Programming Languages

First-Class Functions
Subtyping

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What is functional programming?

"*Functional programming*" can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)

2. Using functions as values (this lecture)

…

- Style encouraging recursion and recursive data structures
- Style closer to mathematical definitions
- Programming idioms using *laziness* (later topic, briefly)
- Anything not OOP or C? (not a good definition)

Not sure a definition of "*functional language*" exists beyond "makes functional programming easy / the default / required"

  – No clear yes/no for a particular language

First-class functions

- *First-class functions*: Can use them *wherever* we use values
  - Functions are values too
  - Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, …

```
fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incr 7))
```

- Most common use is as an argument / result of another function
  - Other function is called a *higher-order function*
  - Powerful way to *factor out* common functionality

Function Closures

- *Function closure*: Functions can use bindings from outside the function definition (in scope where function is defined)
  - Makes first-class functions *much* more powerful
  - Will get to this feature in a bit, after simpler examples

- Distinction between terms *first-class functions* and *function closures* is not universally understood
  - Important conceptual distinction even if terms get muddled

Functions as arguments

- We can pass one function as an argument to another function
  - Not a new feature, just never thought to do it before

```
fun f (g,…) = … g (…) …
fun h1 … = …
fun h2 … = …
…   f(h1,…) … f(h2,…) …
```

- Elegant strategy for factoring out common code
  - Replace *N* similar functions with calls to 1 function where you pass in *N* different (short) functions as arguments

Example

Can reuse **`n_times`** rather than defining many similar functions

- Computes **`f(f(…f(x)))`** where number of calls is **`n`**

```
fun n_times (f,n,x) =




fun double x = x + x
fun increment x = x + 1
val x1 = n_times(double,4,7)
val x2 = n_times(increment,4,7)
val x3 = n_times(tl,2,[4,8,12,16])

fun double_n_times (n,x) = n_times(double,n,x)
fun nth_tail (n,x) = n_times(tl,n,x)
```

Example

Can reuse **`n_times`** rather than defining many similar functions

- Computes **`f(f(…f(x)))`** where number of calls is **`n`**

```
fun n_times (f,n,x) =
   if n=0
   then x
   else f (n_times(f,n-1,x))

fun double x = x + x
fun increment x = x + 1
val x1 = n_times(double,4,7)
val x2 = n_times(increment,4,7)
val x3 = n_times(tl,2,[4,8,12,16])

fun double_n_times (n,x) = n_times(double,n,x)
fun nth_tail (n,x) = n_times(tl,n,x)
```

Relation to types

- Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables
- But there are higher-order functions that are not polymorphic
- And there are non-higher-order (first-order) functions that are polymorphic
- Always a good idea to understand the type of a function, especially a higher-order function

Types for example

```
fun n_times (f,n,x) =
    if n=0
    then x
    else f (n_times(f,n-1,x))
```

- `val n_times : ('a -> 'a )* int * 'a => 'a`

- Two of our examples *instantiated* `'a` with `int`
- One of our examples *instantiated* `'a` with `int list`
- This *polymorphism* makes `n_times` more useful

- Type is *inferred* based on how arguments are used (later lecture)
  - Describes which types must be exactly something (e.g., `int`) and which can be anything but the same (e.g., `'a`)

Types for example

```
fun n_times (f,n,x) =
    if n=0
    then x
    else f (n_times(f,n-1,x))
```

- **`val n_times : ('a -> 'a) * int * 'a -> 'a`**
  - Simpler but less useful: **`(int -> int) * int * int -> int`**

- Two of our examples *instantiated* **`'a`** with **`int`**
- One of our examples *instantiated* **`'a`** with **`int list`**
- This *polymorphism* makes **`n_times`** more useful

- Type is *inferred* based on how arguments are used (later lecture)
  - Describes which types must be exactly something (e.g., **`int`**) and which can be anything but the same (e.g., **`'a`**)

Polymorphism and higher-order functions

- Many higher-order functions are polymorphic because they are so reusable that some types, “can be anything”

- But some polymorphic functions are not higher-order
  - Example: **`length : 'a list -> int`**

- And some higher-order functions are not polymorphic
  - Example: **`times_until_0 : (int -> int) * int -> int`**

```
fun times_until_0 (f,x) =
   if x=0 then 0 else 1 + times_until_0(f, f x)
```

Note: Would be better with tail-recursion

Toward anonymous functions

- Definitions unnecessarily at top-level are still poor style:

```
fun triple x = 3*x
fun triple_n_times (n,x) = n_times(triple,n,x)
```

- So this is better (but not the best):

```
fun triple_n_times (n,x) =
  let fun trip y = 3*y
  in
     n_times(trip,n,x)
  end
```

- And this is even smaller scope
  - It makes sense but looks weird (poor style; see next slide)

```
fun triple_n_times (n,x) =
  n_times(let fun trip y = 3*y in trip end, n, x)
```

Anonymous functions

- This does not work: A function *binding* is not an *expression*

```
fun triple_n_times (f,x) =
  n_times((fun trip y = 3*y), n, x)
```

- This is the best way we were building up to: an expression form for *anonymous functions*

```
fun triple_n_times (f,x) =
  n_times((fn y => 3*y), n, x)
```

  - Like all expression forms, can appear anywhere
  - Syntax:
    - **`fn`** not **`fun`**
    - **`=>`** not **`=`**
    - no function name, just an argument pattern

Using anonymous functions

- Most common use: Argument to a higher-order function
  - Don't need a name just to pass a function

- But: Cannot use an anonymous function for a recursive function
  - Because there is no name for making recursive calls
  - If not for recursion, **`fun`** bindings would be syntactic sugar for **`val`** bindings and anonymous functions

```
fun triple x = 3*x

val triple = fn y => 3*y
```

A style point

Compare:

```
if x then true else false
```

With:

```
(fn x => f x)
```

So don’t do this:

```
n_times((fn y => tl y),3,xs)
```

When you can do this:

```
n_times(tl,3,xs)
```

Map

```
fun map (f,xs) =
    case xs of
      [] => []
    | x::xs' => (f x)::(map(f,xs'))
```

```
val map : ('a -> 'b) * 'a list -> 'b list
```

Map is, without doubt, in the “higher-order function hall-of-fame”

- The name is standard (for any data structure)
- You use it *all the time* once you know it: saves a little space, but more importantly, *communicates what you are doing*
- Similar predefined function: **`List.map`**
  - But it uses currying (coming soon)

Filter

```
fun filter (f,xs) =
    case xs of
      [] => []
    | x::xs' => if f x
                then x::(filter(f,xs'))
                else filter(f,xs')
```

```
val filter : ('a -> bool) * 'a list -> 'a list
```

Filter is also in the hall-of-fame

- So use it whenever your computation is a filter
- Similar predefined function: **`List.filter`**
  - But it uses currying (coming soon)

Generalizing

Our examples of first-class functions so far have all:

- Taken one function as an argument to another function
- Processed a number or a list

But first-class functions are useful anywhere for any kind of data

- Can pass several functions as arguments
- Can put functions in data structures (tuples, lists, etc.)
- Can return functions as results
- Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”

- No new language features

Returning functions

- Remember: Functions are first-class values
  - For example, can return them from functions

- Silly example:

```
fun double_or_triple f =
    if f 7
    then fn x => 2*x
    else fn x => 3*x
```

Has type `(int -> bool) -> (int -> int)`

But the REPL prints `(int -> bool) -> int -> int`
because it never prints unnecessary parentheses and
`t1 -> t2 -> t3 -> t4` means `t1->(t2->(t3->t4))`

Other data structures

- Higher-order functions are not just for numbers and lists
- They work great for common recursive traversals over your own data structures (datatype bindings) too
- Example of a higher-order *predicate*:
  - Are all constants in an arithmetic expression even numbers?
  - Use a more general function of type

    `(int -> bool) * exp -> bool`
  - And call it with **`(fn x => x mod 2 = 0)`**

Type Synonyms

What if I want to call `int * int * int` a `date`?

```
type date = int * int * int
```

Type Synonyms

type vs datatype

Datatype introduces a new type name,

distinct from all existing types

```
datatype suit = Club | Diamond | Heart | Spade
datatype card_value = Jack | Queen | King
                    | Ace | Num of int
```

Type is just another name

```
type card = suit * rank
```

Type Synonym

Why?

For now, just for convenience.

It doesn’t let us do anything new.

Later in the course we will see another use related to modularity.

Type Generality

Write a function that appends two string lists…

Type Generality

We expected

```
string list * string list -> string list
```

But the type checker says

```
'a list * 'a list -> 'a list
```

Why is this okay?

Type Generality

The type ‘a is more general

More general types “can be used” as any less general type.

Type Generality

The “more general” rule

A type *t1* is more general than the type *t2* if you can take *t1*, replace its type variables consistently, and get *t2*

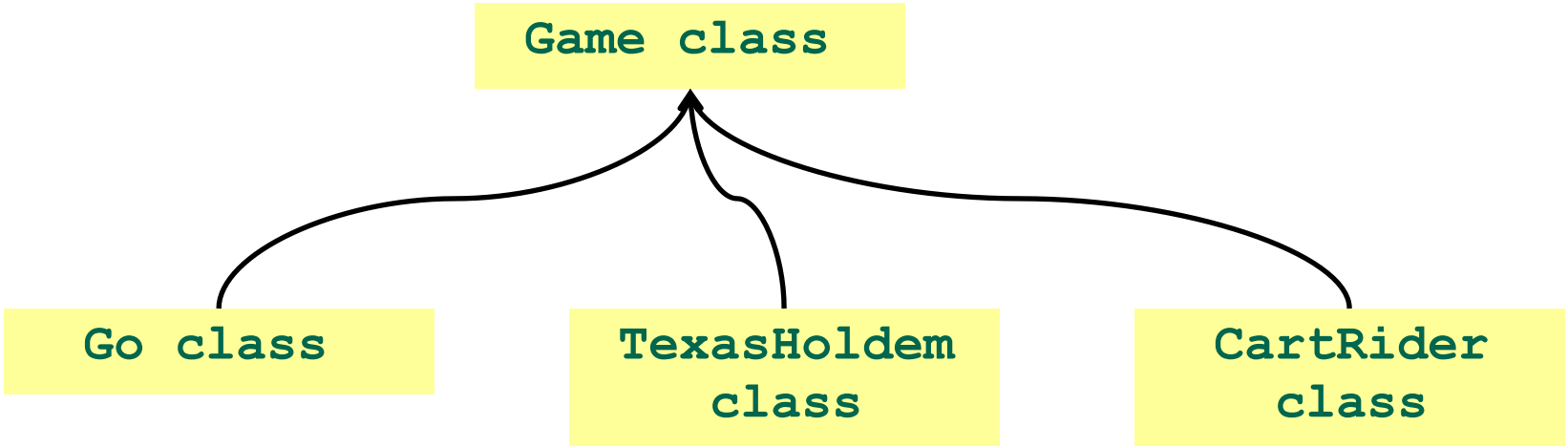
Subtype

If S and T represent type and S is a subtype of T (i.e. S <: T)

then

an instance of type S can be safely used in a context where an instance of type T is expected.

Subtype – example (Java)



Afterthought on Java arrays

Date: Fri, 09 Oct 1998 09:41:05 -0600

From: bill joy

Subject: …[discussion about java genericity]

actually, java array covariance was done for less noble reasons …: it made some generic "bcopy" (memory copy) and like operations much easier to write...

I proposed to take this out in 95, but it was too late (...).

i think it is unfortunate that it wasn't taken out...

it would have made adding genericity later much cleaner, and [array covariance] doesn't pay for its complexity today.

wnj