

# Estimating time-varying transmission rates in the SIR model

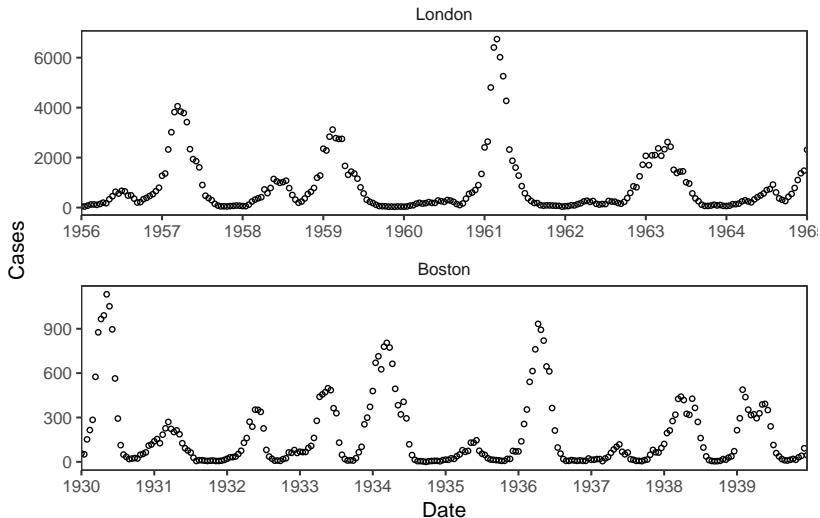
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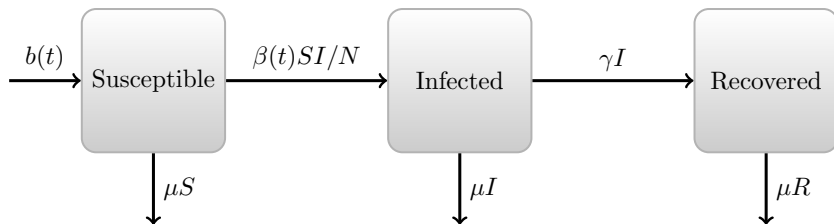
# Epidemic time series

- Measles report (prior to vaccination)



# Susceptible-Infected-Recovered (SIR) model

- Describes how disease spreads in a population (Kermack and McKendrick, 1927)



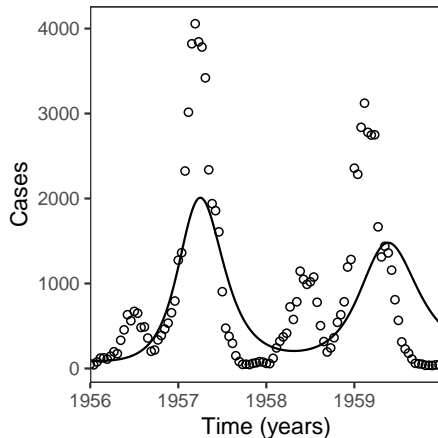
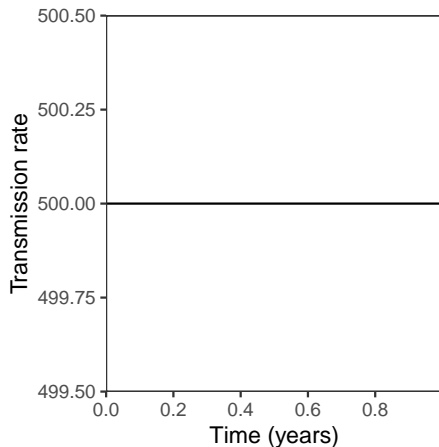
# Susceptible-Infected-Recovered (SIR) model

$$\begin{aligned}\frac{dS}{dt} &= b(t) - \beta(t)S\frac{I}{N} - \mu S \\ \frac{dI}{dt} &= \beta(t)S\frac{I}{N} - (\gamma + \mu)I \\ \frac{dR}{dt} &= \gamma I - \mu R\end{aligned}$$

- Mean infectious period  $1/\gamma$
- Mean life expectancy  $1/\mu$
- Birth rate  $b(t)$
- Contact rate  $\beta(t)$

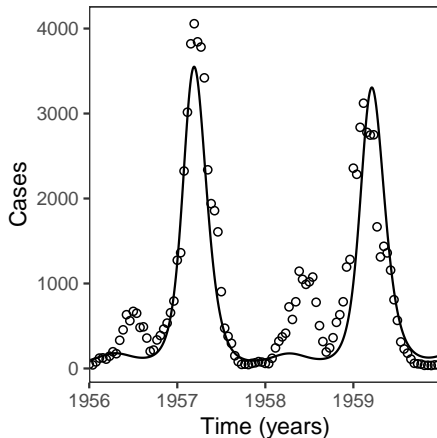
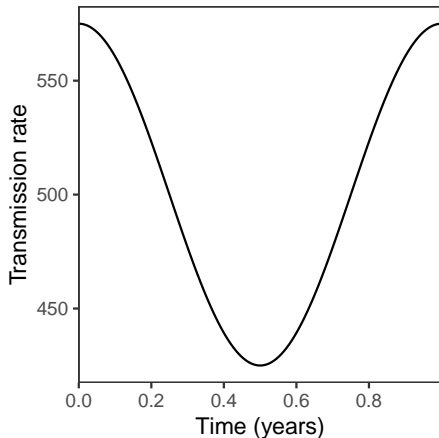
# Trajectory matching

- Try to match the solution of the ODE with the observed time series



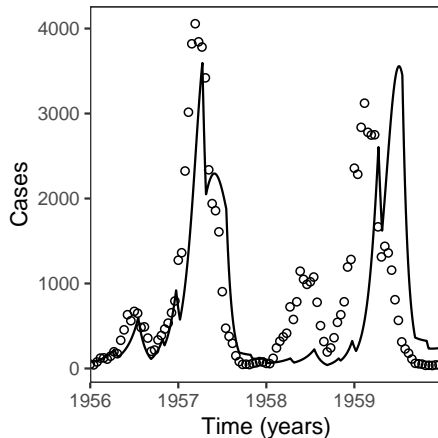
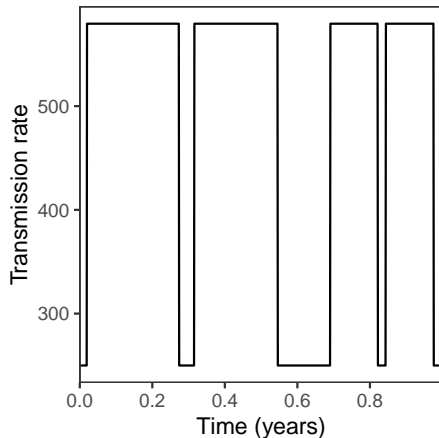
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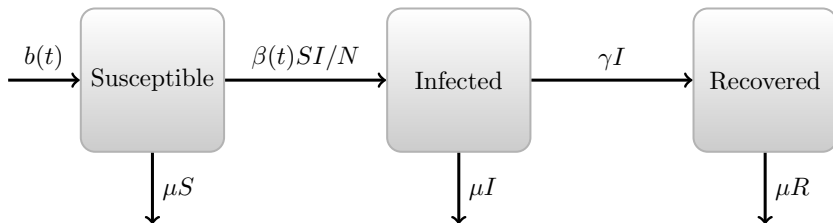
# Trajectory matching

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# Sequential Monte Carlo (SMC)

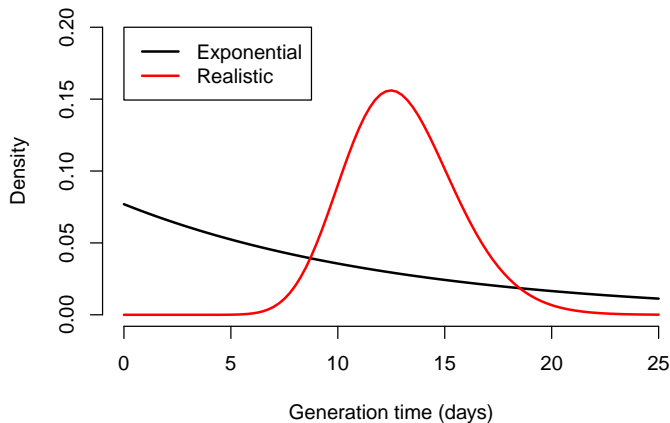
- Probabilistic transition between states based on rate equations
- Use stochastic simulations to integrate over the state space (King et al., 2015)





# Non-exponential distributions

- SIR model assumes exponentially distributed generation time
- Realistic distributions are likely to be narrower (Simpson et al., 1952; Cori et al., 2013)



# The renewal equation

- Current incidence as a function of previous incidence and generation time distribution

$$i(t) = \mathcal{R}(t)S(t) \int i(t-s)g(s)ds$$

- Incidence  $i(t)$
- Generation time distribution  $g(t)$
- Scaled-transmission rate  $\mathcal{R}(t) = \beta(t)/\gamma$

# Time-series SIR model

- Begin by discretizing the system

$$S_{t+1} = B_t + S_t - i_{t+1}$$

$$i_{t+1} = \beta_t S_t \frac{i_t^\alpha}{N}$$

# Time-series SIR model

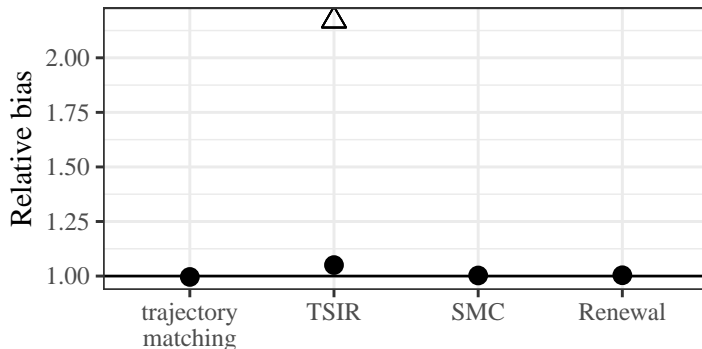
- Estimate the transmission rates using regression (Finkenstädt and Grenfell, 2000)

$$\log i_{t+1} = \log \beta_t + \log S_t + \alpha \log i_t - \log N + \epsilon_t$$

- Key assumptions:
  - Fixed generation time
  - No observation error (only process error)
  - Power-law infection term

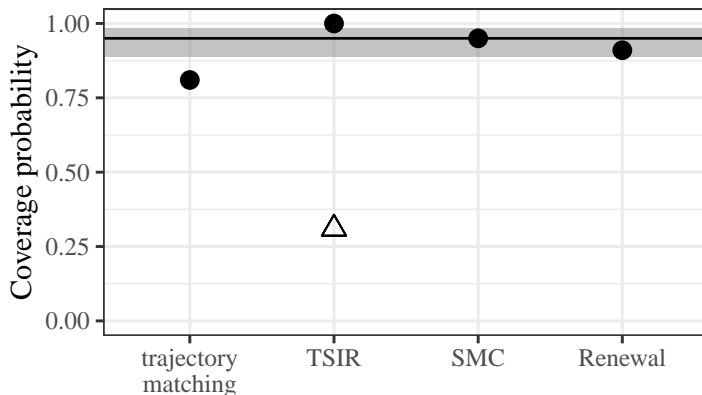
# Relative bias

- Relative bias: mean ratio between the estimates and the true value
- TSIR results in more than twofold bias

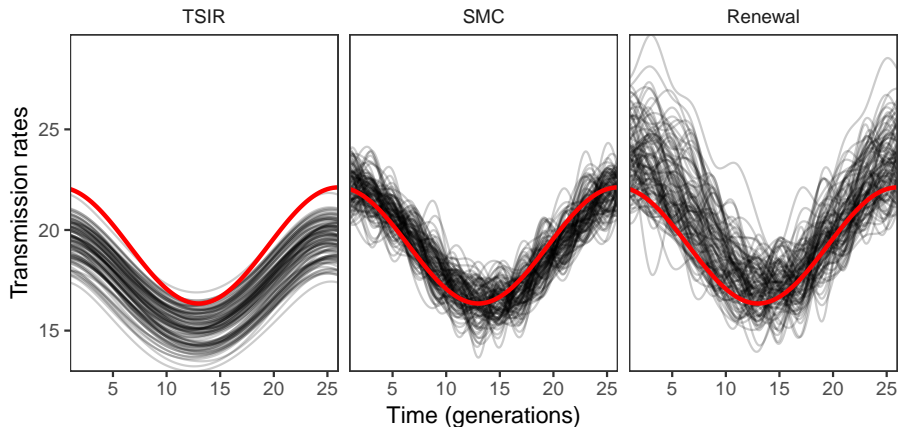


# Coverage

- Coverage: proportion of confidence intervals that contain the true value
- Ignoring process error gives overconfident results (King et al., 2015; Taylor et al., 2016)
- Ignoring observation error may give underconfident results

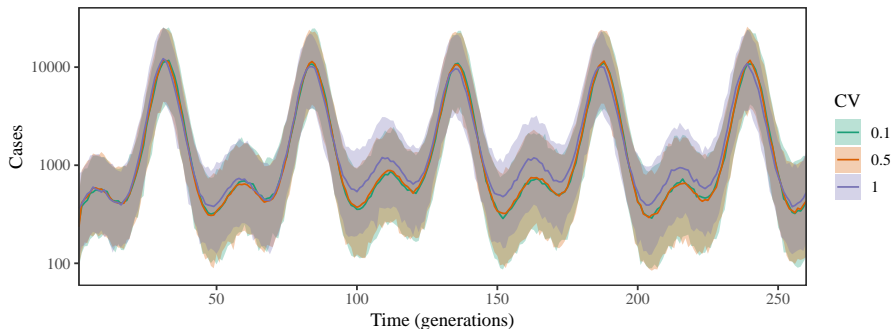


# Estimating time-varying transmission rates



# Generation-time distribution

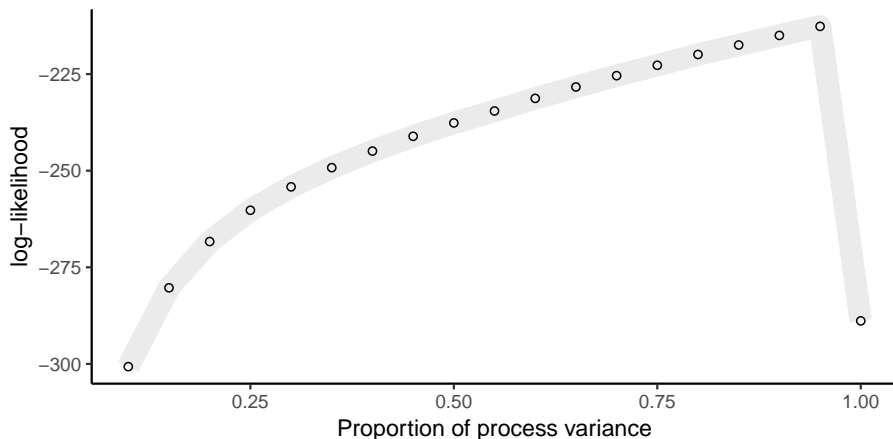
- The shape of the generation-time distribution has little effect on overall dynamics





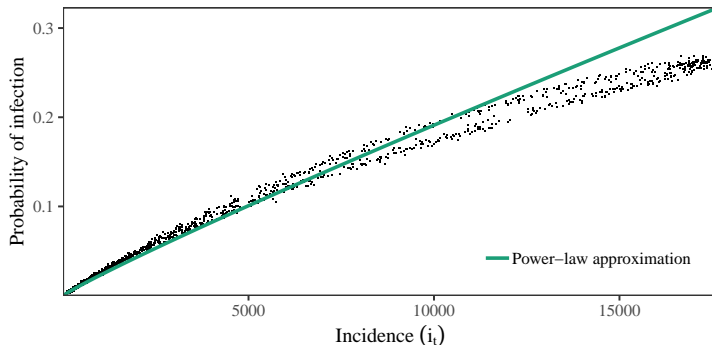
# Amount of process and observation error

- Decompose residual variance from TSIR into process and observation variance
- Measles is largely driven by process error ( $> 95\%$ )



# Probability of infection

- Power-law probability of infection,  $\beta I^\alpha / N$ , does not match the observed patterns
- Small mismatch is expected when the probability of infection is small



- Regression-based methods:
  - Fast and easy to implement
  - Can estimate the exact shape of the transmission rates but give biased answers
  - Good for exploratory analysis
- Simulation-based methods:
  - Computationally expensive
  - Give unbiased answers with good coverage properties
  - Hybrid approach? (Hooker et al., 2010)
- Using simulations can help us assess which assumptions are important

# References

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