

# Class Topics (클래스 홈페이지 참조)

- ❑ Part 1: Fundamental concepts and principles
- ❑ Part 2: 빠른 컴퓨터를 위한 ISA design
  - Topic 1 Computer performance and ISA design (Ch. 1)
  - Topic 2 RISC (MIPS) instruction set (Chapter 2)
    - 2-1 ALU and data transfer instructions
    - 2-2 Branch instructions
    - 2-3 Supporting program execution
  - Topic 3 Computer arithmetic and ALU (Chapter 3)
- ❑ Part 3: ISA 의 효율적인 구현 (pipelining, cache memory)

# Chapter 3

## Arithmetic for Computers (ALU)

Some of authors' slides are modified

# Arithmetic for Computers

- ❑ We have build a 32-bit ALU in “Part 1” (concept only)
- ❑ Can you imagine many algorithms for addition?
  - Multiplication, division, FP operations as well
- ❑ Focus of designing computers in 1950s
  - Algorithms and implementation for faster ALU
- ❑ Field of “Computer Arithmetic”
  - Matured in 1950s and 1960s; improved since then
- ❑ Today, can buy ALU as IP (Intellectual Property)
- ❑ We do not study the design of fast ALU in detail

# Instead, we focus on

- 1) Operations on **integers** (concept only, skip performance)
    - Addition and subtraction
      - Dealing with overflow
    - Multiplication and division
  - 2) **Floating-point real numbers**
    - Representation and operations
  - 3) **Arithmetic for multimedia**
- ❑ **Impact of these operations on ISA**
- This is why Chapter 3 belongs to “Part 2”

# 1) Integer Addition & Subtraction

Overflow (표현범위 벗어남),

Issue of Finite Precision

# Representation of Integers

## □ Unsigned integers

$$000 = +0$$

$$001 = +1$$

$$010 = +2$$

$$011 = +3$$

$$100 = +4$$

$$101 = +5$$

$$110 = +6$$

$$111 = +7$$

# Representing Signed Integers

## Sign Magnitude    One's Complement    Two's Complement

$$000 = +0$$

$$001 = +1$$

$$010 = +2$$

$$011 = +3$$

$$100 = -0$$

$$101 = -1$$

$$110 = -2$$

$$111 = -3$$

$$000 = +0$$

$$001 = +1$$

$$010 = +2$$

$$011 = +3$$

$$100 = -3$$

$$101 = -2$$

$$110 = -1$$

$$111 = -0$$

$$000 = +0$$

$$001 = +1$$

$$010 = +2$$

$$011 = +3$$

$$100 = -4$$

$$101 = -3$$

$$110 = -2$$

$$111 = -1$$

- ❑ Issues: balance, number of zeros, ease of operations
- ❑ Which one is best? Why?

# Representing Signed Integers

Biased Notation:

$$000 = -3$$

$$001 = -2$$

$$010 = -1$$

$$011 = 0$$

$$100 = +1$$

$$101 = +2$$

$$110 = +3$$

$$111 = +4$$

(Bias 3)

Biased Notation:

$$000 = -4$$

$$001 = -3$$

$$010 = -2$$

$$011 = -1$$

$$100 = 0$$

$$101 = +1$$

$$110 = +2$$

$$111 = +3$$

(Bias 4)



# Integer Addition

## □ Addition (unsigned and signed)

- Example:  $9 + 12$

$$\begin{array}{r}
 \phantom{9_{10}} \phantom{=} \phantom{0000} 0001000 \\
 9_{10} = 00001001 \\
 12_{10} = 00001100 \\
 \hline
 \phantom{9_{10}} \phantom{=} \phantom{0000} 00010101 = 21_{10}
 \end{array}$$

carry

*(Note: In the original image, a dashed box highlights the carry bits from the 4th and 5th positions of the addends to the 6th position of the result, with arrows indicating the carry path.)*

- Subtraction for signed integers (add 2's complement)

## □ Subtraction for unsigned integers

# C/B in Unsigned Arithmetic

**au = bu + cu**      // au, bu, cu are unsigned number

- Carry and borrow (표현범위 벗어남)

$$\begin{array}{r}
 \text{1} \leftarrow \\
 192 \quad 1100 \ 0000 \\
 65 \quad 0100 \ 0001 \\
 \hline
 1(?) \ 0000 \ 0001
 \end{array}$$

$$\begin{array}{r}
 \text{1} \rightarrow \\
 128 \quad 1000 \ 0000 \\
 -192 \quad 1100 \ 0000 \\
 \hline
 192(?) \ 1100 \ 0000
 \end{array}$$

- Programmers are entirely responsible (processor 개입 불가)
  - If problem, add correctional code or use long data type
    - If (carry bit), then ...      // error handling
  - Otherwise, ignore (e.g., address arithmetic by compiler)

# Overflow in Signed Arithmetic

**a = b + c** // a, b, c are signed numbers

□ (2's complement) arithmetic overflow (표현범위 벗어남)

- Two patterns: can detect using a single XOR gate

carry: 0 1

70	0100	0110
+ 80	0101	0000
<hr/>		
150(?)	<b>1</b> 001	0110

carry: 1 0

-70	1011	1010
-80	1011	0000
<hr/>		
-150(?)	<b>0</b> 110	1010

□ Overflow occur when adding two positives yields a negative

- Or, adding two negatives gives a positive

# Overflow in Signed Arithmetic

- ❑ If problem, add correctional code or use long data type
  - If (overflow), then ... // error handling
- ❑ Otherwise, ignore
  - But 2's C. arithmetic overflow almost always a problem!
    - Processor 개입 여지 (should it detect overflow?)
- ❖ If we ignore 2's complement arithmetic overflow
  - Programmers responsible for everything
    - Consistent with the case of unsigned arithmetic

# Overflow in Signed Arithmetic

- ❑ **C language**: leave all to programmers
  - Compilers use “**addu, addiu, subu, ...**”
- ❑ **Fortran** (& Ada): catch 2's complement overflow (exception)
  - Use “**add, addi, sub, ...**” for signed numbers
  - Use “**addu, addiu, subu, ...**” for unsigned numbers
- ❑ MIPS solution: two kinds of arithmetic instructions

**addu, addiu, subu, ...** // ignore overflow

// “u” in “addu” does not mean unsigned

**add, addi, sub, ...** // detect overflow (exception)

# Overflow in Signed Arithmetic

- ❑ Detecting overflow means invoking exception handler
  - Save PC in exception program counter (EPC) register
  - Jump to predefined handler address
- ❑ What can an exception handler do?

# Overflow in Signed Arithmetic

## □ 32 bit signed numbers:

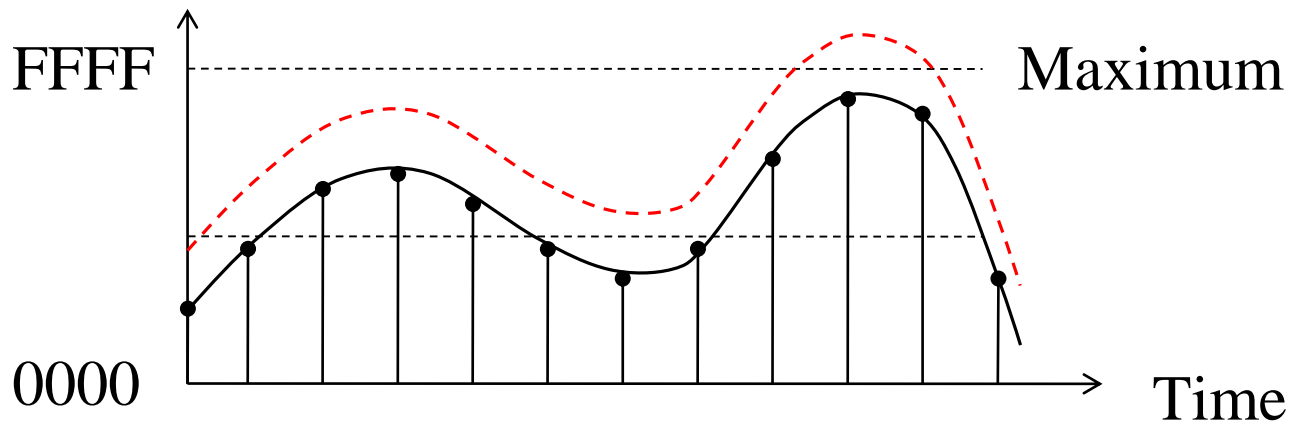
0000 0000 0000 0000 0000 0000 0000 0000	<sub>two</sub>	=	0 <sub>ten</sub>	
0000 0000 0000 0000 0000 0000 0000 0001	<sub>two</sub>	=	+ 1 <sub>ten</sub>	
0000 0000 0000 0000 0000 0000 0000 0010	<sub>two</sub>	=	+ 2 <sub>ten</sub>	
...				
0111 1111 1111 1111 1111 1111 1111 1110	<sub>two</sub>	=	+ 2,147,483,646 <sub>ten</sub>	
0111 1111 1111 1111 1111 1111 1111 1111	<sub>two</sub>	=	+ 2,147,483,647 <sub>ten</sub>	<i>maxint</i>
1000 0000 0000 0000 0000 0000 0000 0000	<sub>two</sub>	=	- 2,147,483,648 <sub>ten</sub>	
1000 0000 0000 0000 0000 0000 0000 0001	<sub>two</sub>	=	- 2,147,483,647 <sub>ten</sub>	<i>minint</i>
1000 0000 0000 0000 0000 0000 0000 0010	<sub>two</sub>	=	- 2,147,483,646 <sub>ten</sub>	
...				
1111 1111 1111 1111 1111 1111 1111 1101	<sub>two</sub>	=	- 3 <sub>ten</sub>	
1111 1111 1111 1111 1111 1111 1111 1110	<sub>two</sub>	=	- 2 <sub>ten</sub>	
1111 1111 1111 1111 1111 1111 1111 1111	<sub>two</sub>	=	- 1 <sub>ten</sub>	

## □ Finite precision; how can we represent big numbers?

- Double precision, floating point

# Arithmetic for Multimedia

- ❑ Saturating operations
  - On overflow, result is largest (or smallest) representable value
    - e.g., clipping in audio, saturation in video



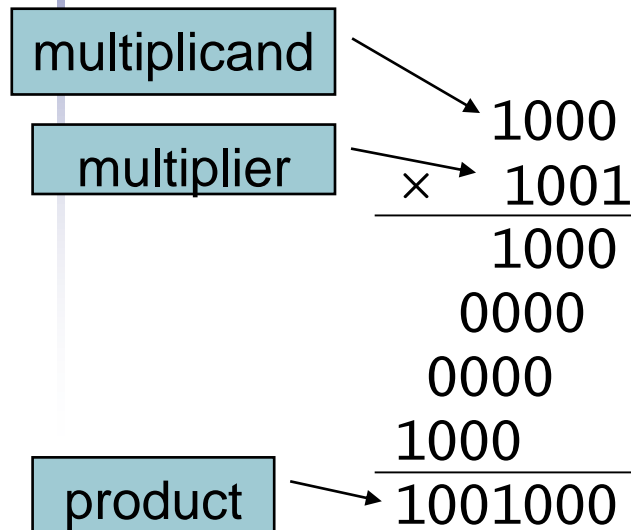


# Integer Multiplication & Division

(가법계)

(concept only; not for performance)

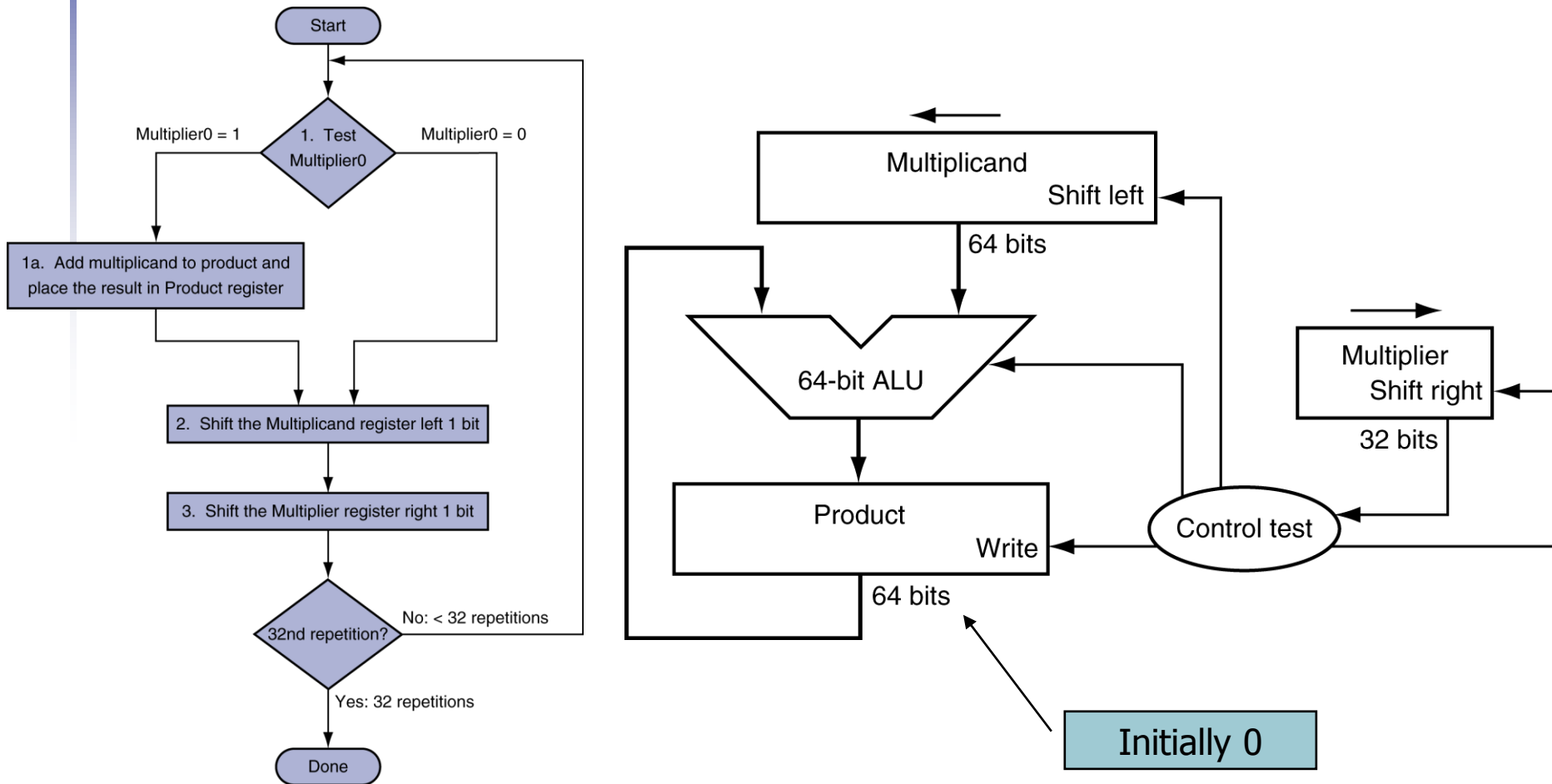
# Multiplication



Length of product  
is the sum of  
operand lengths

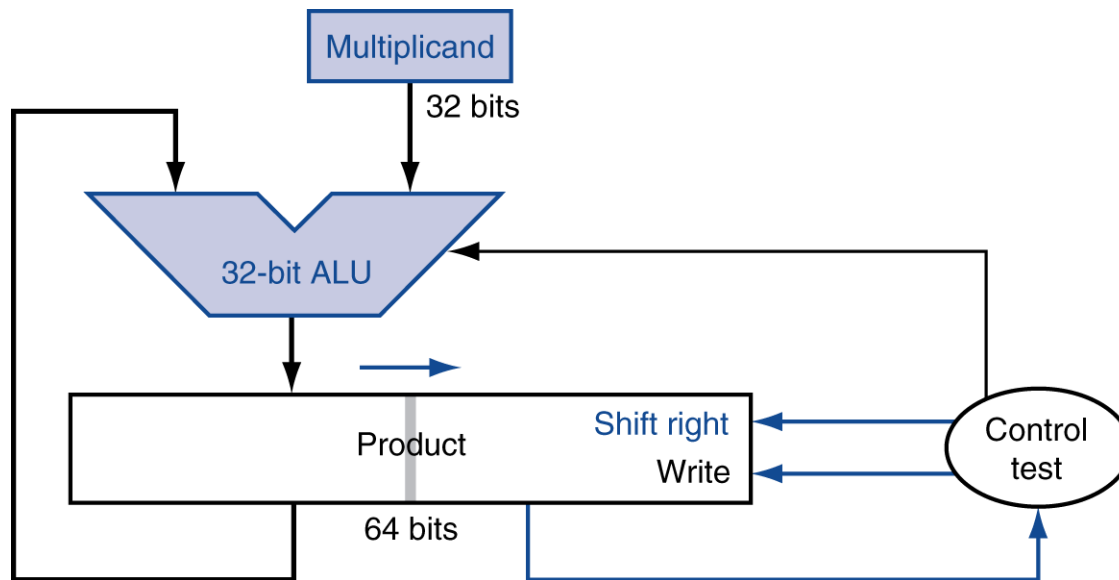
- ❑ More complicated than addition
  - Accomplished via shifting and addition
- ❑ More time and more area
- ❑ Negative numbers: convert and multiply
  - There are better techniques

# Multiplication Hardware (참고)



# Optimized Multiplier (참고)

- ❑ Perform steps in parallel: add/shift

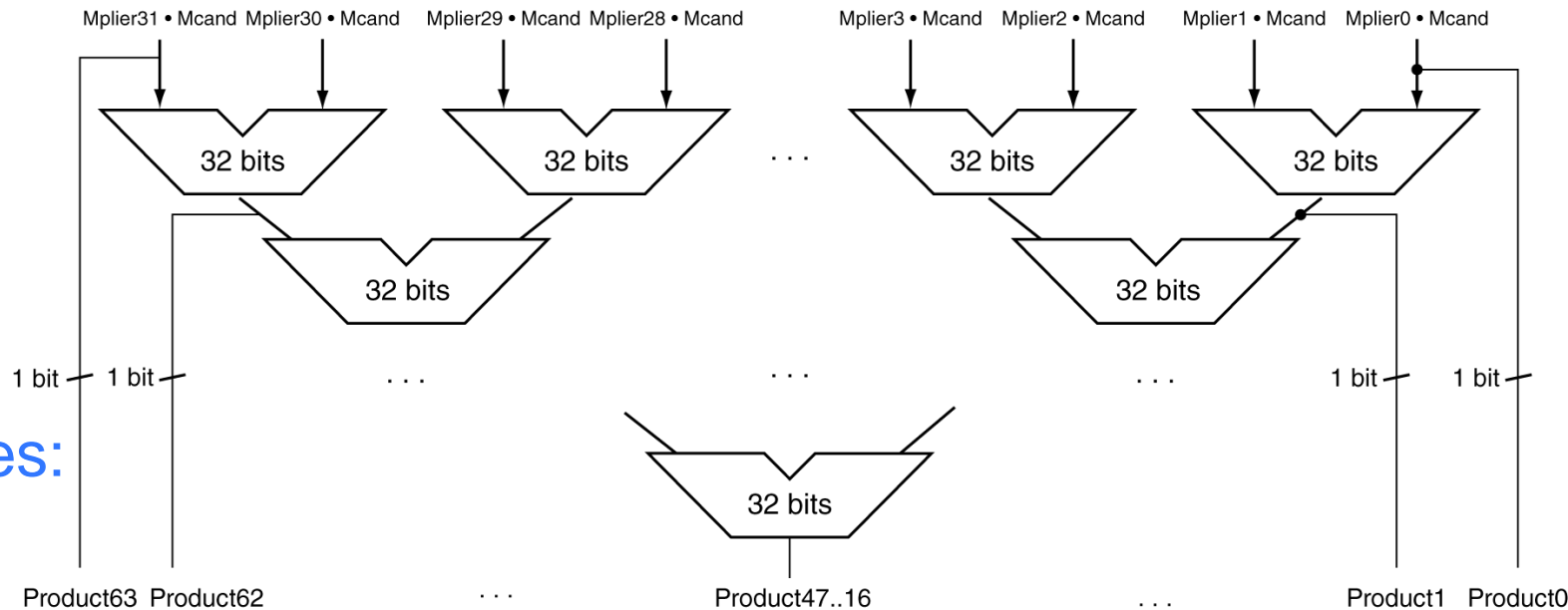


- ❑ One cycle per partial-product addition
  - That's ok, if frequency of multiplications is low

# Faster Multiplier (참고)

❑ Uses multiple adders (Moore's law)

- Cost/performance tradeoff



No. of stages:  
 $\log_2 32 = 5$

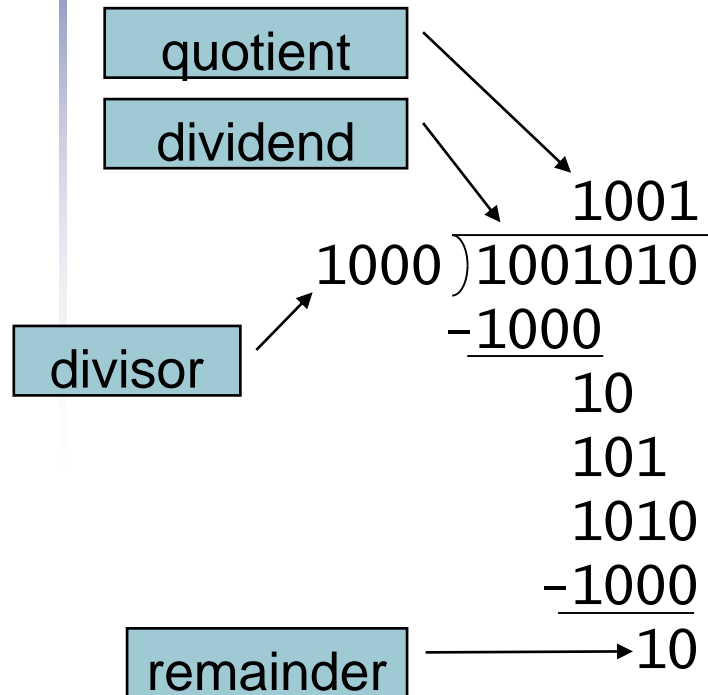
❑ Can be pipelined

- Several multiplication performed in parallel

# MIPS Multiplication

- ❑ Two 32-bit registers for product
  - HI: most-significant 32 bits
  - LO: least-significant 32-bits
- ❑ Instructions
  - `mult rs, rt` / `multu rs, rt`
    - 64-bit product in HI/LO
  - `mghi rd` / `mflo rd`
    - Move from HI/LO to rd
  - `mul rd, rs, rt`
    - Least-significant 32 bits of product → rd

# Division (slower than mult)



$n$ -bit operands yield  $n$ -bit quotient and remainder

- ❑ Check for 0 divisor
- ❑ Long division approach
  - If divisor  $\leq$  dividend bits
    - 1 bit in quotient, subtract
  - Otherwise
    - 0 bit in quotient, bring down next dividend bit
- ❑ Restoring division
  - Do the subtract, and if remainder goes  $< 0$ , add divisor back
- ❑ Signed division
  - Divide using absolute values
  - Adjust sign of quotient and remainder as required

# MIPS Division

- ❑ Use HI/LO registers for result
  - HI: 32-bit remainder
  - LO: 32-bit quotient
- ❑ Instructions
  - `div rs, rt` / `divu rs, rt`
  - Use `mfhi`, `mflo` to access result
  - No overflow or divide-by-0 checking
    - Software must perform checks if required



## 2) Floating-Point Real Numbers

# Floating Point

- ❑ Representation for non-integral numbers
  - Including very small and very big numbers
- ❑ Scientific numbers

$$-2.34 \times 10^{56}$$

$$+0.002 \times 10^{-94}$$

$$+9.5$$

// small numbers, no problem

- ❑ 지수표현
  - Significand and exponent

-2.34	56
-------	----

32-bit

# Floating Point

## ❑ Floating point Representation **not unique**

- Use normalized form

$$-2.34 \times 10^{56}$$

normalized

$$-0.234 \times 10^{57}$$

not normalized

$$-23.4 \times 10^{55}$$

## ❑ In binary


- $\pm 1.xxxxxxx_2 \times 2^{yyyy}$



32-bit

- Implicit 1 and fraction

# Floating Point Standard

- ❑ Defined by IEEE Std 754-1985 
- ❑ Developed in response to divergence of representations
  - Portability issues for scientific code
- ❑ Now almost universally adopted
- ❑ Two representations
  - Single precision (32-bit): type **float** in C
  - Double precision (64-bit): type **double** in C

# IEEE Floating-Point Format



3 1	3 0	2 9	2 8	2 7	2 6	2 5	2 4	2 3	2 2	2 1	2 0	1 9	1 8	1 7	1 6	1 5	1 4	1 3	1 2	1 1	1 0	9	8	7	6	5	4	3	2	1	0
S	exponent								fraction																						

1 bit

8 bits

23 bits



3 1	3 0	2 9	2 8	2 7	2 6	2 5	2 4	2 3	2 2	2 1	2 0	1 9	1 8	1 7	1 6	1 5	1 4	1 3	1 2	1 1	1 0	9	8	7	6	5	4	3	2	1	0
S	exponent											fraction																			

1 bit

11 bits

20 bits

fraction (continued)																													
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32 bits



# Floating-Point Example

```
float x = -0.75;  
double x = -0.75;
```

□ Represent  $-0.75$

- $-0.75 = -0.11_2 = (-1)^1 \times 1.1_2 \times 2^{-1}$

- $S = 1$

- Fraction =  $1000\dots00_2$

- Exponent =  $-1 + \text{Bias}$  // bias 127 or 1023

- Single:  $-1 + 127 = 126 = \underline{01111110}_2$

- Double:  $-1 + 1023 = 1022 = 01111111110_2$

□ Single:  $10111111 \ 01000000 \ 00000000 \ 00000000$

□ Double:  $10111111 \ 11101000 \ 00000000 \ 00000000 \dots00$

# Floating-Point Example

❑ What number is represented by the single-precision float

11000000 10100000 00000000 00000000

- $S = 1$
- Fraction =  $01000\dots00_2$
- Exponent =  $10000001_2 = 129$

$$\begin{aligned}\text{❑ } x &= (-1)^1 \times (1 + .01_2) \times 2^{(129 - 127)} && // \text{ bias } 127 \\ &= (-1) \times 1.25 \times 2^2 \\ &= -5.0\end{aligned}$$

# IEEE Floating-Point Format (부연)

single: 8 bits  
double: 11 bits

single: 23 bits  
double: 52 bits

S	Exponent	Fraction
---	----------	----------

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- ❑ S: sign bit (0  $\Rightarrow$  non-negative, 1  $\Rightarrow$  negative)
- ❑ Normalize significand:  $1.0 \leq |\text{significand}| < 2.0$ 
  - Implicit 1; Significand is Fraction + 1
- ❑ Exponent: excess representation: actual exponent + Bias
  - Ensures exponent is unsigned
  - Single: Bias = 127; Double: Bias = 1023



# Single-Precision Range

- ❑ Exponents 00000000 and 11111111 reserved
- ❑ Smallest value
  - Exponent: 00000001  
 $\Rightarrow$  actual exponent =  $1 - 127 = -126$
  - Fraction: 000...00  $\Rightarrow$  significand = 1.0
  - $\pm 1.0 \times 2^{-126} \approx \pm 1.2 \times 10^{-38}$
- ❑ Largest value
  - exponent: 11111110  
 $\Rightarrow$  actual exponent =  $254 - 127 = +127$
  - Fraction: 111...11  $\Rightarrow$  significand  $\approx 2.0$
  - $\pm 2.0 \times 2^{+127} \approx \pm 3.4 \times 10^{+38}$

# Double-Precision Range

- ❑ Exponents 0000...00 and 1111...11 reserved
- ❑ Smallest value
  - Exponent: 000000000001  
 $\Rightarrow$  actual exponent =  $1 - 1023 = -1022$
  - Fraction: 000...00  $\Rightarrow$  significand = 1.0
  - $\pm 1.0 \times 2^{-1022} \approx \pm 2.2 \times 10^{-308}$
- ❑ Largest value
  - Exponent: 11111111110  
 $\Rightarrow$  actual exponent =  $2046 - 1023 = +1023$
  - Fraction: 111...11  $\Rightarrow$  significand  $\approx 2.0$
  - $\pm 2.0 \times 2^{+1023} \approx \pm 1.8 \times 10^{+308}$

# Floating Point Standard



## ❑ Members of the committee

- 과학 계산 전문가
- Computer arithmetic (ALU 설계/구현) 전문가


## ❑ 과학계산 issues (표현 범위, 유효숫자)

- Total number of bits
- Number of bits for significand/exponent
  - More bits for significand gives more accuracy
  - More bits for exponent increases range

## ❑ The rest are determined by ALU specialists

- For efficient implementation

# Floating Point Standard

- ❑ When you do C, Java programming
  - When do you use **float**? 
  - When do you use **double**?
- ❑ **FP numbers** and FP ALU (or FP coprocessor)
  - 근사계산
    - Too many bits for fraction: rounded
  - No beauty **of integers**
    - Not accurate from mathematics perspective
  - Think about overflow, underflow

# Floating Point Standard

- ❑ How do we represent 0?

$$x = (-1)^s \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

- ❑ Exponents 0000...00 and 1111...11 reserved

- Exp = 255, sig != 0 → NaN (not a number)
- Exp = 255, sig = 0 → +, - infinity
- Exp = 0, sig = 0 → 0
- Exp = 0, sig != 0 → even smaller numbers

- ❑ IEEE 754-2008

- Binary 32, 64, 128, (16, 256)

# Associativity

- ❑ Order of computation in “ $x + y + z$ ” (problem?)

		$(x+y)+z$	$x+(y+z)$
x	-1.50E+38		-1.50E+38
y	1.50E+38	0.00E+00	
z	1.0	1.0	1.50E+38
		1.00E+00	0.00E+00

Y: 1.50 E+38

Z: 0.00000 00000 00000 00000 00000 00000 00000 001 E+38

Y+Z: 1.50000 00000 00000 00000 00000 00000 00000 001 E+38

// too many bits in fraction: rounded

Y+Z = 1.50 E38

# Floating Point Comparison

- ❑ Floating-point “beq”? What will happen? Why?

```
float x, y;
```

```
x = 1.5; y = 2.0;
```

```
if ((x * y) == 3.0) printf(“equal\n”);
```

```
x = 0.1; y = 10.0; // no. of fraction bits
```

```
❏ if ((x * y) == 1.0) printf(“equal\n”);
```

- ❑ Floating point equality check: use “  $|x - \text{constant}| < \epsilon$  ”

# Floating-Point Addition

- Now consider a 4-digit binary example

$$1.000_2 \times 2^{-1} + -1.110_2 \times 2^{-2} \quad // \quad 0.5 + -0.4375$$

## 1. Align binary points

- Shift number with smaller exponent
- $1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-1}$

## 2. Add significands

- $1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-1} = 0.001_2 \times 2^{-1}$

## 3. Normalize result & check for over/underflow

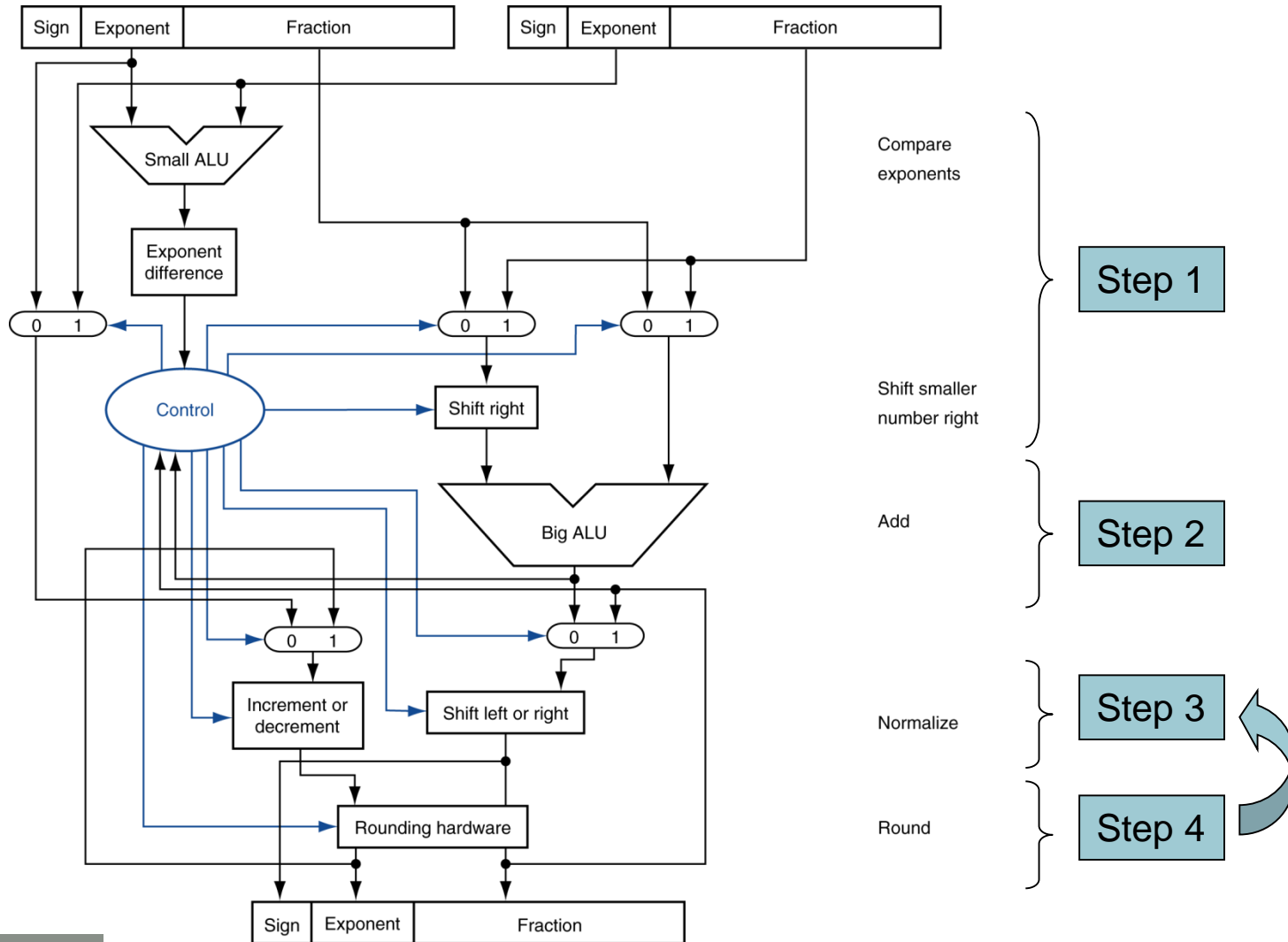
- $1.000_2 \times 2^{-4}$ , with no over/underflow

## 4. Round and renormalize if necessary

- $1.000_2 \times 2^{-4}$  (no change) = 0.0625



# FP Adder Hardware (참고)



# FP Adder Hardware

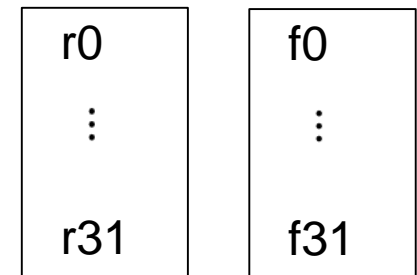
- ❑ Much more complex than integer adder
- ❑ Doing it in one clock cycle would take too long
  - Much longer than integer operations
  - Slower clock would penalize all instructions
- ❑ FP adder usually takes several cycles
  - Can be pipelined

# FP Arithmetic Hardware

- ❑ FP multiplier is of similar complexity to FP adder
- ❑ FP arithmetic hardware usually does
  - Addition, subtraction, multiplication, division, reciprocal, square-root
  - $\text{FP} \leftrightarrow \text{integer}$  conversion
- ❑ Operations usually takes several cycles
  - Can be pipelined

# FP Instructions in MIPS

- ❑ FP hardware is adjunct processor that extends the ISA
- ❑ Separate FP registers
  - 32 single-precision: \$f0, \$f1, ... \$f31
  - Paired for double-precision: \$f0/\$f1, \$f2/\$f3, ...
- ❑ FP instructions operate only on FP registers
  - Generally no integer ops on FP data, or vice versa
  - More registers with minimal code-size impact
- ❑ FP load and store instructions
  - lwc1, ldc1, swc1, sdc1
    - e.g., ldc1 \$f8, 32(\$sp)



# FP Instructions in MIPS

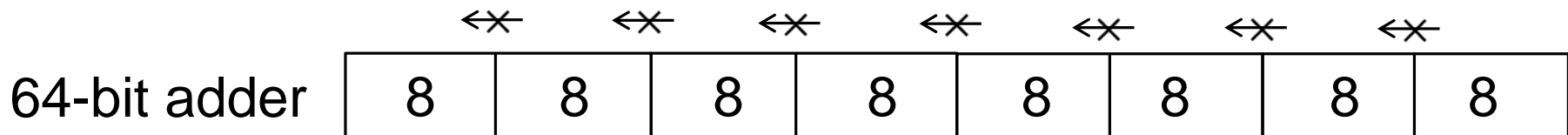
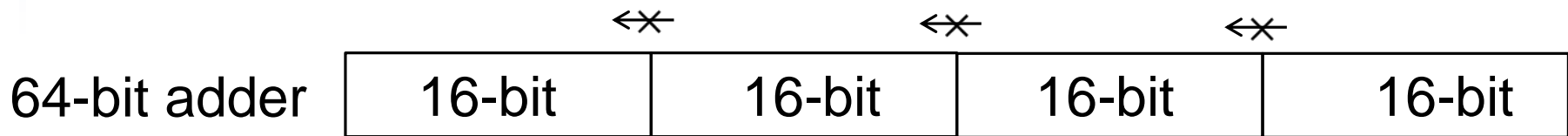
- ❑ Single-precision arithmetic: `add.s`, `sub.s`, `mul.s`, `div.s`
  - e.g., `add.s $f0, $f1, $f6`
- ❑ Double-precision arithmetic: `add.d`, `sub.d`, `mul.d`, `div.d`
  - e.g., `mul.d $f4, $f4, $f6`
- ❑ Single- and double-precision comparison
  - `c.xx.s`, `c.xx.d` (`xx` is `lt`, `le`, ...)
  - Sets or clears FP condition-code bit
    - e.g. `c.lt.s $f3, $f4`
- ❑ Branch on FP condition code true or false: `bc1t`, `bc1f`
  - e.g., `bc1t TargetLabel`

# 3) Arithmetic for Multimedia (and Scientific Computing)

(Textbook Sections 3.6 – 3.8)

# Arithmetic for Multimedia

- ❑ Graphics and media processing operates on vectors of 8-bit and 16-bit data
  - Use 64-bit adder, with partitioned carry chain
    - Operate on  $8 \times 8$ -bit,  $4 \times 16$ -bit, or  $2 \times 32$ -bit vectors



# Subword Parallelism

- ❑ Also called data-level parallelism (DLP),  
vector parallelism,  
or Single Instruction Multiple Data (SIMD)
  - Instructions operate on them simultaneously
- ❑ Moore's law for graphics and audio applications
  - 128-bit adder and wide registers
    - 16×8-bit adds, 8×16-bit adds, 4×32-bit adds



# MMX and SSE (Intel 사례)

## ❑ MMX (1993) // multimedia

- Use (64 bits of) existing FP registers
- Small integer data types (8bit color, 16bit audio)
  - $(8 \times 8)$ ,  $(4 \times 16)$ ,  $(2 \times 32)$ ,  $(1 \times 64)$

## ❑ SSE (1999) // scientific computing (Moore's law)

- Add new  $8 \times 128$ -bit FP registers (XMM0-XMM7)

32-bit FP	32-bit FP	32-bit FP	32-bit FP
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- Packed single precision FP ( $4 \times 32$ ) with more FP units

# SSE2, ..., AVX (Intel 사례)


## ❑ SSE2 (2001)

- Generalize SSE ( $8 \times 128$ -bit registers)
- Can be used for multiple FP/INT operands
  - $2 \times 64$ -bit FP (addpd %xmm0, %xmm4)
  - $4 \times 32$ -bit FP (addps %xmm0, %xmm4)
  - $2 \times 64$ -bit,  $4 \times 32$ -bit,  $8 \times 16$ -bit,  $16 \times 8$ -bit INT

## ❑ AVX (advanced vector extension; 2008)

- Double the width of registers
  - e.g.,  $4 \times 64$ -bit double precision FP

# Going Faster

- ❑ DGEMM (Double Precision General Matrix Multiply) with Intel Core i7
  - AVX version 3.85 times fast
    - 4 double precision FP operations in parallel
- ❑ Matrix computation (과학 계산) 
  - Differential equations
  - Deep learning (vectorization with GPU)

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ & \cdots & \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & \end{bmatrix} = \begin{bmatrix} * & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{bmatrix}$$

# Matrix Multiply, DGEMM (참고)

## ❑ Unoptimized code:

```
1. void dgemm (int n, double* A, double* B, double* C)
2. {
3.   for (int i = 0; i < n; ++i)
4.     for (int j = 0; j < n; ++j)
5.       {
6.         double cij = C[i+j*n]; /* cij = C[i][j] */
7.         for(int k = 0; k < n; k++ )
8.           cij += A[i+k*n] * B[k+j*n]; /* cij += A[i][k]*B[k][j] */
9.         C[i+j*n] = cij; /* C[i][j] = cij */
10.      }
11. }
```

# Matrix Multiply, DGEMM (참고)

## □ x86 assembly code:

```

1. vmovsd (%r10),%xmm0    # Load 1 element of C into %xmm0
2. mov %rsi,%rcx          # register %rcx = %rsi
3. xor %eax,%eax          # register %eax = 0
4. vmovsd (%rcx),%xmm1    # Load 1 element of B into %xmm1
5. add %r9,%rcx           # register %rcx = %rcx + %r9
6. vmulsd (%r8,%rax,8),%xmm1,%xmm1 # Multiply %xmm1,
   element of A
7. add $0x1,%rax          # register %rax = %rax + 1
8. cmp %eax,%edi          # compare %eax to %edi
9. vaddsd %xmm1,%xmm0,%xmm0 # Add %xmm1, %xmm0
10. jg 30 <dgemm+0x30>    # jump if %eax > %edi
11. add $0x1,%r11d        # register %r11 = %r11 + 1
12. vmovsd %xmm0, (%r10)  # Store %xmm0 into C element

```

# Matrix Multiply, DGEMM (참고)

## ❑ Optimized C code:

```

1. #include <x86intrin.h>
2. void dgemm (int n, double* A, double* B, double* C)
3. {
4.     for ( int i = 0; i < n; i+=4 )
5.         for ( int j = 0; j < n; j++ ) {
6.             __m256d c0 = _mm256_load_pd(C+i+j*n); /* c0 = C[i][j]
               */
7.             for( int k = 0; k < n; k++ )
8.                 c0 = _mm256_add_pd(c0, /* c0 += A[i][k]*B[k][j] */
9.                                     _mm256_mul_pd(_mm256_load_pd(A+i+k*n),
10.                                     _mm256_broadcast_sd(B+k+j*n)));
11.             _mm256_store_pd(C+i+j*n, c0); /* C[i][j] = c0 */
12.         }
13. }

```

# Matrix Multiply, DGEMM (참고)

## ❑ Optimized x86 assembly code:

```

1. vmovapd (%r11),%ymm0      # Load 4 elements of C into %ymm0
2. mov %rbx,%rcx             # register %rcx = %rbx
3. xor %eax,%eax             # register %eax = 0
4. vbroadcastsd (%rax,%r8,1),%ymm1 # Make 4 copies of B element
5. add $0x8,%rax             # register %rax = %rax + 8
6. vmulpd (%rcx),%ymm1,%ymm1 # Parallel mul %ymm1, 4 A elements
7. add %r9,%rcx              # register %rcx = %rcx + %r9
8. cmp %r10,%rax             # compare %r10 to %rax
9. vaddpd %ymm1,%ymm0,%ymm0  # Parallel add %ymm1, %ymm0
10. jne 50 <dgemm+0x50>      # jump if not %r10 != %rax
11. add $0x1,%esi            # register %esi = %esi + 1
12. vmovapd %ymm0, (%r11)    # Store %ymm0 into 4 C elements

```

# C Intrinsics

- ❑ Intrinsic functions
  - Functions available for use in a given programming language whose implementation is specially handled by compiler
  - Often used to explicitly implement vectorization and parallelization in languages which do not address such constructs
    - e.g., MMX, SSE, OpenMP
- ❑ Compiler target?
- ❑ Cost-performance-energy tradeoff



# SIMD Parallelism

- ❑ Three variations (Chapter 6) of DLP
  - SIMD extensions
  - Vector architectures
  - Graphics Processing Units (GPUs)
    - 3D video games since 2000, extensive parallelism
    - Also used by scientific computing
- ❑ Deep learning and GPU computing

## 4) Summary of MIPS ISA

(Textbook section 3.10)

# Concluding Remarks

- ❑ ISAs support arithmetic
  - Signed and unsigned integers
  - Floating-point approximation to reals
- ❑ Bounded range and precision
  - Operations can overflow and underflow
- ❑ MIPS ISA
  - Core instructions: 56 most frequently used
    - 100% of SPECINT, 97% of SPECFP
  - Other instructions: less frequent

# MIPS Core, MIPS-32, Pseudo MIPS

MIPS core instructions	Name	Format	MIPS arithmetic core	Name	Format
add	add	R	multiply	mult	R
add immediate	addi	I	multiply unsigned	multu	R
add unsigned	addu	R	divide	div	R
add immediate unsigned	addiu	I	divide unsigned	divu	R
subtract	sub	R	move from Hi	mfhi	R
subtract unsigned	subu	R	move from Lo	mflo	R
AND	AND	R	move from system control (EPC)	mfc0	R
AND immediate	ANDi	I	floating-point add single	add.s	R
OR	OR	R	floating-point add double	add.d	R
OR immediate	ORi	I	floating-point subtract single	sub.s	R
NOR	NOR	R	floating-point subtract double	sub.d	R
shift left logical	sll	R	floating-point multiply single	mul.s	R
shift right logical	srl	R	floating-point multiply double	mul.d	R
load upper immediate	lui	I	floating-point divide single	div.s	R
load word	lw	I	floating-point divide double	div.d	R
store word	sw	I	load word to floating-point single	lwc1	I
load halfword unsigned	lhu	I	store word to floating-point single	swc1	I
store halfword	sh	I	load word to floating-point double	ldc1	I
load byte unsigned	lbu	I	store word to floating-point double	sdc1	I
store byte	sb	I	branch on floating-point true	bc1t	I
load linked ( <i>atomic update</i> )	ll	I	branch on floating-point false	bc1f	I
store cond. ( <i>atomic update</i> )	sc	I	floating-point compare single	c.x.s	R
branch on equal	beq	I	(x = eq, neq, lt, le, gt, ge)		
branch on not equal	bne	I	floating-point compare double	c.x.d	R
jump	j	J	(x = eq, neq, lt, le, gt, ge)		
jump and link	jal	J			
jump register	jr	R			
set less than	slt	R			
set less than immediate	slti	I			
set less than unsigned	sltu	R			
set less than immediate unsigned	sltiu	I			

Figure 3.26

# MIPS Core, MIPS-32, Pseudo MIPS

Figure 3.27

Remaining MIPS-32	Name	Format	Pseudo MIPS	Name	Format
exclusive or ( $rs \oplus rt$ )	xor	R	absolute value	abs	rd,rs
exclusive or immediate	xori	I	negate ( <i>signed or unsigned</i> )	negs	rd,rs
shift right arithmetic	sra	R	rotate left	rol	rd,rs,rt
shift left logical variable	sllv	R	rotate right	ror	rd,rs,rt
shift right logical variable	srlv	R	multiply and don't check oflw ( <i>signed or uns.</i> )	mul <sub>s</sub>	rd,rs,rt
shift right arithmetic variable	srav	R	multiply and check oflw ( <i>signed or uns.</i> )	mulo <sub>s</sub>	rd,rs,rt
move to Hi	mt <sub>hi</sub>	R	divide and check overflow	div	rd,rs,rt
move to Lo	mt <sub>lo</sub>	R	divide and don't check overflow	divu	rd,rs,rt
load halfword	lh	I	remainder ( <i>signed or unsigned</i> )	rem <sub>s</sub>	rd,rs,rt
load byte	lb	I	load immediate	li	rd,imm
load word left ( <i>unaligned</i> )	lwl	I	load address	la	rd,addr
load word right ( <i>unaligned</i> )	lwr	I	load double	ld	rd,addr
store word left ( <i>unaligned</i> )	swl	I	store double	sd	rd,addr
store word right ( <i>unaligned</i> )	swr	I	unaligned load word	ulw	rd,addr
load linked ( <i>atomic update</i> )	ll	I	unaligned store word	usw	rd,addr
store cond. ( <i>atomic update</i> )	sc	I	unaligned load halfword ( <i>signed or uns.</i> )	ulh <sub>s</sub>	rd,addr
move if zero	movz	R	unaligned store halfword	ush	rd,addr
move if not zero	movn	R	branch	b	Label
multiply and add ( <i>S or uns.</i> )	madd <sub>s</sub>	R	branch on equal zero	beqz	rs,L
multiply and subtract ( <i>S or uns.</i> )	msub <sub>s</sub>	I	branch on compare ( <i>signed or unsigned</i> )	bx <sub>s</sub>	rs,rt,L
branch on $\geq$ zero and link	bgezal	I	( $x = lt, le, gt, ge$ )		
branch on $<$ zero and link	bltzal	I	set equal	seq	rd,rs,rt
jump and link register	jalr	R	set not equal	sne	rd,rs,rt
branch compare to zero	bxz	I	set on compare ( <i>signed or unsigned</i> )	sx <sub>s</sub>	rd,rs,rt
branch compare to zero likely	bxzl	I	( $x = lt, le, gt, ge$ )		
( $x = lt, le, gt, ge$ )			load to floating point ( <u>s</u> or <u>d</u> )	l. <sub>f</sub>	rd,addr
branch compare reg likely	bxl	I	store from floating point ( <u>s</u> or <u>d</u> )	s. <sub>f</sub>	rd,addr
trap if compare reg	tx	R			
trap if compare immediate	txi	I			
( $x = eq, neq, lt, le, gt, ge$ )					
return from exception	rfe	R			
system call	syscall	I			
break ( <i>cause exception</i> )	break	I			
move from FP to integer	mfc <sub>l</sub>	R			
move to FP from integer	mtc <sub>l</sub>	R			
FP move ( <u>s</u> or <u>d</u> )	mov. <sub>f</sub>	R			
FP move if zero ( <u>s</u> or <u>d</u> )	movz. <sub>f</sub>	R			
FP move if not zero ( <u>s</u> or <u>d</u> )	movn. <sub>f</sub>	R			
FP square root ( <u>s</u> or <u>d</u> )	sqr <sub>t</sub> . <sub>f</sub>	R			
FP absolute value ( <u>s</u> or <u>d</u> )	abs. <sub>f</sub>	R			
FP negate ( <u>s</u> or <u>d</u> )	neg. <sub>f</sub>	R			
FP convert ( <u>w</u> , <u>s</u> , or <u>d</u> )	cvt. <sub>f</sub> <sub>f</sub>	R			
FP compare un ( <u>s</u> or <u>d</u> )	c.xn. <sub>f</sub>	R			

# MIPS Core, MIPS-32, Pseudo MIPS

- ❑ Frequency of use in SPEC CPU2006 benchmark

Instruction subset	Integer	Fl. pt.
MIPS core	98%	31%
MIPS arithmetic core	2%	66%
Remaining MIPS-32	0%	3%

- ❑ For the rest of book, focus on MIPS core (integer instruction set excluding multiply and divide)
  - To make the explanation of computer design easier

# Homework #10 (see Class Homepage)

- 1) Write a report summarizing the materials discussed in Topic 3
- 2) Write a report summarizing the materials discussed in Topic 4-1 (이번 주 수업에서 공부하는 부분만 요약함)

\*\* 문장으로 써도 좋고 파워포인트 형태의 개조식 정리도 좋음

- Due: see Blackboard
  - Submit electronically to Blackboard

# Class Topics (클래스 홈페이지 참조)

- ❑ Part 1: Fundamental concepts and principles
- ❑ Part 2: 빠른 컴퓨터를 위한 ISA design
  - Topic 1 Computer performance and ISA design (Ch. 1)
  - Topic 2 RISC (MIPS) instruction set (Chapter 2)
    - 2-1 ALU and data transfer instructions
    - 2-2 Branch instructions
    - 2-3 Supporting program execution
  - Topic 3 Computer arithmetic and ALU (Chapter 3)
- ❑ Part 3: ISA 의 효율적인 구현 (pipelining, cache memory)