Programming Languages

Thunks, Laziness, Streams, Memoization

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Delayed evaluation

For each language construct, the semantics specifies when subexpressions get evaluated. In ML, Racket, Java, C:

- Function arguments are *eager* (call-by-value)
  - Evaluated once before calling the function
- Conditional branches are not eager

It matters: calling **`factorial-bad`** never terminates:

```
(define (my-if-bad x y z)
  (if x y z))

(define (factorial-bad n)
  (my-if-bad (= n 0)
             1
             (* n (factorial-bad (- n 1)))))
```

Thunks delay

We know how to delay evaluation: put expression in a function!

- Thanks to closures, can use all the same variables later

A zero-argument function used to delay evaluation is called a *thunk*

- As a verb: *thunk the expression* *(also freeze and thaw)*

This works (but it is silly to wrap **`if`** like this):

```
(define (my-if x y z)
  (if x (y) (z)))

(define (fact n)
    (my-if (= n 0)
           (lambda() 1)
           (lambda() (* n (fact (- n 1))))))
```

The key point

- Evaluate an expression **`e`** to get a result:

```
e
```

- A function that *when called*, evaluates **`e`** and returns result
  - Zero-argument function for "thunking"

```
(lambda () e)
```

- Evaluate **`e`** to some thunk and then call the thunk

```
(e)
```

- Next: Powerful idioms related to delaying evaluation and/or avoided repeated or unnecessary computations
  - Some idioms also use mutation in encapsulated ways

Avoiding expensive computations

Thunks let you skip expensive computations if they are not needed

Great if take the true-branch:

```
(define (f th)
  (if (…) 0 (…  (th) …)))
```

But worse if you end up using the thunk more than once:

```
(define (f th)
  (… (if (…) 0 (… (th) …))
     (if (…) 0 (… (th) …))
     …
     (if (…) 0 (… (th) …))))
```

In general, might not know many times a result is needed

Best of both worlds

Assuming some expensive computation has no side effects, ideally we would:

- Not compute it *until needed*
- *Remember the answer* so future uses complete immediately

Called *lazy evaluation*

Languages where most constructs, including function arguments, work this way are *lazy languages*

- Haskell

Racket predefines support for *promises*, but we can make our own

- Thunks and mutable pairs are enough

Delay and force

```
(define (my-delay th)
  (mcons #f th))

(define (my-force p)
  (if (mcar p)
      (mcdr p)
      (begin (set-mcar! p #t)
             (set-mcdr! p ((mcdr p)))
             (mcdr p))))
```

An ADT represented by a mutable pair

- **`#f`** in *car* means *cdr* is unevaluated thunk
  - Really a one-of type: thunk or result-of-thunk
- Ideally hide representation in a module

Using promises

```
(define (f p)
  (… (if (…) 0 (… (my-force p) …))
     (if (…) 0 (… (my-force p) …))
     …
     (if (…) 0 (… (my-force p) …))))
```

```
(f (my-delay (lambda () e)))
```

Lessons From Example

See code file for example that does multiplication using a very slow addition helper function

- With thunking second argument:
  - *Great* if first argument 0
  - Okay if first argument 1
  - *Worse* otherwise

- With precomputing second argument:
  - *Okay* in all cases

- With thunk that uses a promise for second argument:
  - *Great* if first argument 0
  - *Okay* otherwise

Streams

- A stream is an *infinite sequence* of values
  - So cannot make a stream by making all the values
  - Key idea: Use a thunk to delay creating most of the sequence
  - Just a programming idiom

A powerful concept for division of labor:

- Stream producer knows how create any number of values
- Stream consumer decides how many values to ask for

Some examples of streams you might (not) be familiar with:

- User actions (mouse clicks, etc.)
- UNIX pipes: **`cmd1 | cmd2`** has **`cmd2`** "pull" data from **`cmd1`**
- Output values from a sequential feedback circuit

Using streams

We will represent streams using pairs and thunks

Let a stream be a thunk that *when called* returns a pair:

```
'(next-answer . next-thunk)
```

So given a stream **`s`**, the client can get any number of elements

- First: **`(car (s))`**
- Second: **`(car ((cdr (s))))`**
- Third: **`(car ((cdr ((cdr (s))))))`**

(Usually bind **`(cdr (s))`** to a variable or pass to a recursive function)

Example using streams

This function returns how many stream elements it takes to find one for which tester does not return **`#f`**

- Happens to be written with a tail-recursive helper function

```
(define (number-until stream tester)
  (letrec ([f (lambda (stream ans)
                (let ([pr (stream)])
                  (if (tester (car pr))
                      ans
                      (f (cdr pr) (+ ans 1)))))])
    (f stream 1)))
```

- **`(stream)`** generates the pair
- So recursively pass **`(cdr pr)`**, the thunk for the rest of the infinite sequence

Streams

Coding up a stream in your program is easy

- We will do functional streams using pairs and thunks

Let a stream be a thunk that *when called* returns a pair:

```
'(next-answer . next-thunk)
```

Saw how to use them, now how to make them…

- Admittedly mind-bending, but uses what we know

Making streams

- How can one thunk create the right next thunk? Recursion!
  - Make a thunk that produces a pair where cdr is next thunk
  - A recursive function can return a thunk where recursive call does not happen until thunk is called

Making streams

- How can one thunk create the right next thunk? Recursion!
  - Make a thunk that produces a pair where cdr is next thunk
  - A recursive function can return a thunk where recursive call does not happen until thunk is called

```
(define ones (lambda () (cons 1 ones)))

(define nats
  (letrec ([f (lambda (x)
                (cons x (lambda () (f (+ x 1)))))])
     (lambda () (f 1))))

(define powers-of-two
  (letrec ([f (lambda (x)
                (cons x (lambda () (f (* x 2)))))])
     (lambda () (f 2))))
```

Getting it wrong

- This uses a variable before it is defined

```
(define ones-really-bad (cons 1 ones-really-bad))
```

- This goes into an infinite loop making an infinite-length list

```
(define ones-bad (lambda () (cons 1 (ones-bad))))
(define (ones-bad) (cons 1 (ones-bad)))
```

- This is a stream: thunk that returns a pair with cdr a thunk

```
(define ones (lambda () (cons 1 ones)))
(define (ones) (cons 1 ones))
```

Memoization

- If a function has no side effects and does not read mutable memory, no point in computing it twice for the same arguments
  - Can keep a *cache* of previous results
  - Net win if (1) maintaining cache is cheaper than recomputing and (2) cached results are reused

- Similar to promises, but if the function takes arguments, then there are multiple “previous results”

- For recursive functions, this *memoization* can lead to *exponentially* faster programs
  - Related to algorithmic technique of dynamic programming

How to do memoization: see example

- Need a (mutable) cache that all calls using the cache share
  – So must be defined *outside* the function(s) using it

- See code for an example with Fibonacci numbers

  – Good demonstration of the idea because it is short, but, as shown in the code, there are also easier less-general ways to make **`fibonacci`** efficient

  – (An association list (list of pairs) is a simple but sub-optimal data structure for a cache; okay for our example)

Fibonacci

```
(define (fib1 x)
  (if (or (= x 1) (= x 2))
      1
      (+ (fib1 (- x 1))
         (fib1 (- x 2)))))
```

Fibonacci

```
(define (fib1 x)
  (if (or (= x 1) (= x 2))
      1
      (+ (fib1 (- x 1))
         (fib1 (- x 2)))))
```

```
(define (fib2 x)
  (letrec ([f (lambda (acc1 acc2 y)
                (if (= y x)
                    (+ acc1 acc2)
                    (f (+ acc1 acc2) acc1 (+ y 1))))])
    (if (or (= x 1) (= x 2))
        1
        (f 1 1 3))))
```

Fibonacci

```
(define (fib2 x)
  (letrec ([f (lambda (acc1 acc2 y)
                 (if (= y x)
                     (+ acc1 acc2)
                     (f (+ acc1 acc2) acc1 (+ y 1))))])
    (if (or (= x 1) (= x 2))
        1
        (f 1 1 3))))
```

```
(define (fib3 x)
  (letrec ([memo null] ;memo=(4 . 3) (3 . 2) (1 . 1) (2 . 1))
           [f (lambda (x)
                 (let ([ans (assoc x memo)])
                  (if ans (cdr ans)
                    (let ([new-ans (if (or (= x 1) (= x 2))
                                       1
                                   (+ (f (- x 1)) (f (- x 1))))])
                     (begin
                       (set! memo (cons (cons x new-ans) memo))
                        new-ans)))))])
  (f x)))
```

assoc

- Example uses **`assoc`**, which is just a library function you could look up in the Racket reference manual:

  `(assoc v lst)` takes a list of pairs and locates the first element of **`lst`** whose car is equal to **`v`** according to **`is-equal?`**. If such an element exists, the pair (i.e., an element of **`lst`**) is returned. Otherwise, the result is **`#f`**.

- Returns **`#f`** for not found to distinguish from finding a pair with **`#f`** in cdr