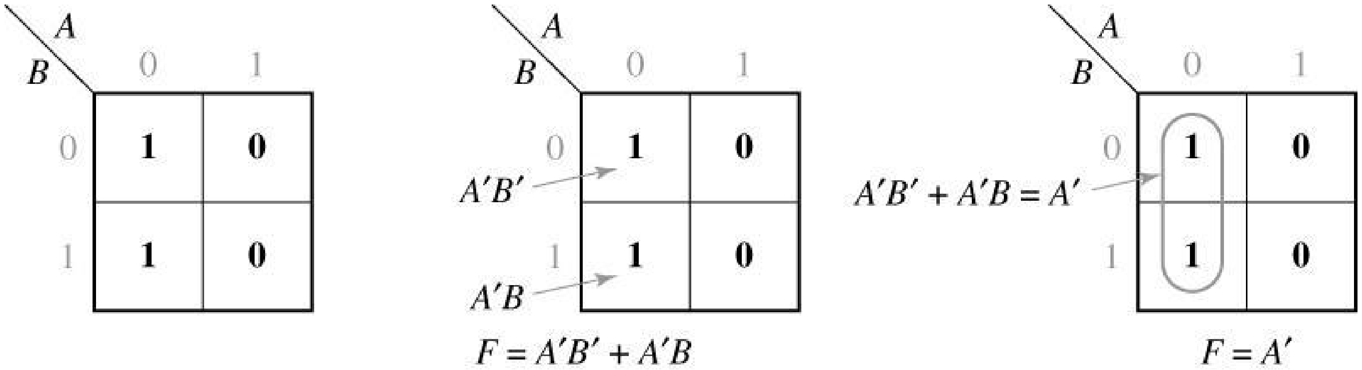


5.2 Two- and Three-Variable Karnaugh Maps

Truth Table for a function F

A B	F
0 0	1
0 1	1
1 0	0
1 1	0

(a)



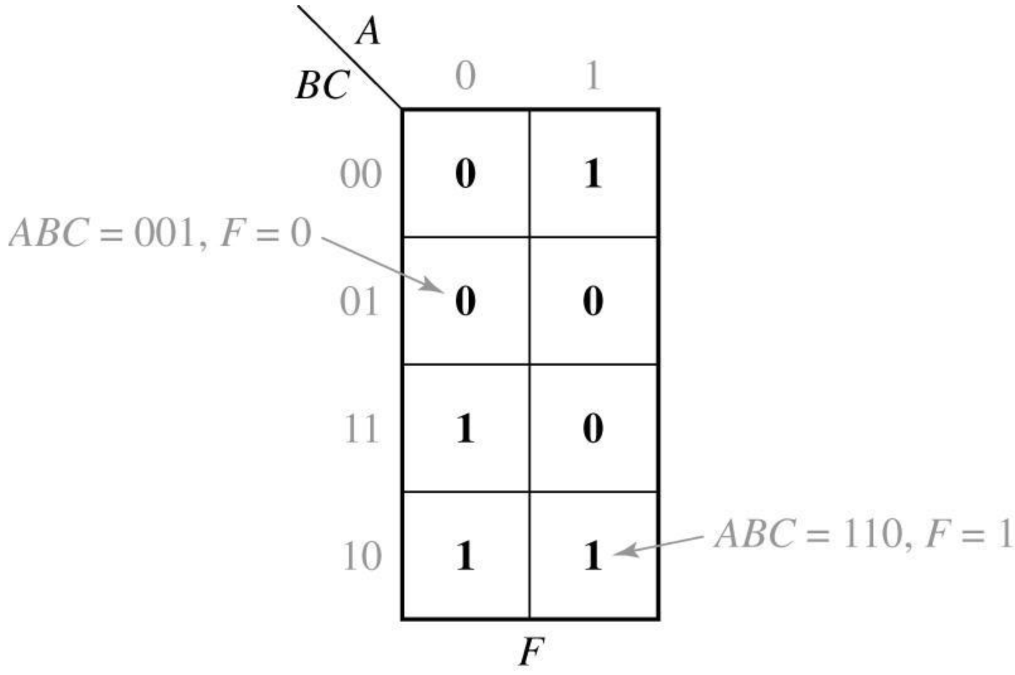
(b) (c) (d)

5.2 Two- and Three-Variable Karnaugh Maps

Truth Table and Karnaugh Map for Three-Variable Function

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

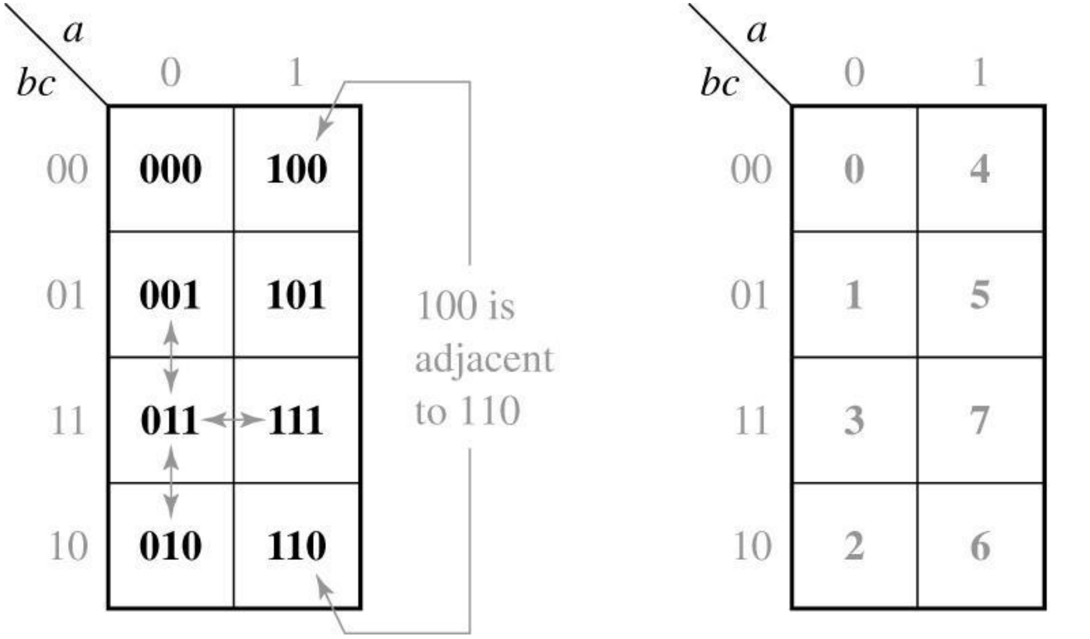
(a)



(b)

5.2 Two- and Three-Variable Karnaugh Maps

Location of Minterms on a Three-Variable Karnaugh Map



(a) Binary notation (b) Decimal notation

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Map of F(*a*, *b*, *c*) = $\sum$m(1, 3, 5) = $\prod$(0, 2, 4, 6, 7)

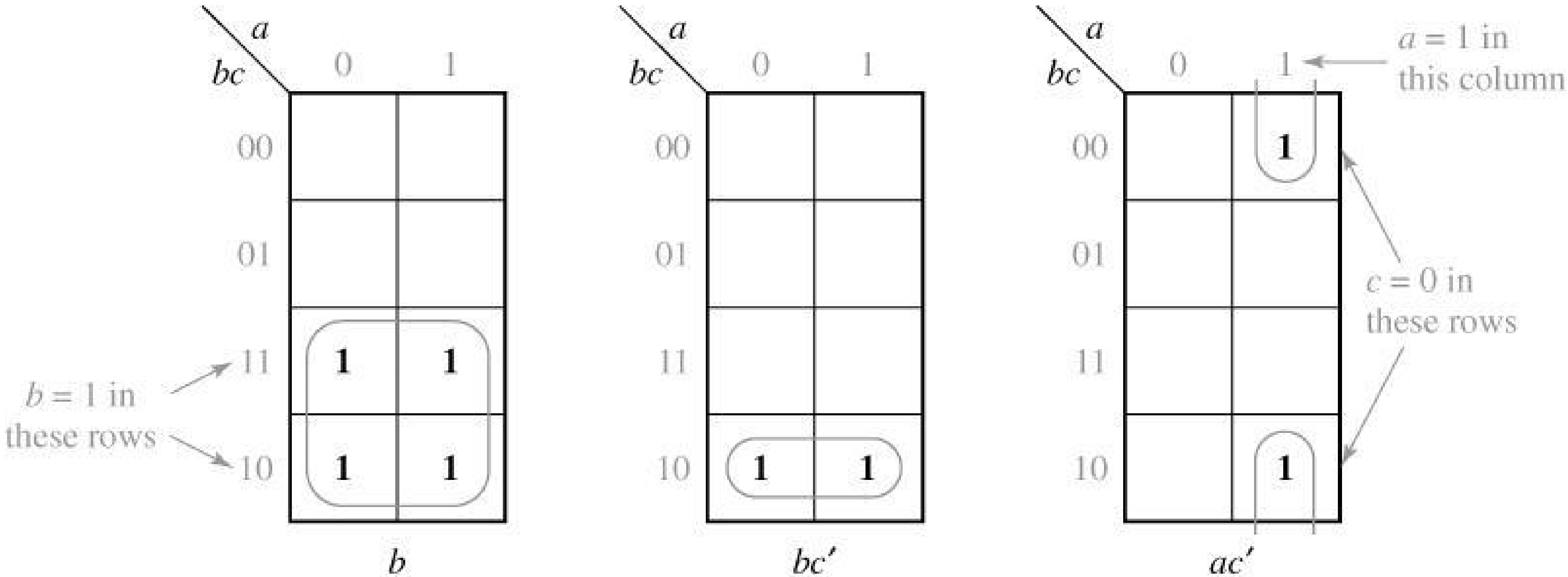
bc \ *a*	0	1
00	**0** (0)	**0** (4)
01	**1** (1)	**1** (5)
11	**1** (3)	**0** (7)
10	**0** (2)	**0** (6)

$$F(a,b,c) = m_1 + m_3 + m_5$$

$$= M_0 \cdot M_2 \cdot M_4 \cdot M_6 \cdot M_7$$

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Maps for Product Terms

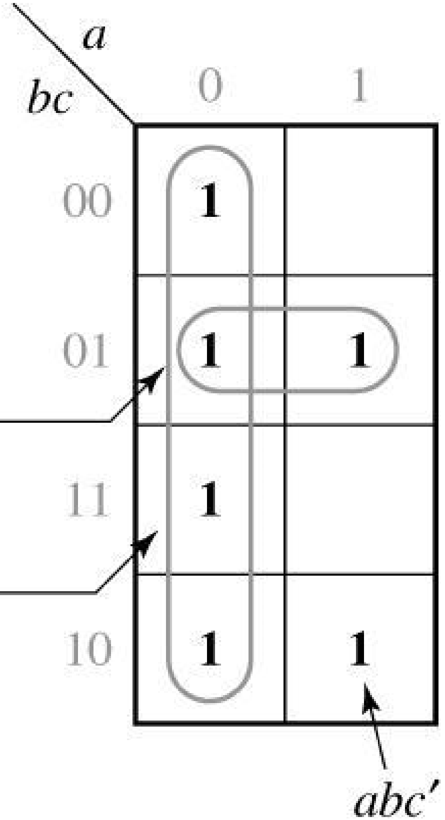


5.2 Two- and Three-Variable Karnaugh Maps

Given Function

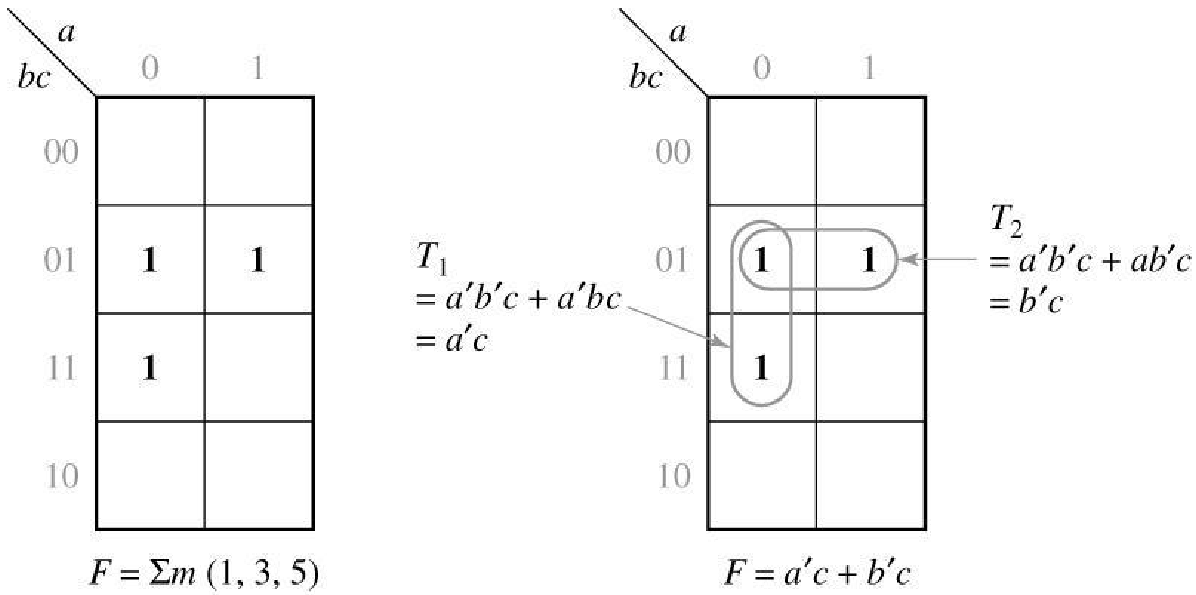
$$f(a,b,c) = abc' + b'c + a'$$

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the abc = 001 square, we do not have to place a second 1 there because $x + x = x$.)



5.2 Two- and Three-Variable Karnaugh Maps

Simplification of a Three-Variable Function

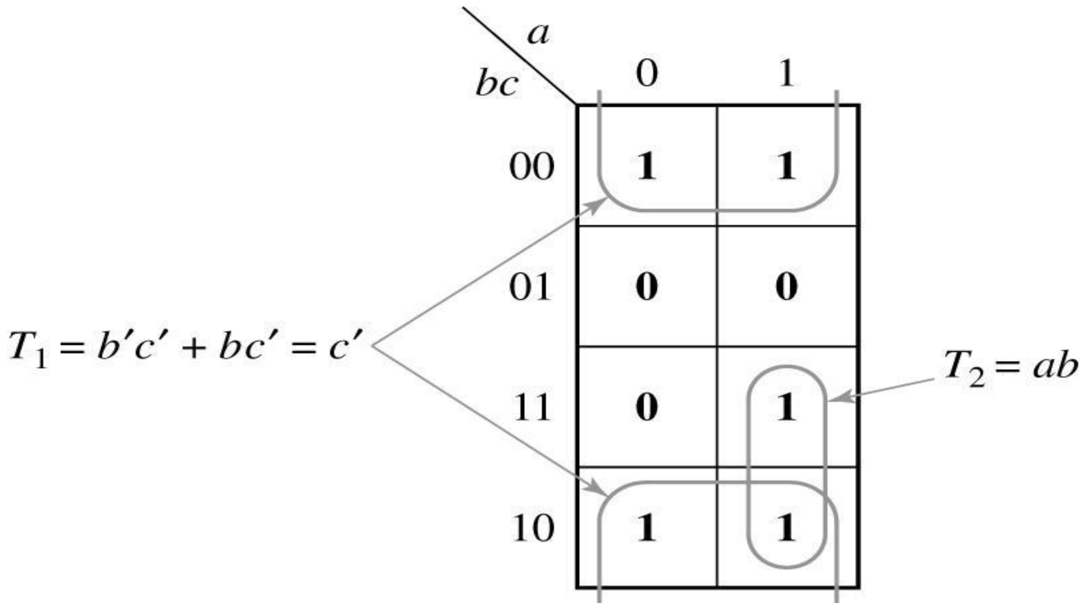


(a) Plot of minterms (b) Simplified form of F

$$F = T_1 + T_2 = a'c + b'c$$

5.2 Two- and Three-Variable Karnaugh Maps

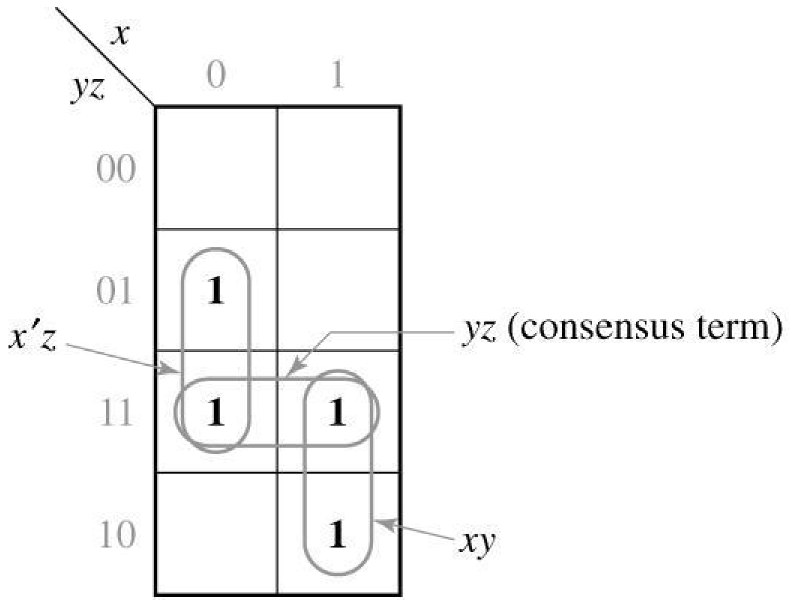
Complement of Map in Figure 5-6(a)

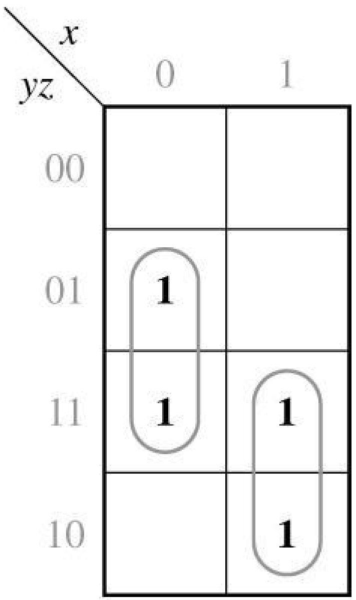


$$F = T_1 + T_2 = c' + ab$$

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Maps Which Illustrate the Consensus Theorem





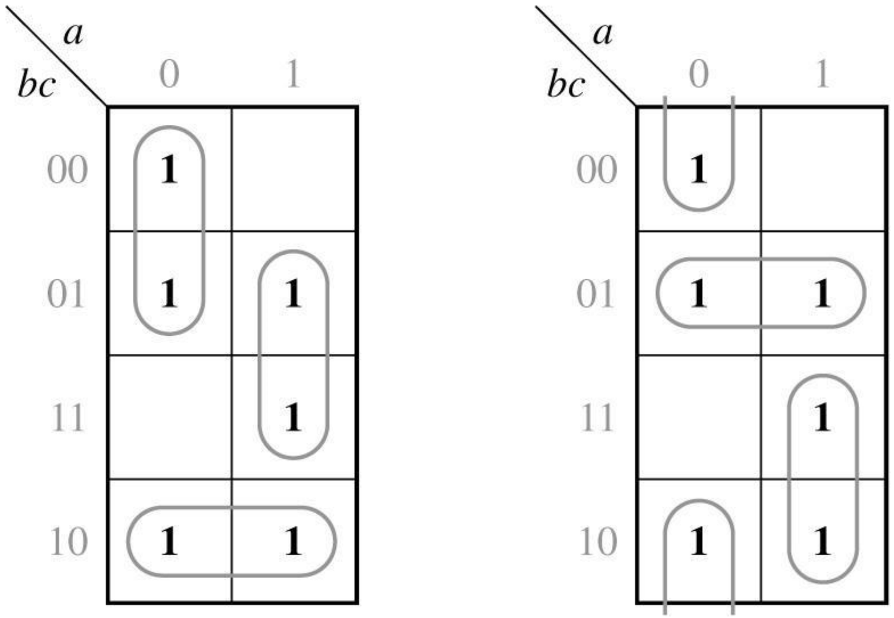
$$xy + x'z + yz = xy + x'z$$

Consensus term is redundant

5.2 Two- and Three-Variable Karnaugh Maps

Function with Two Minimal Forms

$$F = \sum m(0,1,2,5,6,7)$$



$F = a'b' + bc' + ac$ $\qquad$ $F = a'c' + b'c + ab$

5.3 Four-Variable Karnaugh Maps

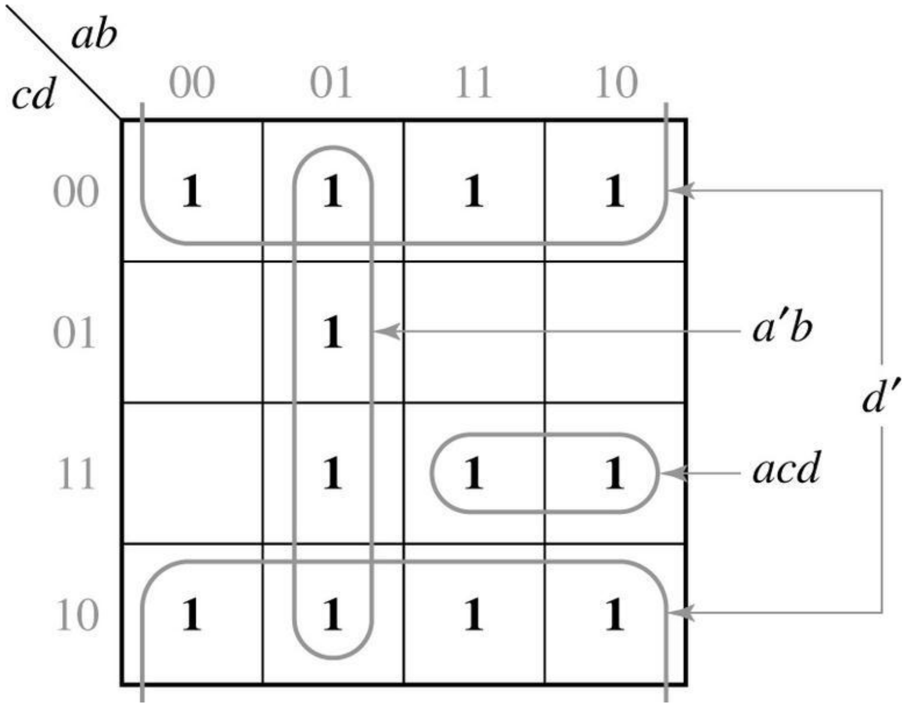
Location of Minterms on Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

5.3 Four-Variable Karnaugh Maps

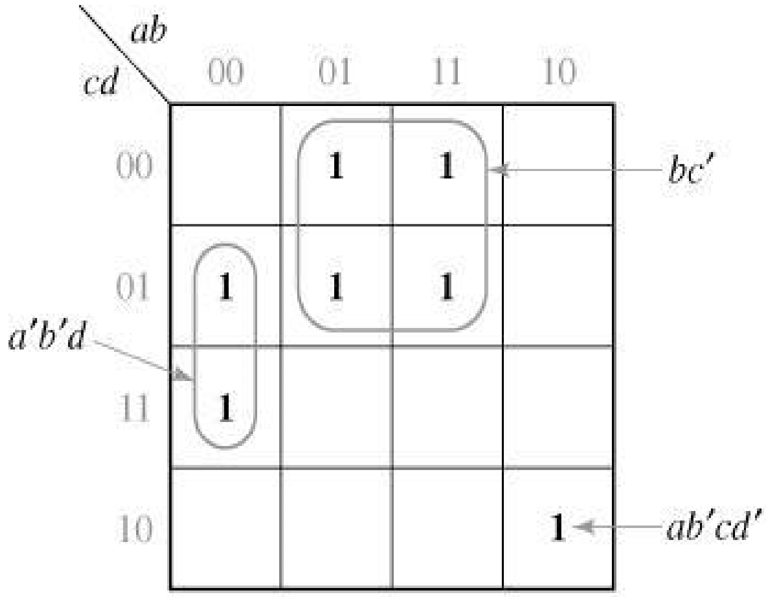
Plot of ***acd* + *a'b* + *d'***

$$f(a,b,c,d) = acd + a'b + d'$$



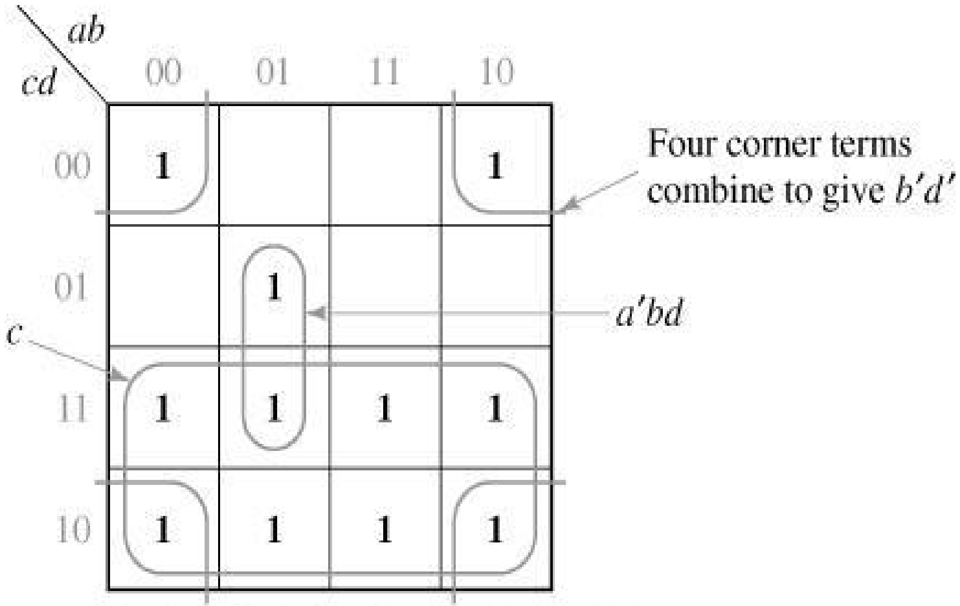
5.3 Four-Variable Karnaugh Maps

Simplification of Four-Variable Functions



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

(a)

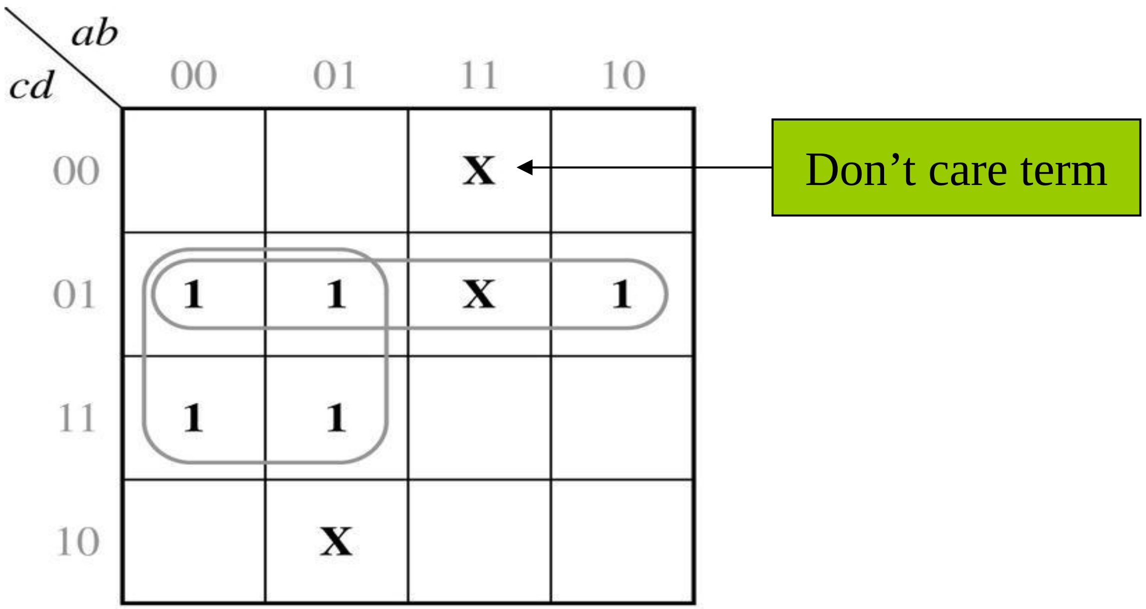


$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

5.3 Four-Variable Karnaugh Maps

Simplification of an Incompletely Specified Function



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

5.3 Four-Variable Karnaugh Maps

Figure 5-14

1's of f

$$f = x'z' + wyz + w'y'z' + x'y$$

0's of f

$$f' = y'z + wxz' + w'xy$$

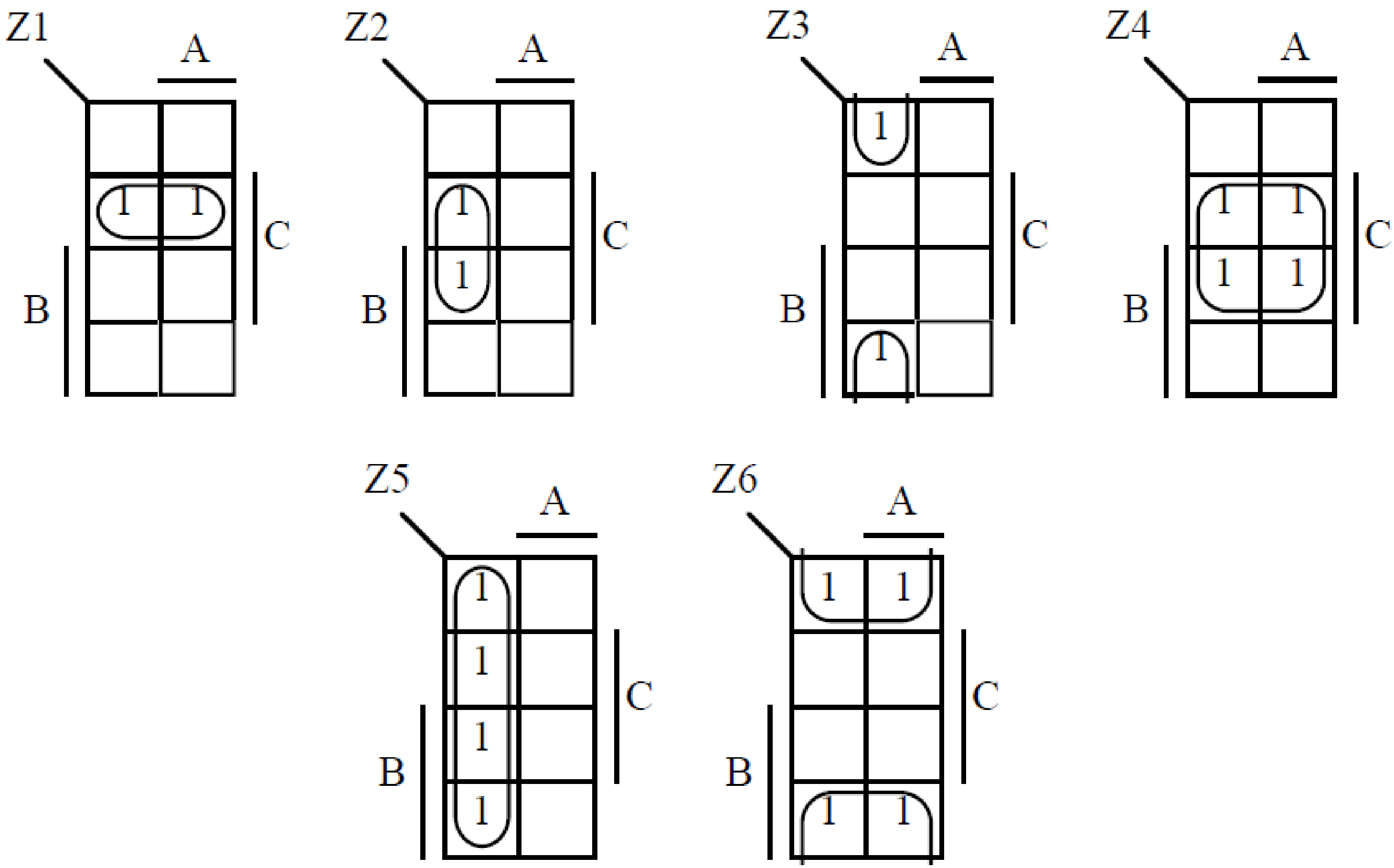
$$f = (y + z')(w' + x' + z)(w + x' + y')$$

minimum product of sums for f

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

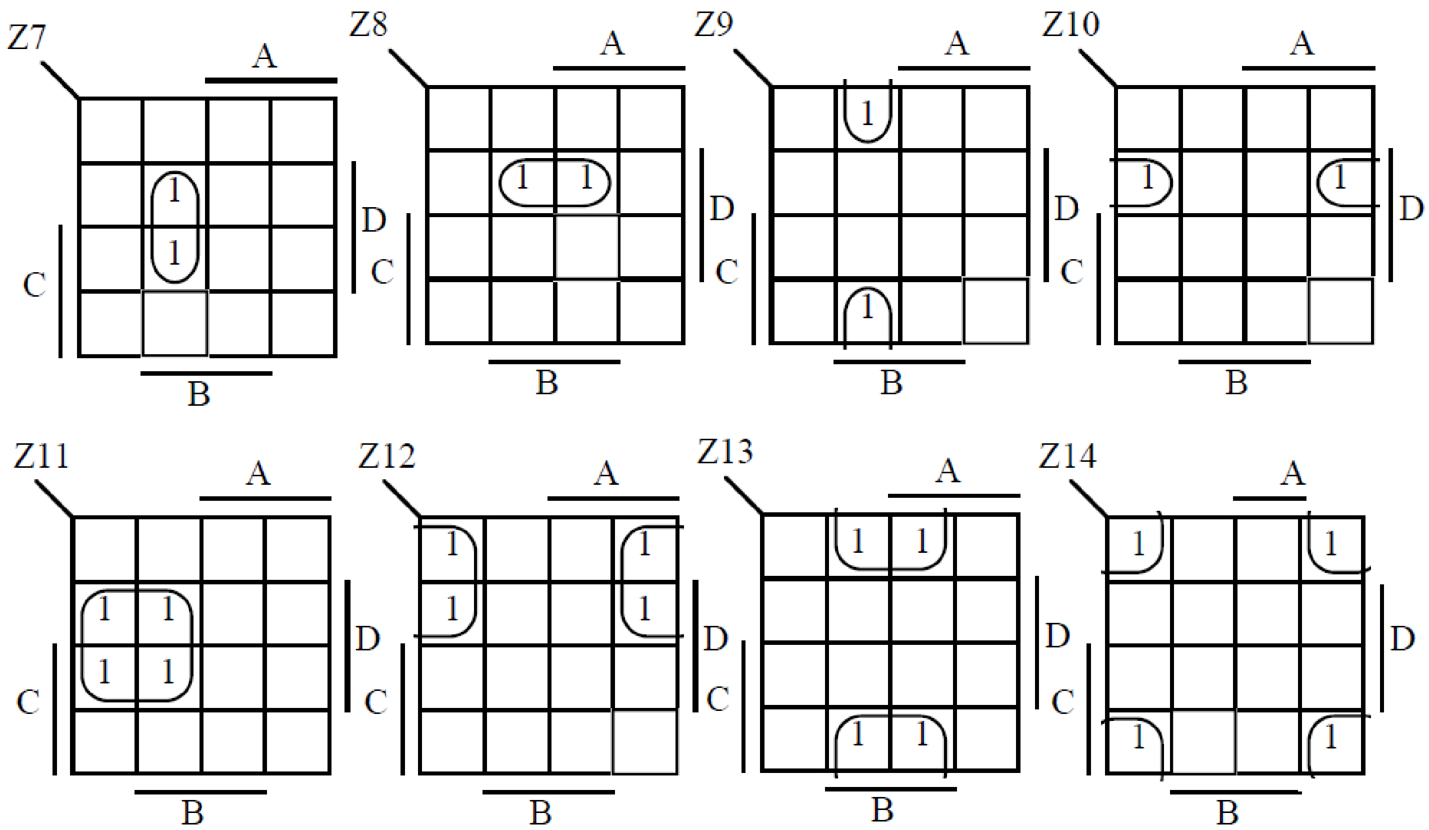
Basic Karnaugh Map Groupings

For Three-Variable Maps



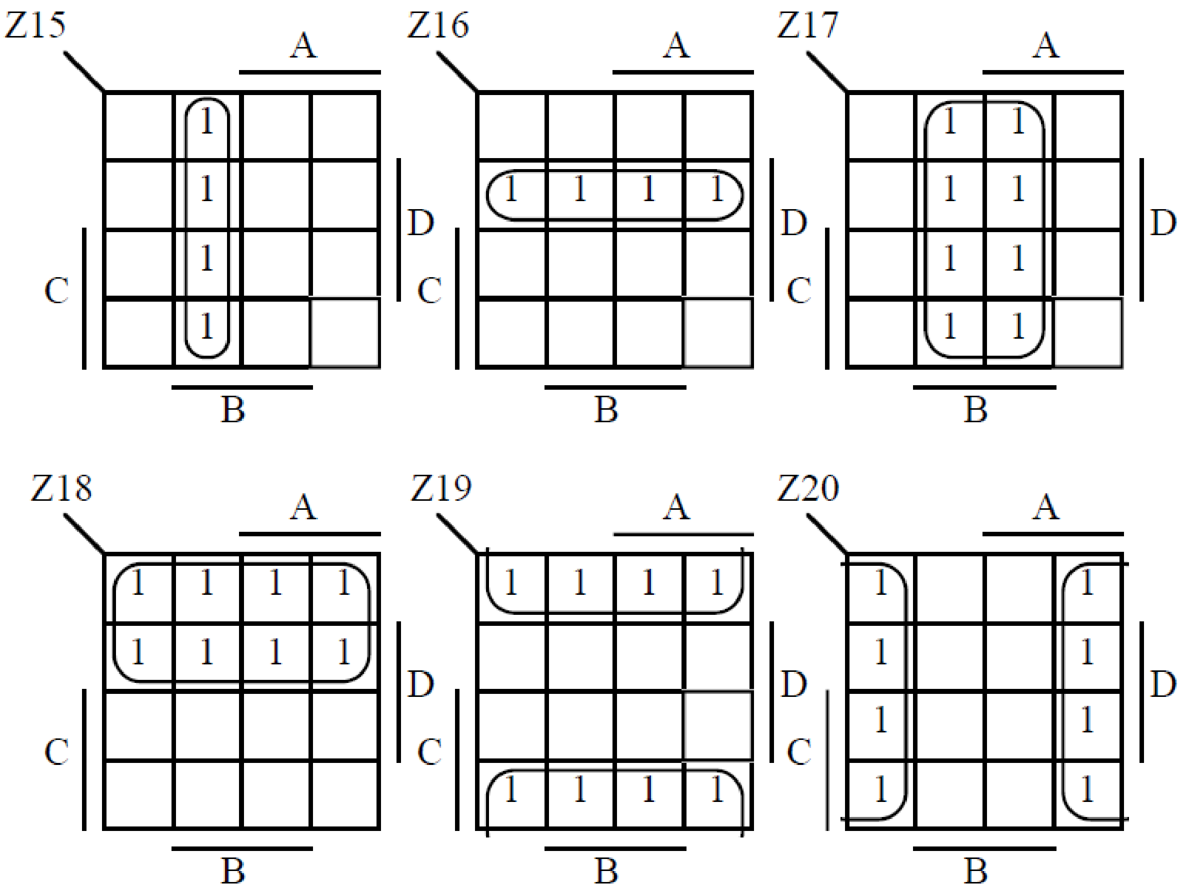
Basic Karnaugh Map Groupings

For Four-Variable Maps



Basic Karnaugh Map Groupings

For Four-Variable Maps



5.4 Determination of Minimum Expressions Using Essential Prime Implicants

- ***Implicants of F*** : Any single '1' or any group of "1's which can be combined together on a Map ➔ each grouping of any size is thus an implicant

- ***Prime Implicants of F*** : A product term if it can not be combined with other terms to eliminate variable ➔ a largest possible grouping

- ***Essential Prime Implicants of F*** : A prime implicant that is the ONLY cover for some 1's on the map (essential is relative to a particular minterm)

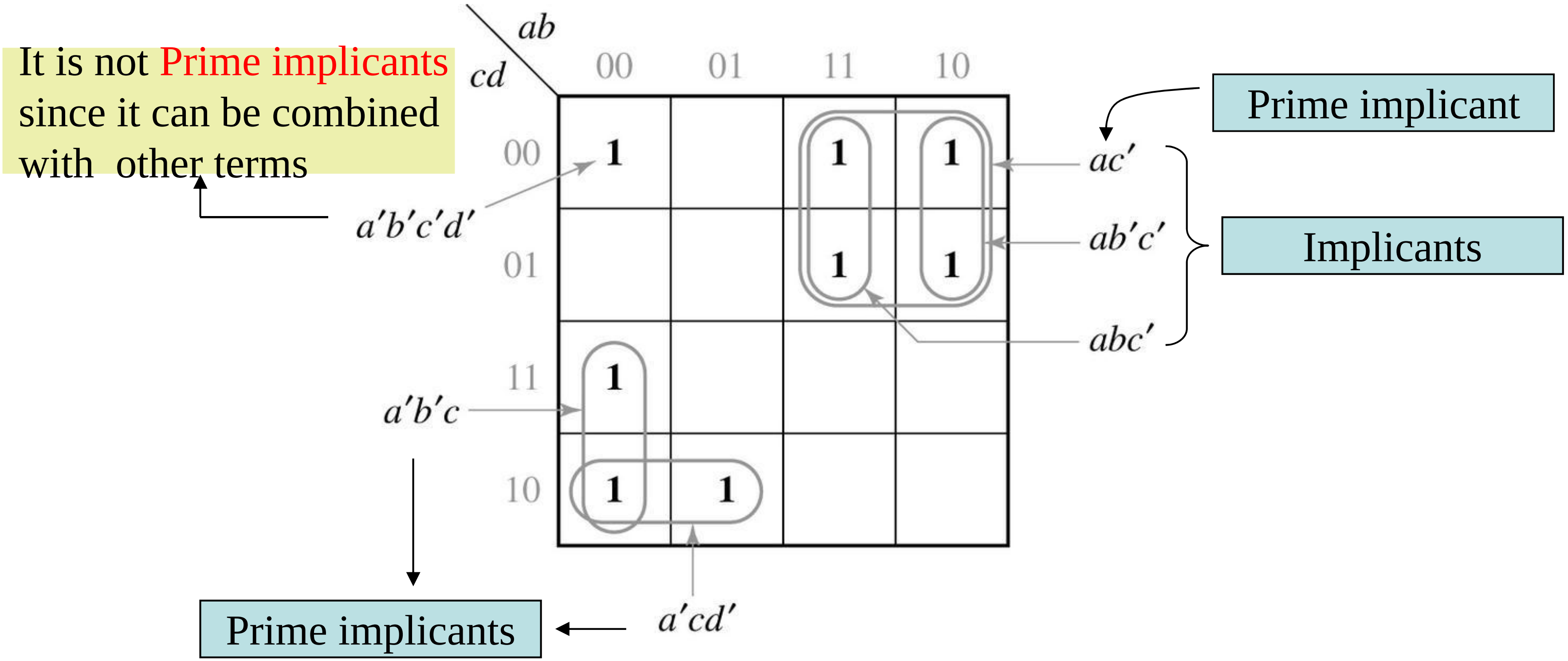
➔ always look for E.P.I. first in simplication

Simplification Procedure

Step 1) Identify those groupings that are maximal

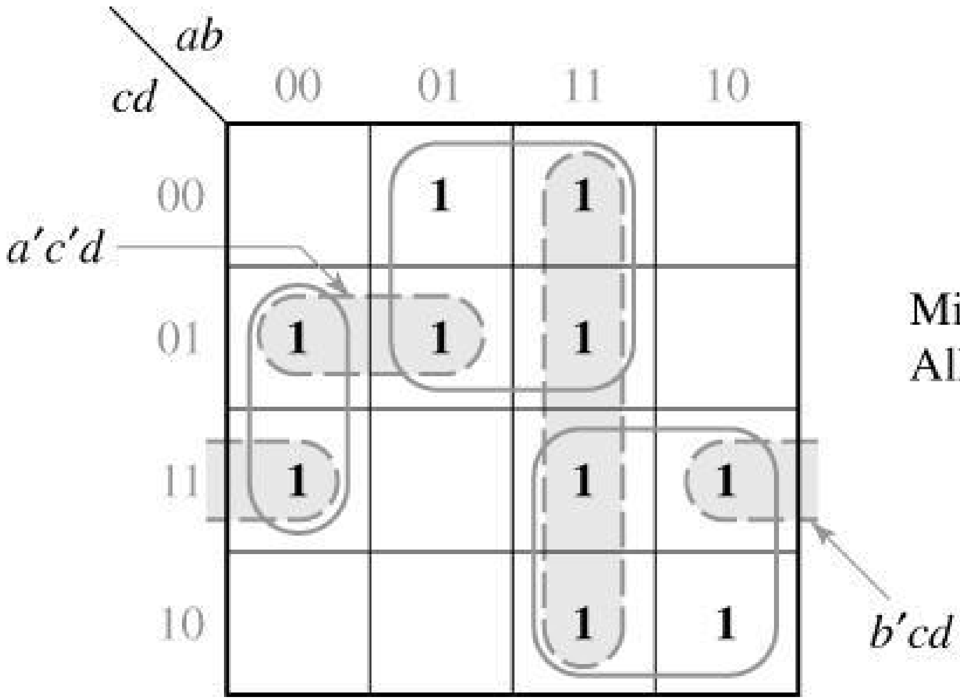
Step 2) Use the fewest possible number of maximal groupings

5.4 Determination of Minimum Expressions Using Essential Prime Implicants



5.4 Determination of Minimum Expressions Using Essential Prime Implicants

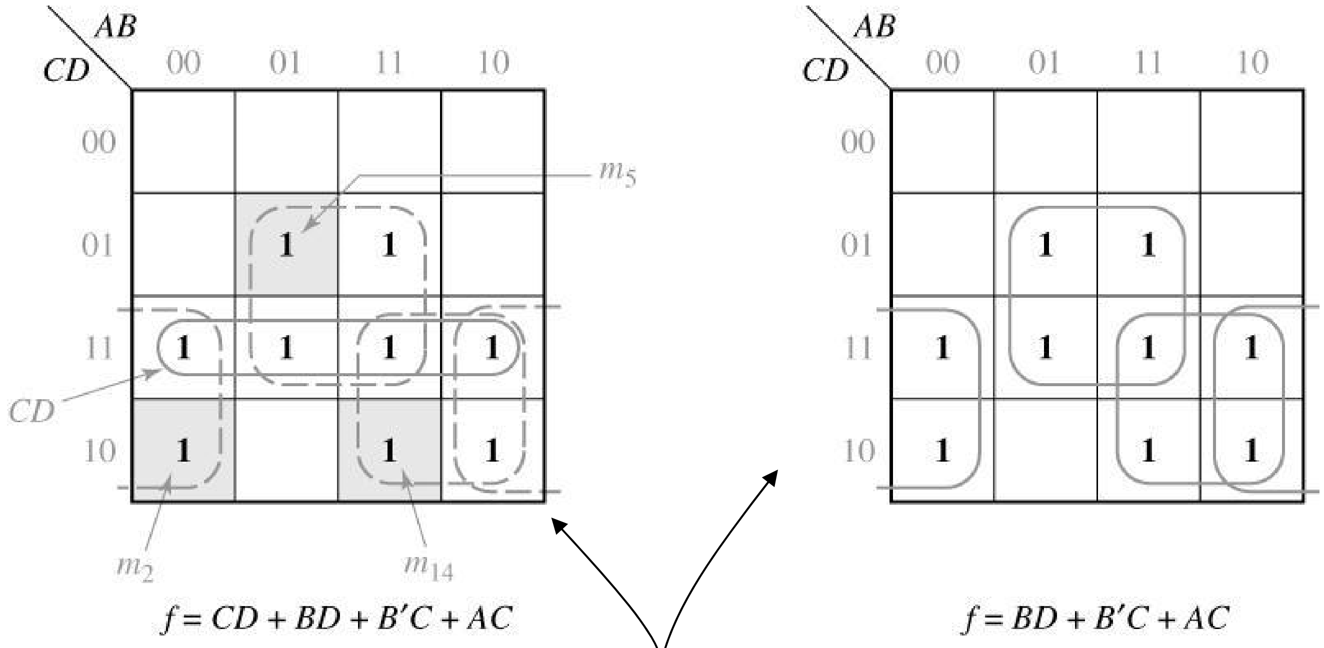
Determination of All Prime Implicants



Minimum solution: $F = a'b'd + bc' + ac$
All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

Because all of the prime implicants of a function are generally not needed in forming the minimum sum of products, selecting prime implicants is needed.



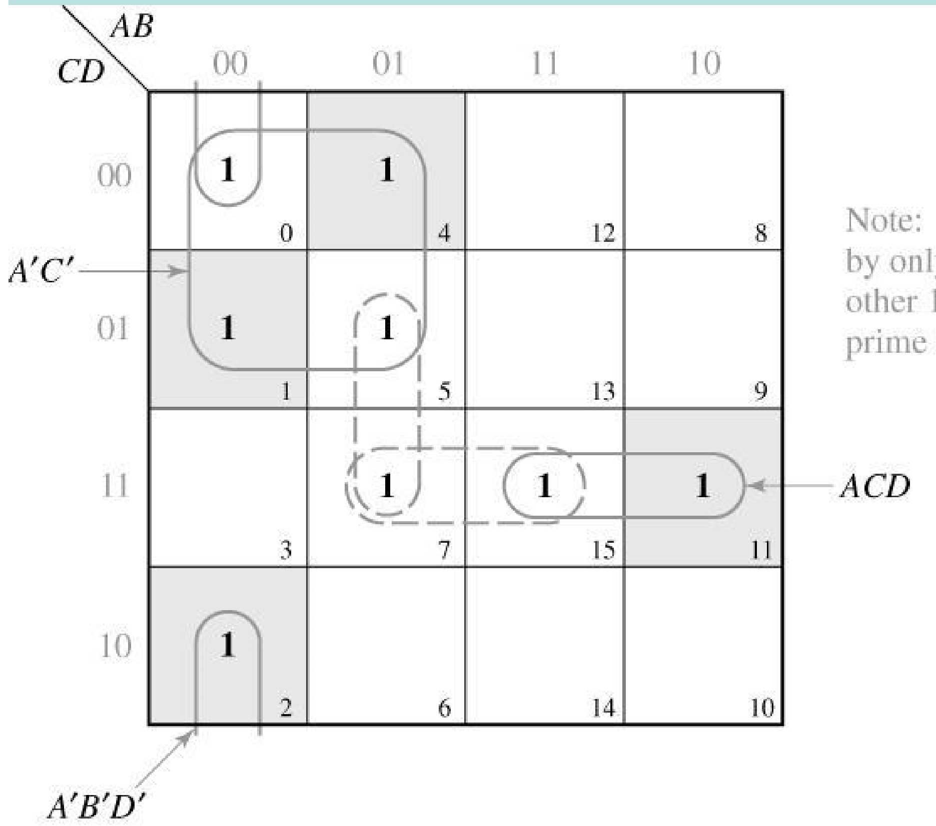
- ***CD*** **is not needed to cover for minimum expression**

-*B'C* , *AC* , *BD* are "essential" prime implicants

- *CD* is not an "essential " prime implicant

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

1. **First, find essential prime implicants**
2. **If minterms are not covered by essential prime implicants only, more prime implicants must be added to form minimum expression.**

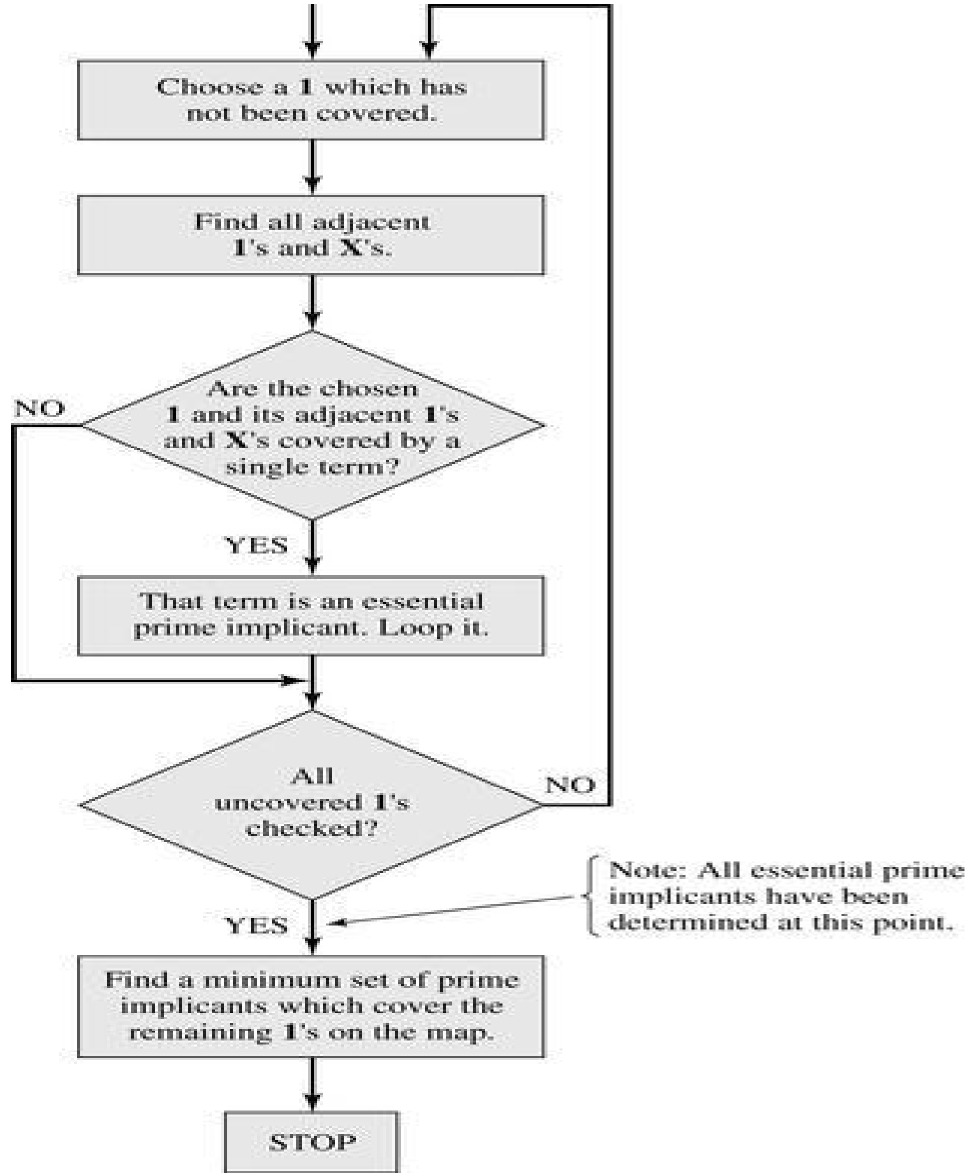


Note: 1's shaded in blue are covered by only one prime implicant. All other 1's are covered by at least two prime implicants.

$$A'C' + A'B'D' + ACD + \begin{cases} A'BD \\ \text{or} \\ BCD \end{cases}$$

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

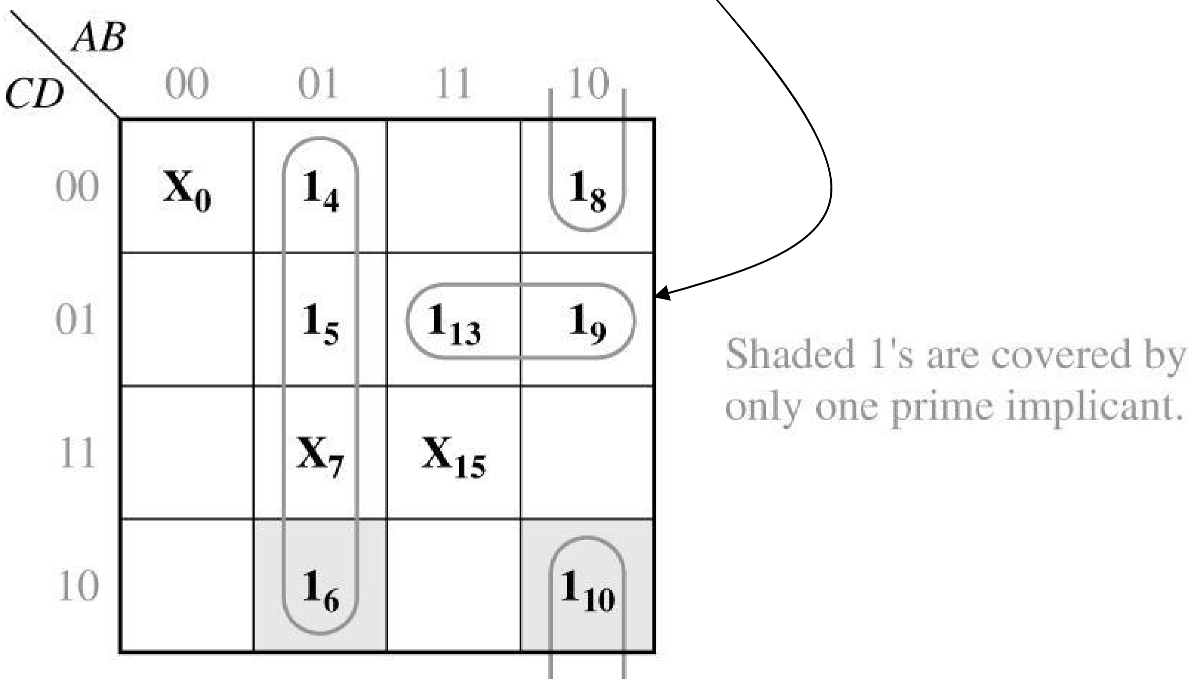


5.4 Determination of Minimum Expressions Using Essential Prime Implicants

1) *A'B* **covers** $\mathbf{I_6}$ **and its adjacent → essential PI**

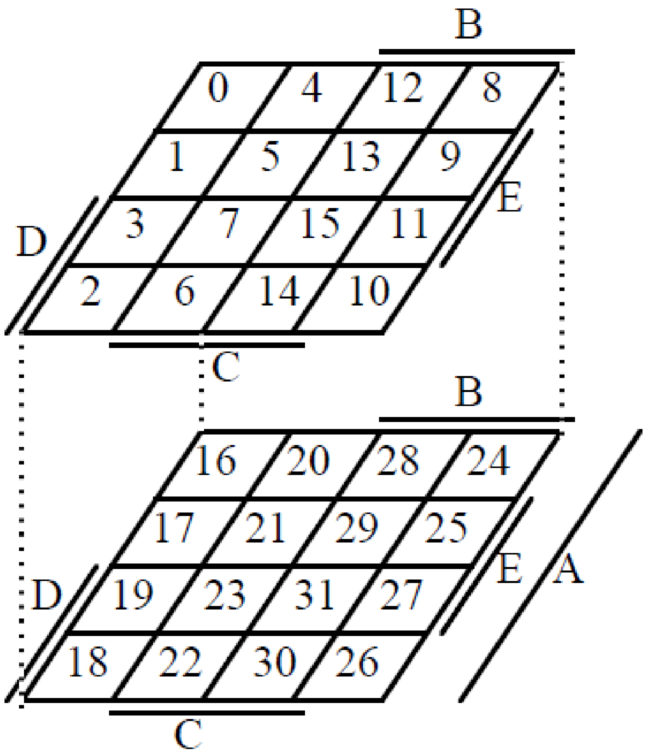
2) *AB'D'* **covers** $\mathbf{I_{10}}$ **and its adjacent → essential PI**

3) *AC'D* **is chosen for minimal cover →** *AC'D* **is not an essential PI**

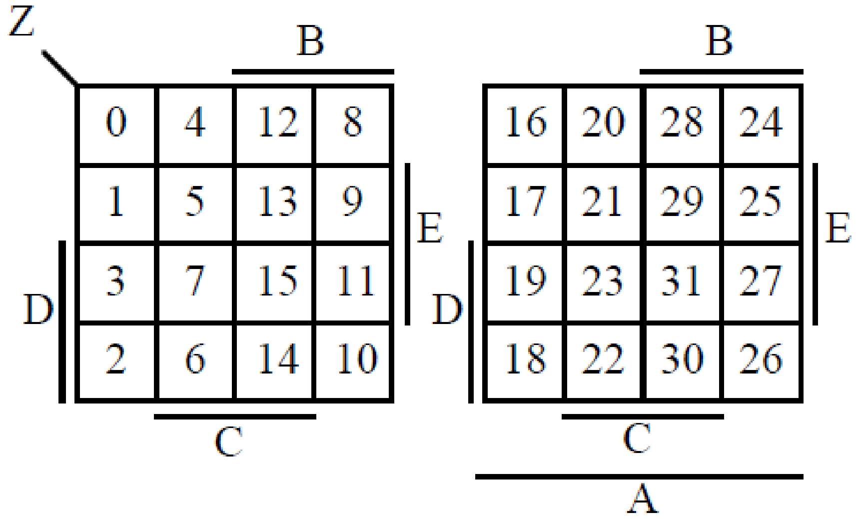


5.5 Five-Variable Karnaugh Maps

Five-Variable Karnaugh Map



Five-Variable Map Structure

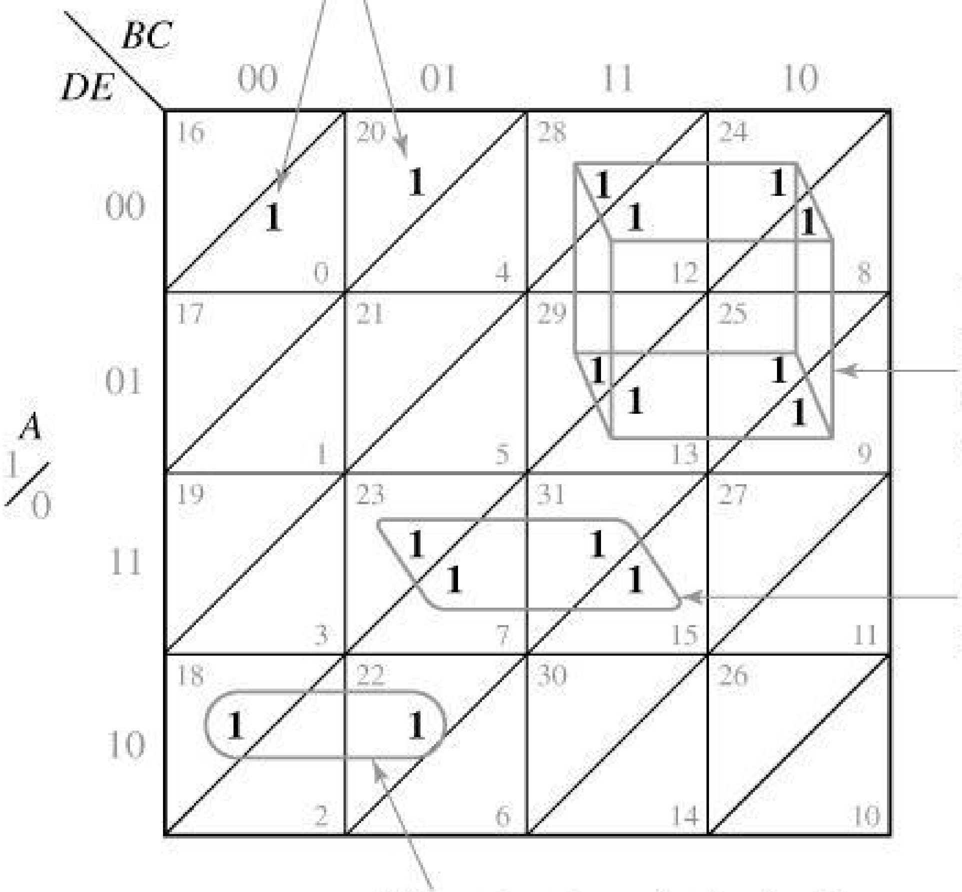


Alternate Version of Five-Variable Map

5.5 Five-Variable Karnaugh Maps

Five-Variable Karnaugh Map

These terms do not combine because they are in different layers and different columns (they differ in two variables).



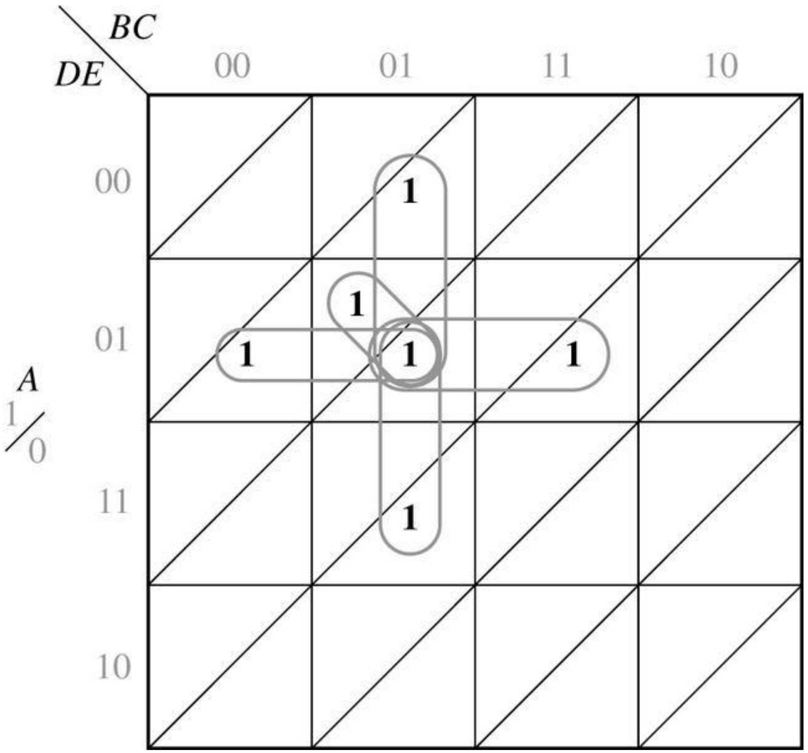
These eight terms combine to give BD' (B from last two columns and D' from top two rows; A is eliminated because four terms are in the top layer and four in the bottom).

These four terms (two from top layer and two from bottom) combine to yield CDE (C from the middle two columns and DE from the row).

These two terms in the top layer combine to give $AB'DE'$.

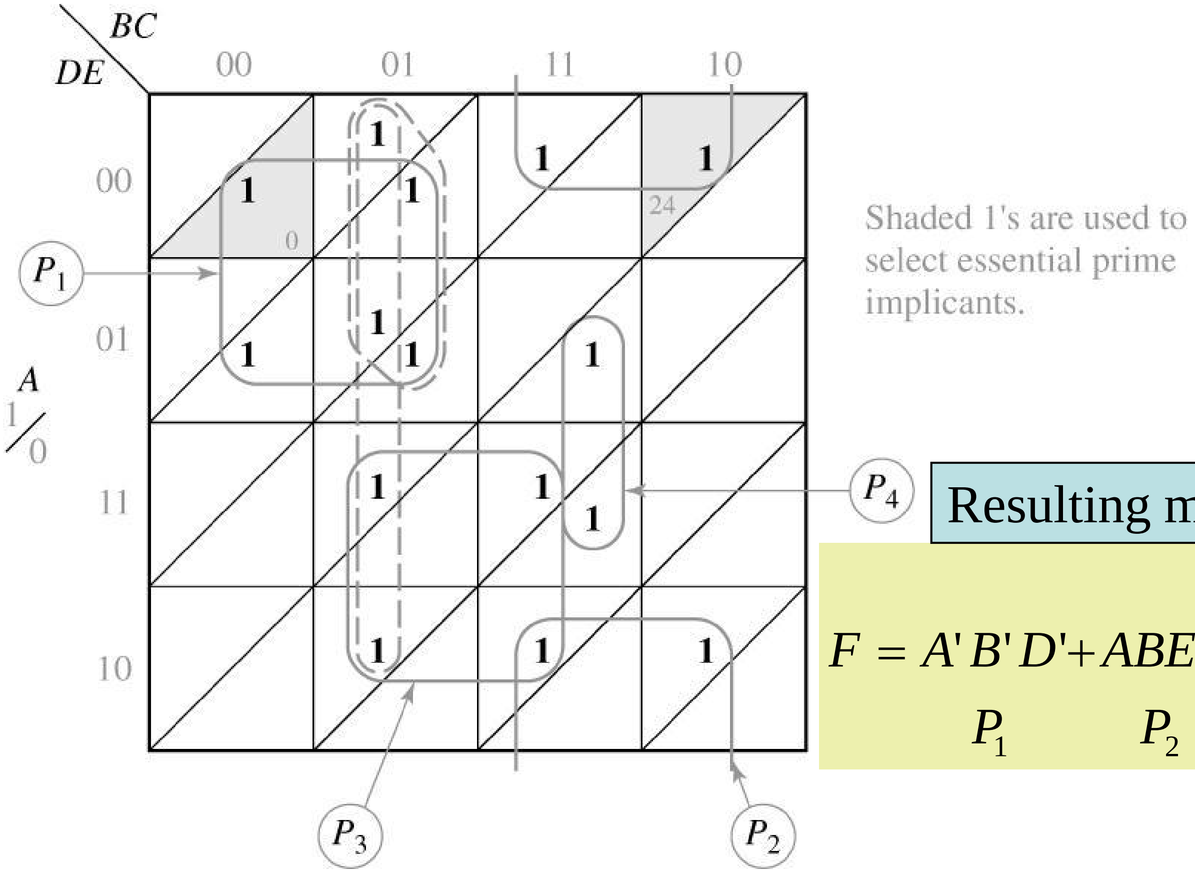
5.5 Five-Variable Karnaugh Maps

Figure 5-22



5.5 Five-Variable Karnaugh Maps

Figure 5-23 $F(A,B,C,D,E)=\sum m(0,1,4,5,13,15,20,21,22,23,24,26,28,30,31)$



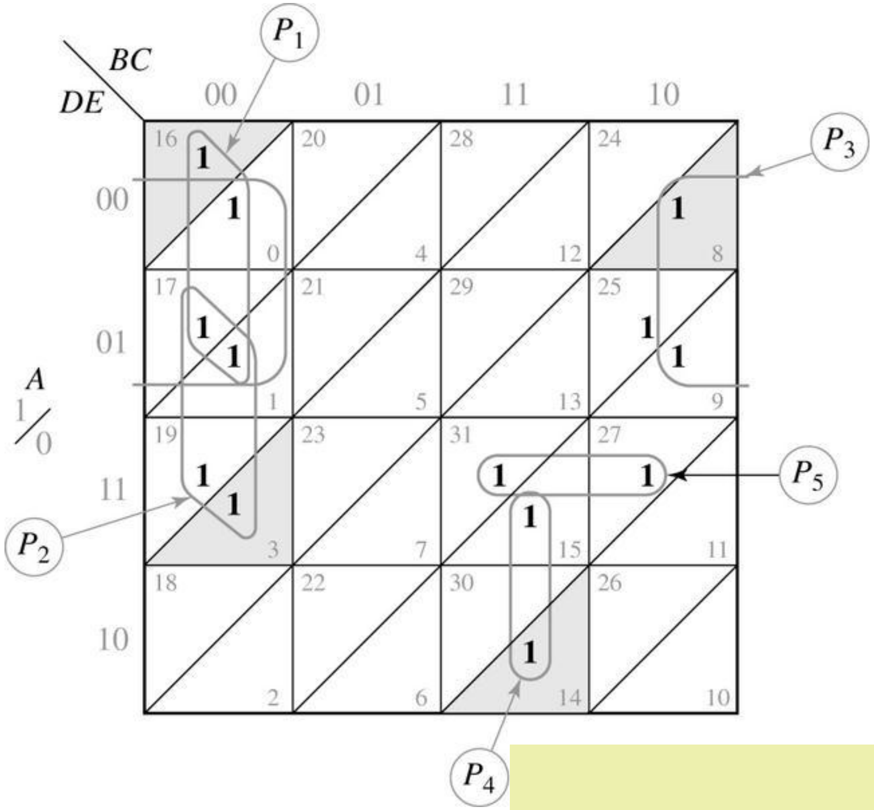
Resulting minimum solution

$$F = \underbrace{A'B'D'}_{P_1} + \underbrace{ABE'}_{P_2} + \underbrace{ACD}_{P_3} + \underbrace{A'BCE}_{P_4} + \begin{Bmatrix} AB'C \\ \text{or} \\ B'CD' \end{Bmatrix}$$

5.5 Five-Variable Karnaugh Maps

Figure 5-24

$$F(A,B,C,D,E)=\sum m(0,1,3,8,9,14,15,16,17,19,25,27,31)$$

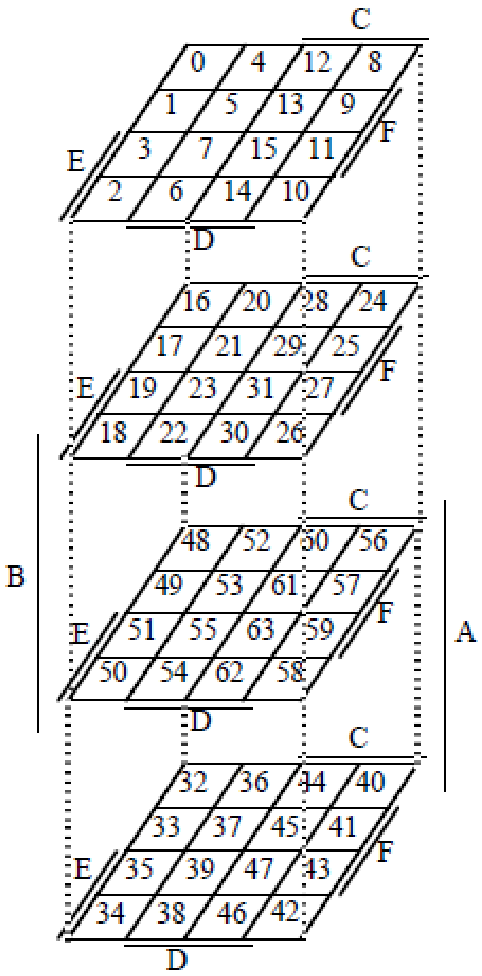


Final solution

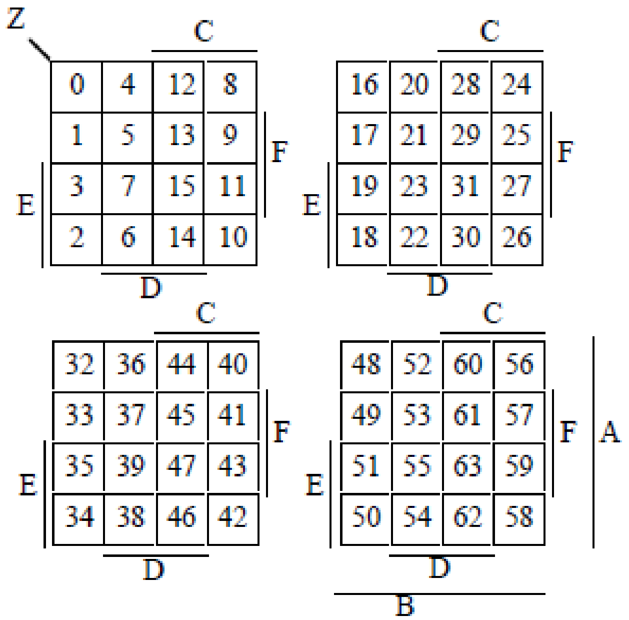
$$F = \underbrace{B'C'D'}_{P_1} + \underbrace{B'C'E}_{P_2} + \underbrace{A'C'D'}_{P_3} + \underbrace{A'BCD}_{P_4} + \underbrace{ABDE}_{P_5} + \left\{ \begin{array}{c} C'D'E \\ \text{or} \\ AC'E \end{array} \right\}$$

Six-Variable Karnaugh Maps

Six-Variable Karnaugh Map



Six Variable Map Structure



Alternate Version of Six-Variable Map

5.6 Other Uses of Karnaugh Maps

$$\left.\begin{array}{l}\text{minterm expansion of } f \text{ is } \quad f = \sum m(0,2,3,4,8,10,11,15) \\ \text{maxterm expansion of } f \text{ is } \quad f = \prod M(1,5,6,7,9,12,13,14)\end{array}\right\} \boxed{\text{same}}$$

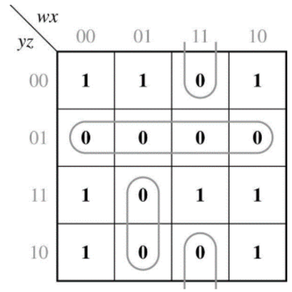
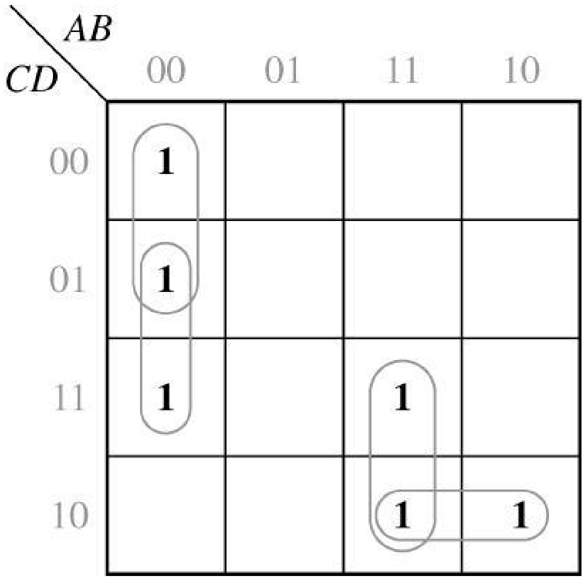


Figure 5-25



$$F = A'B'(C' + D) + AC(B + D')$$

5.6 Other Uses of Karnaugh Maps

Figure 5-26

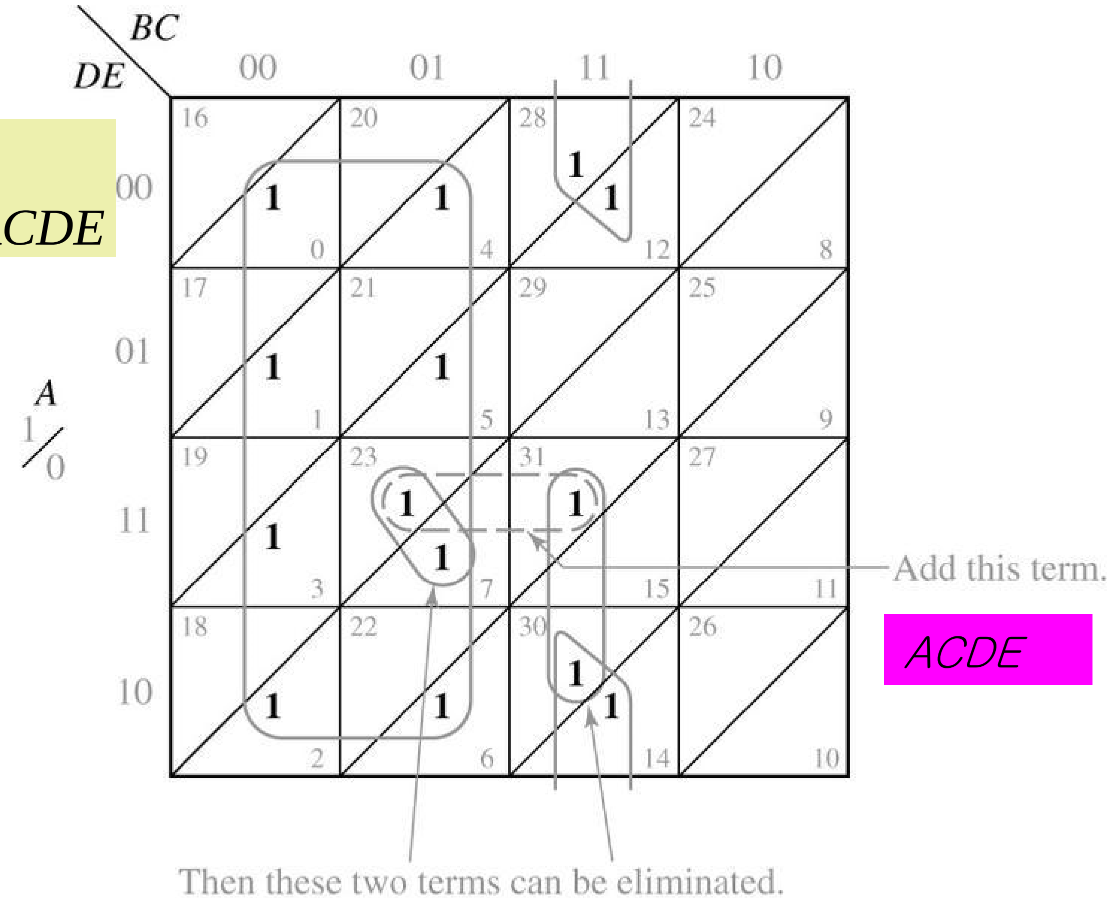
$$F = ABCD + B'CDE + A'B' + BCE'$$

Using the consensus theorem:

$$F = ABCD + B'CDE + A'B' + BCE' + ACDE$$

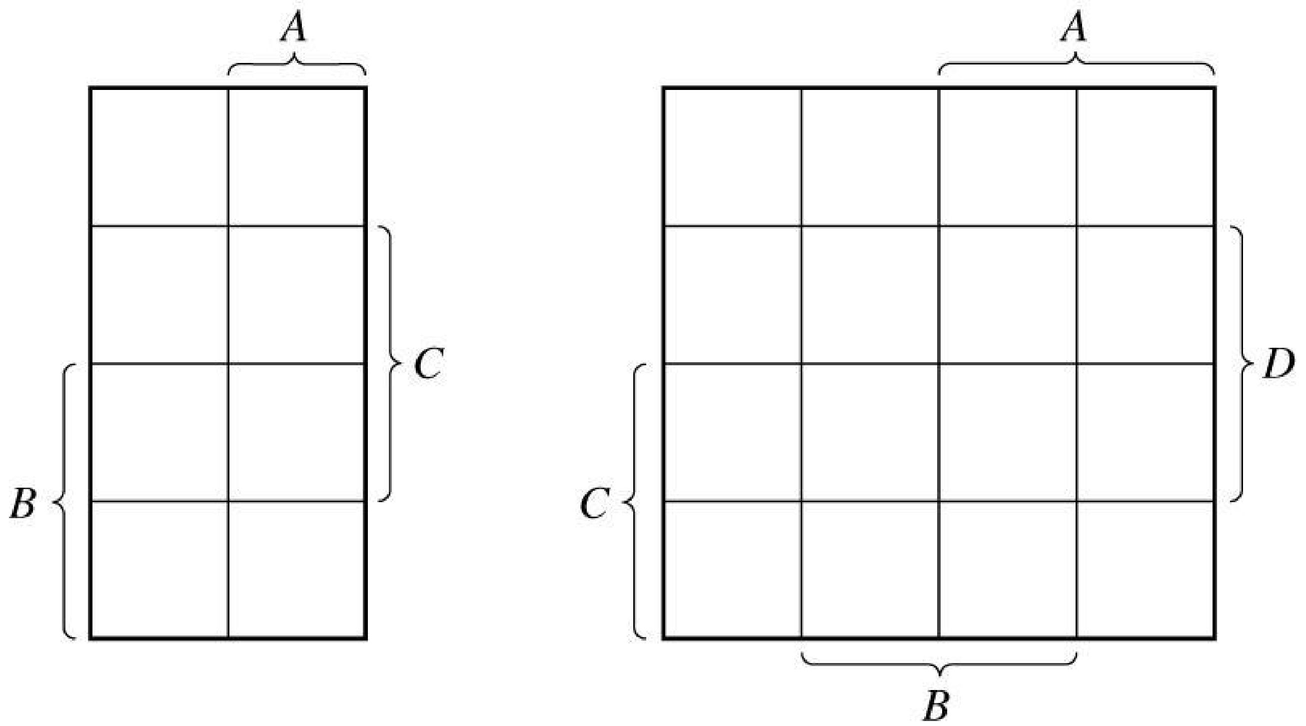
minimum solution:

$$F = A'B' + BCE' + ACDE$$



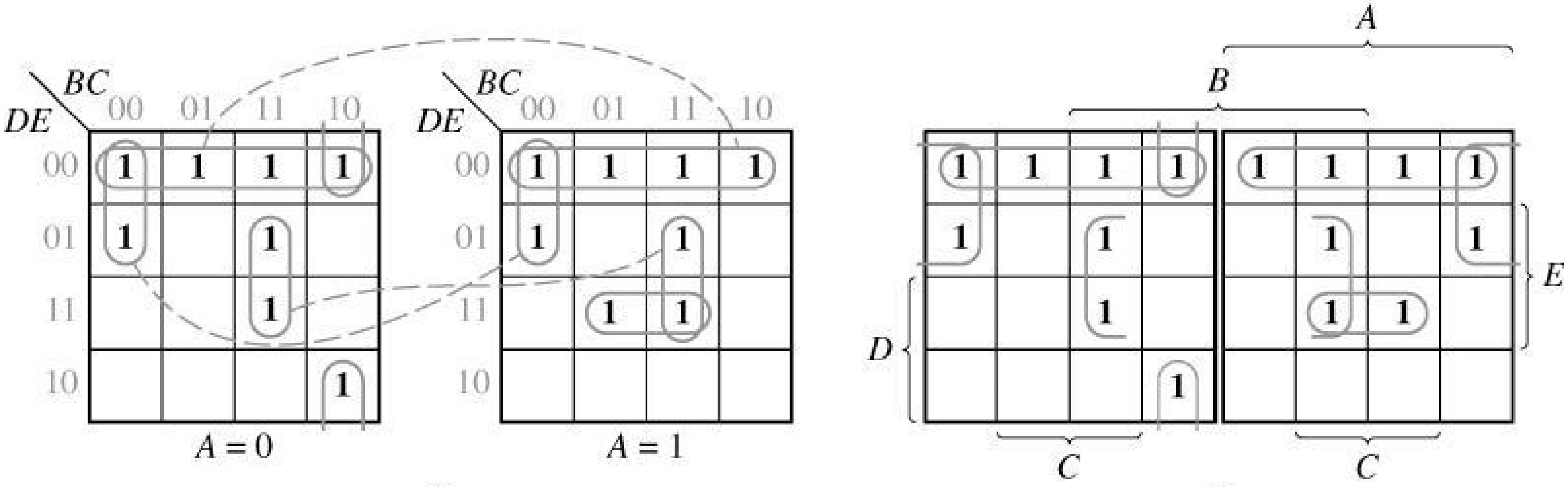
5.7 Other Forms of Karnaugh Maps

Figure 5-27. Veitch Diagrams



5.7 Other Forms of Karnaugh Maps

Figure 5-28. Other Forms of Five-Variable Karnaugh Maps



(a) (b)

$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$