Estimating time-varying transmission rates of the SIR model

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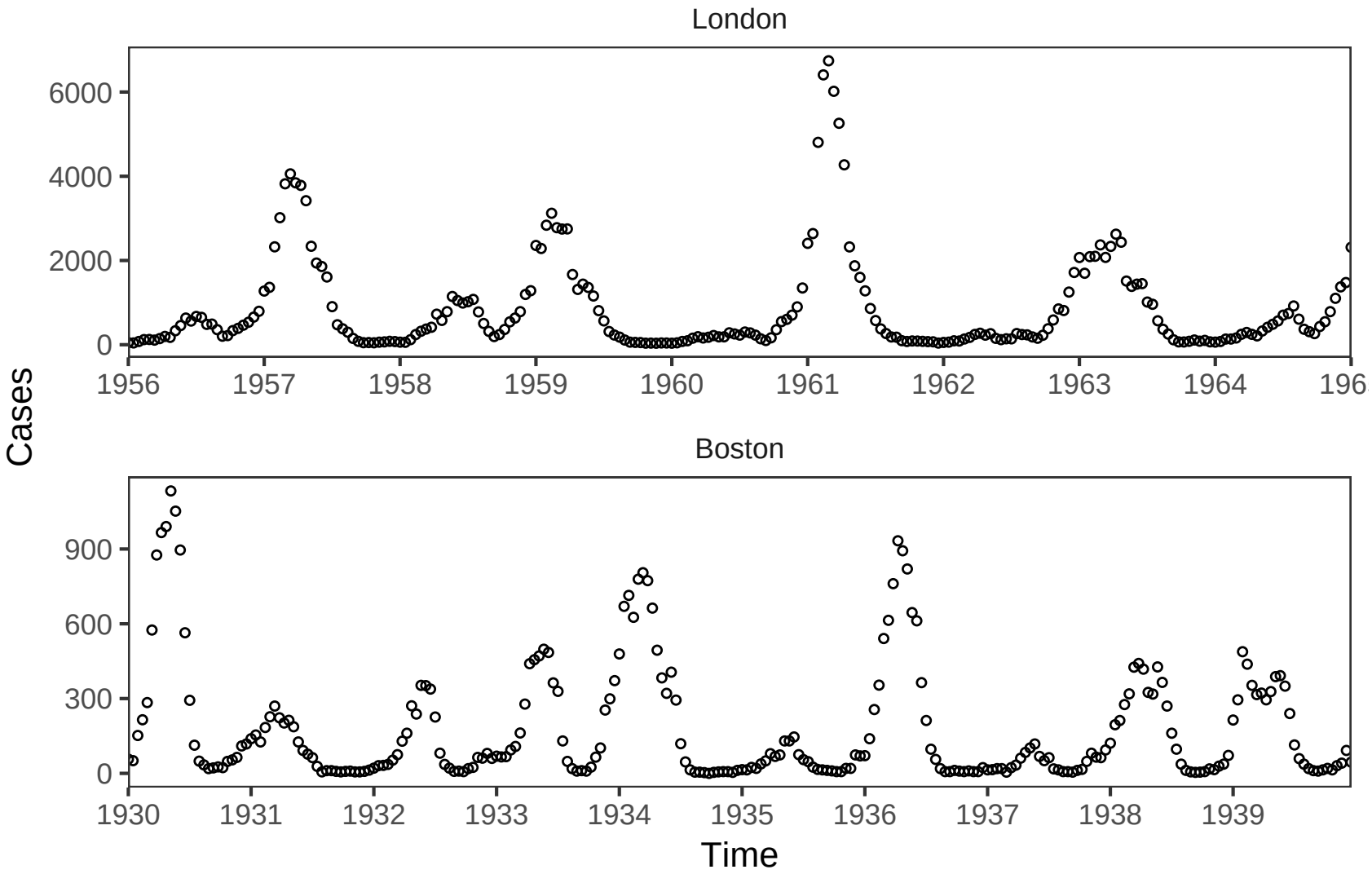
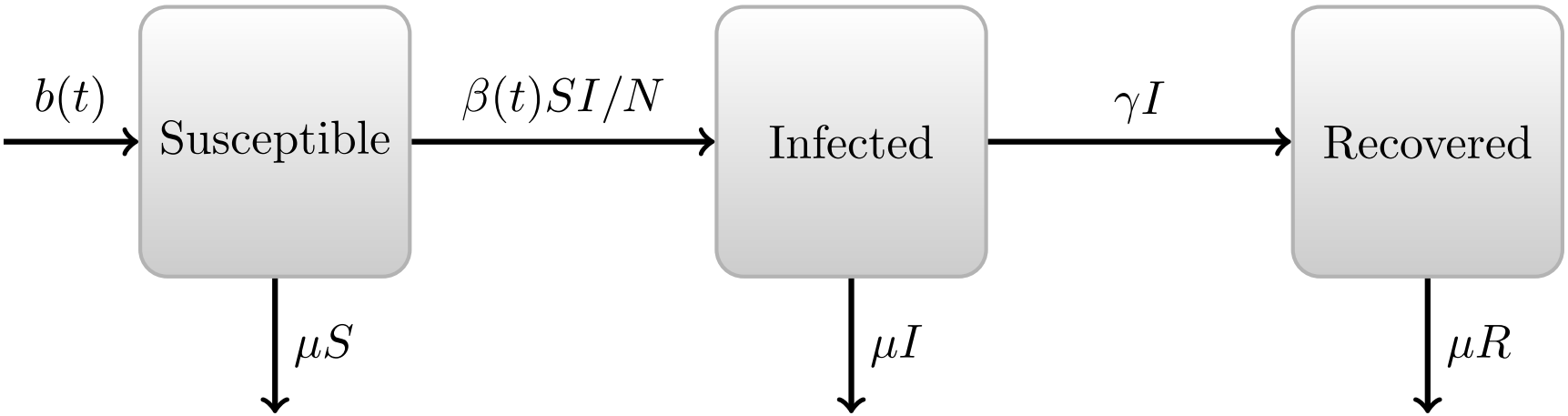
Epidemic time series

- Measles report from the prevaccination era



Susceptible-Infected-Recovered (SIR) model

- Describes how disease spreads in a population
- Individuals move through compartments (Susceptible-Infected-Recovered)



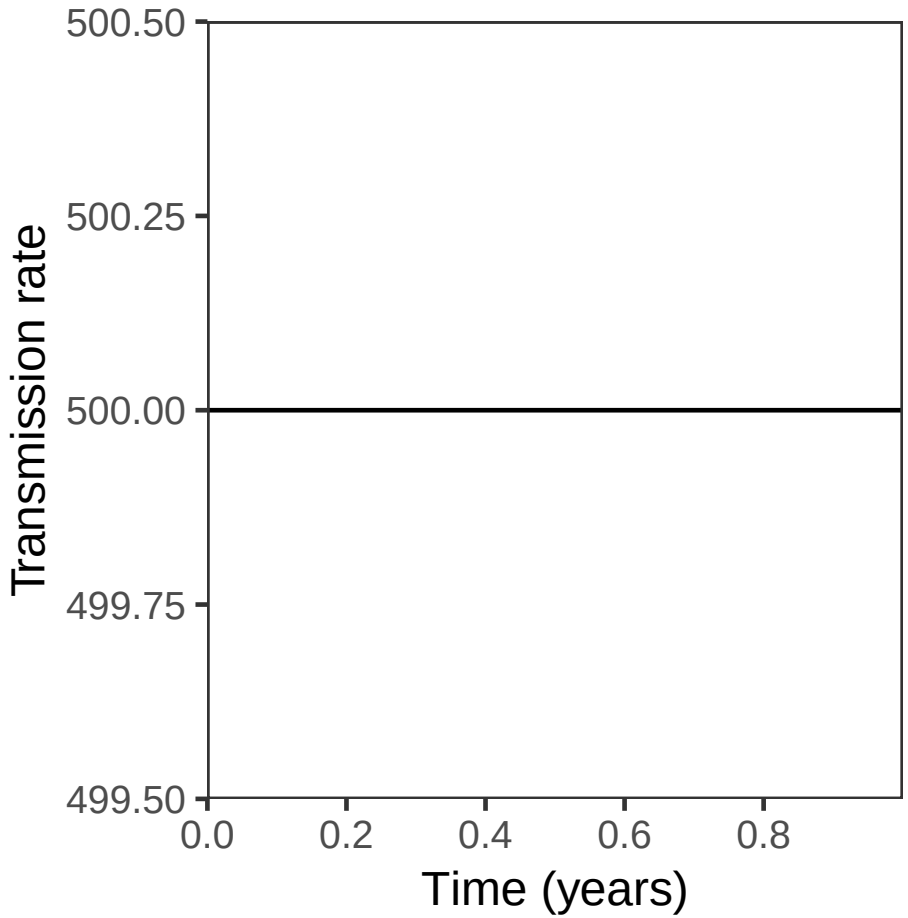
Susceptible-Infected-Recovered (SIR) model

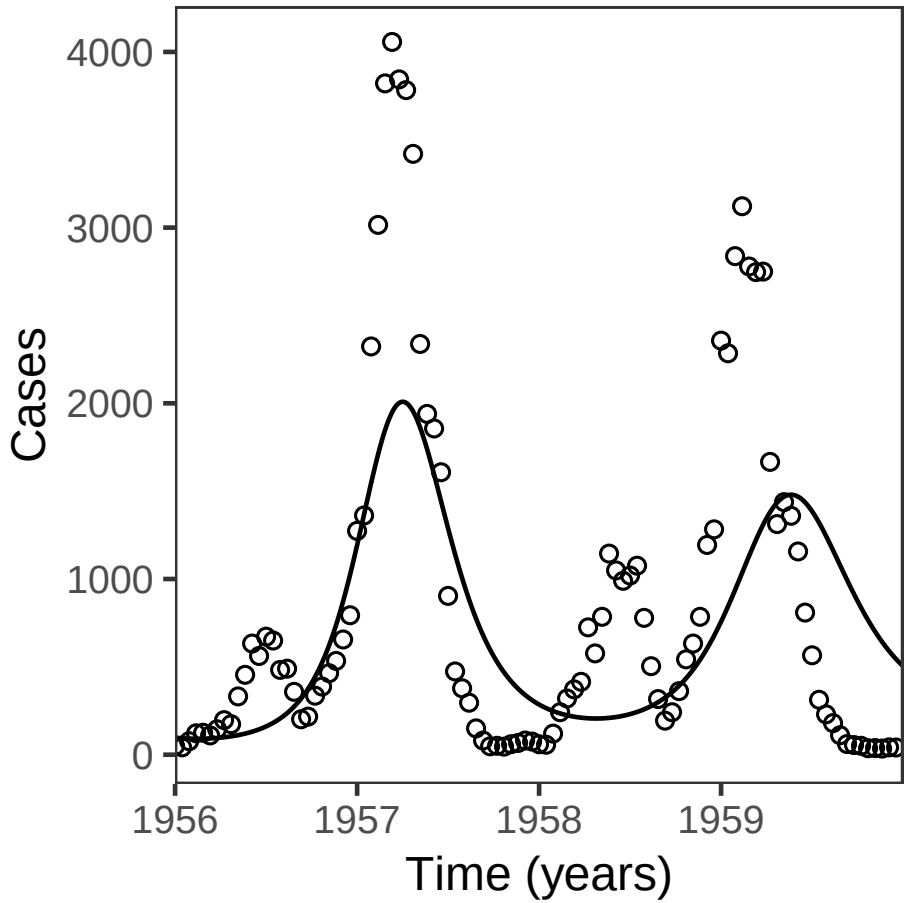
$$\frac{dS}{dt} = b(t) - \beta(t)S\frac{I}{N} - \mu S$$
$$\frac{dI}{dt} = \beta(t)S\frac{I}{N} - (\gamma + \mu)I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

- Mean infectious period $1/\gamma$
- Mean life expectancy $1/\mu$
- Birth rate $b(t)$
- Transmission rate $\beta(t)$

Trajectory matching

- Try to match the solution of the ODE with the observed time series





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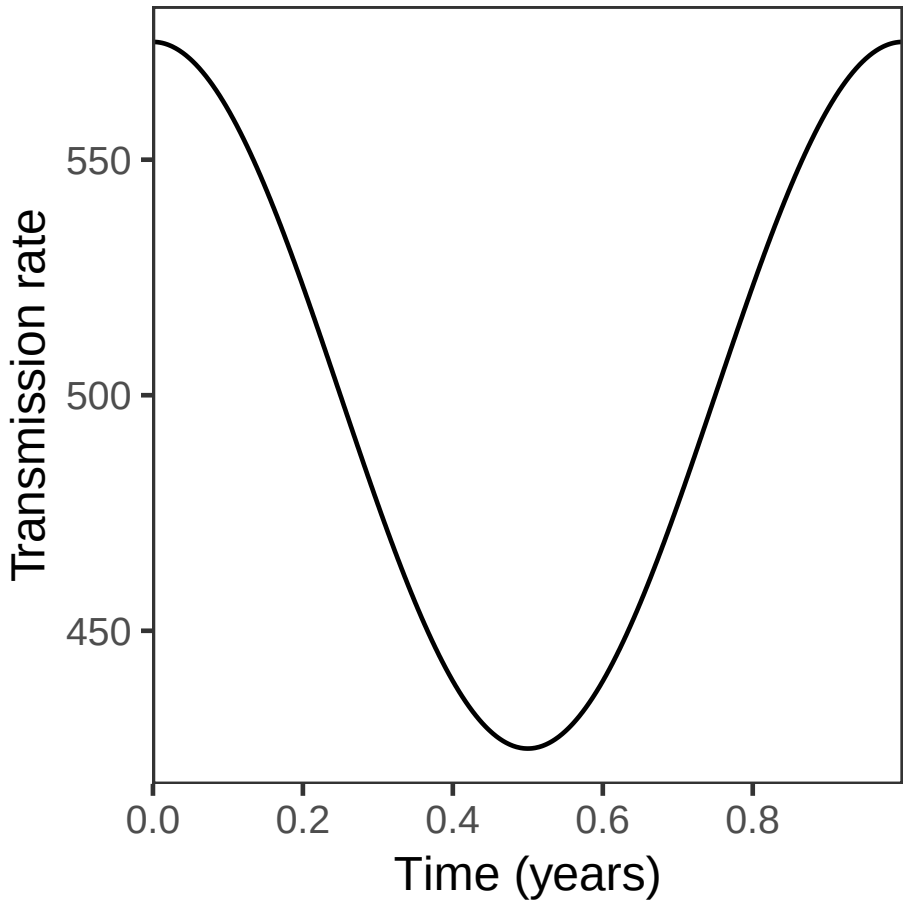

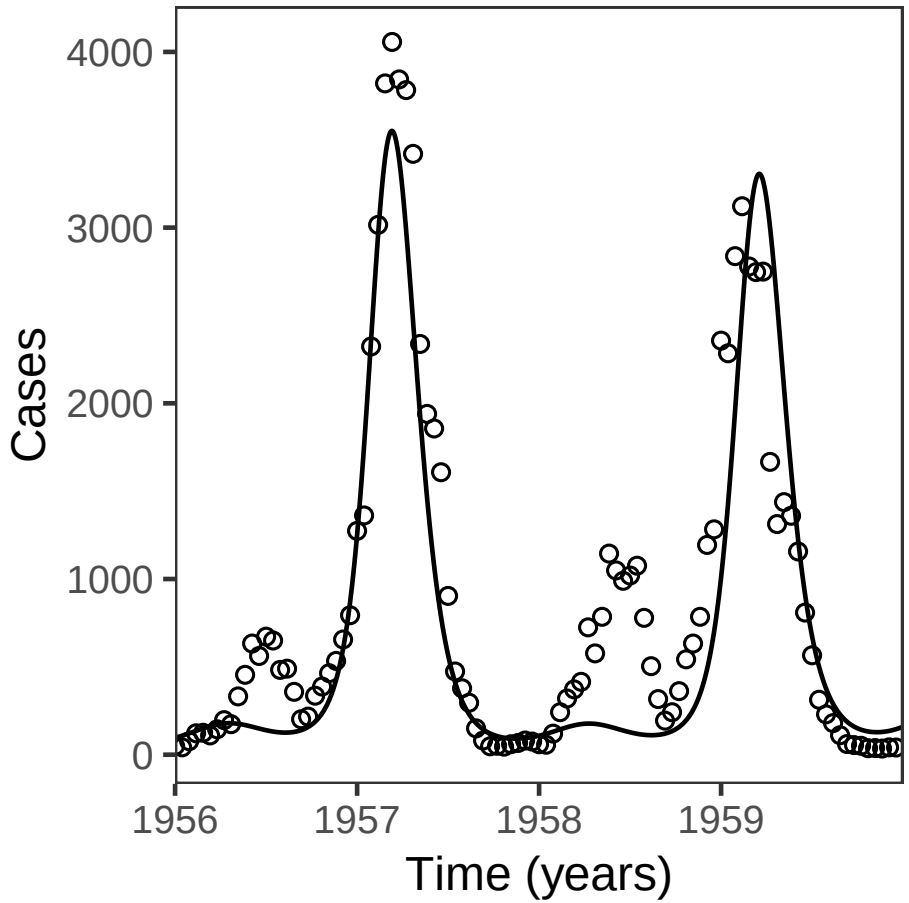

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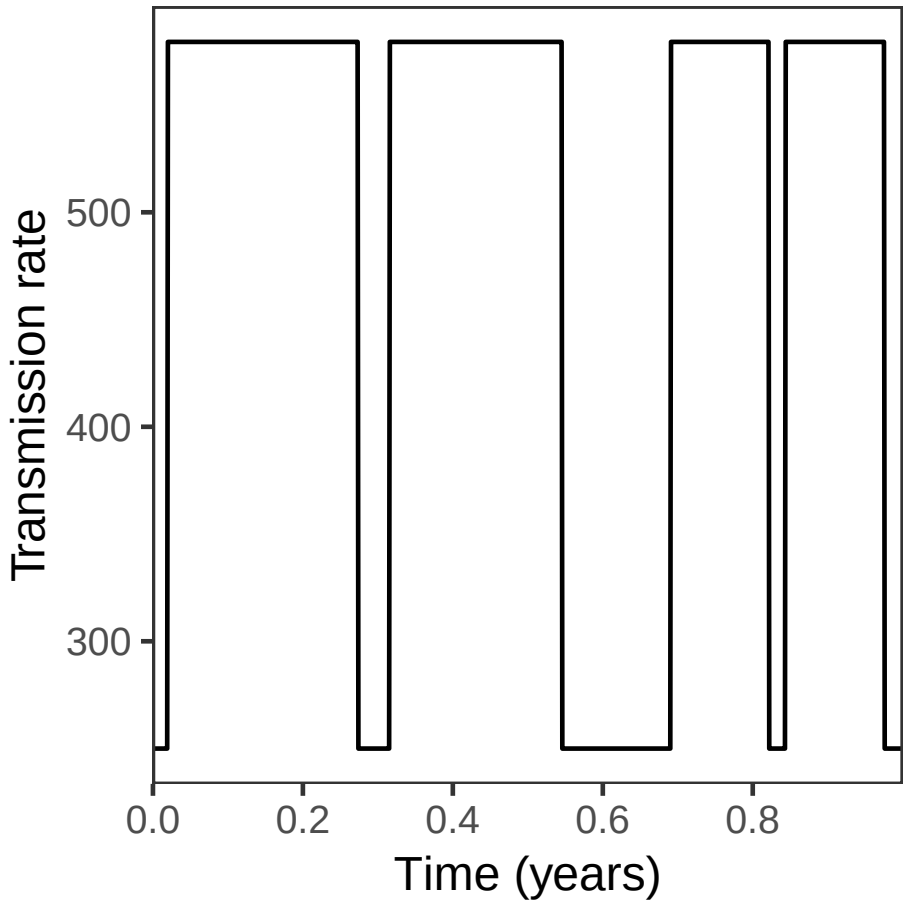
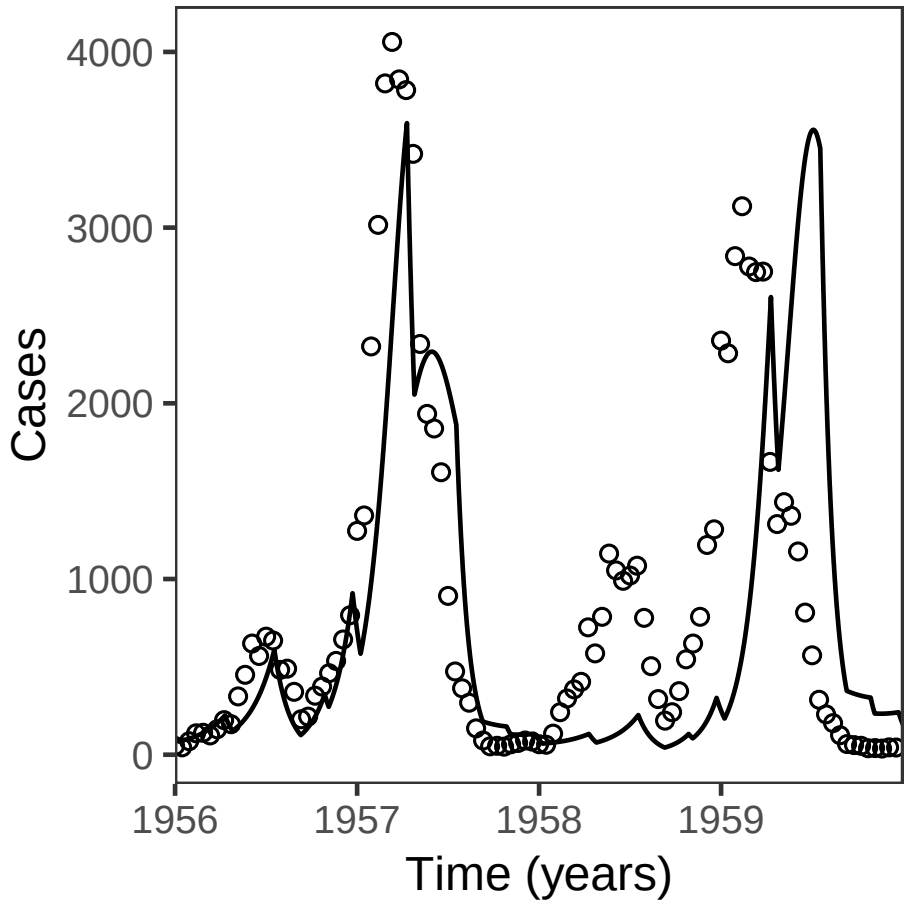
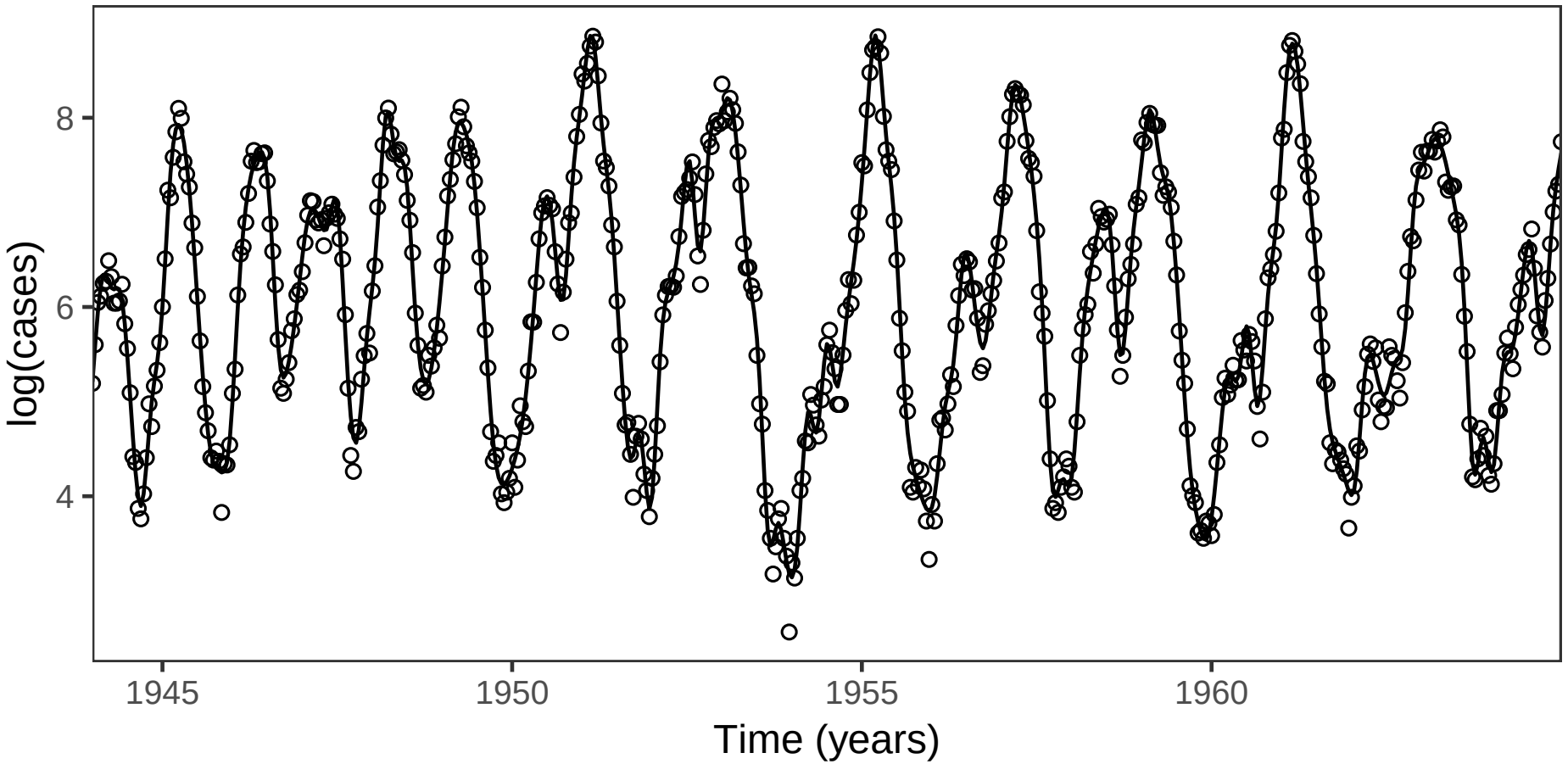




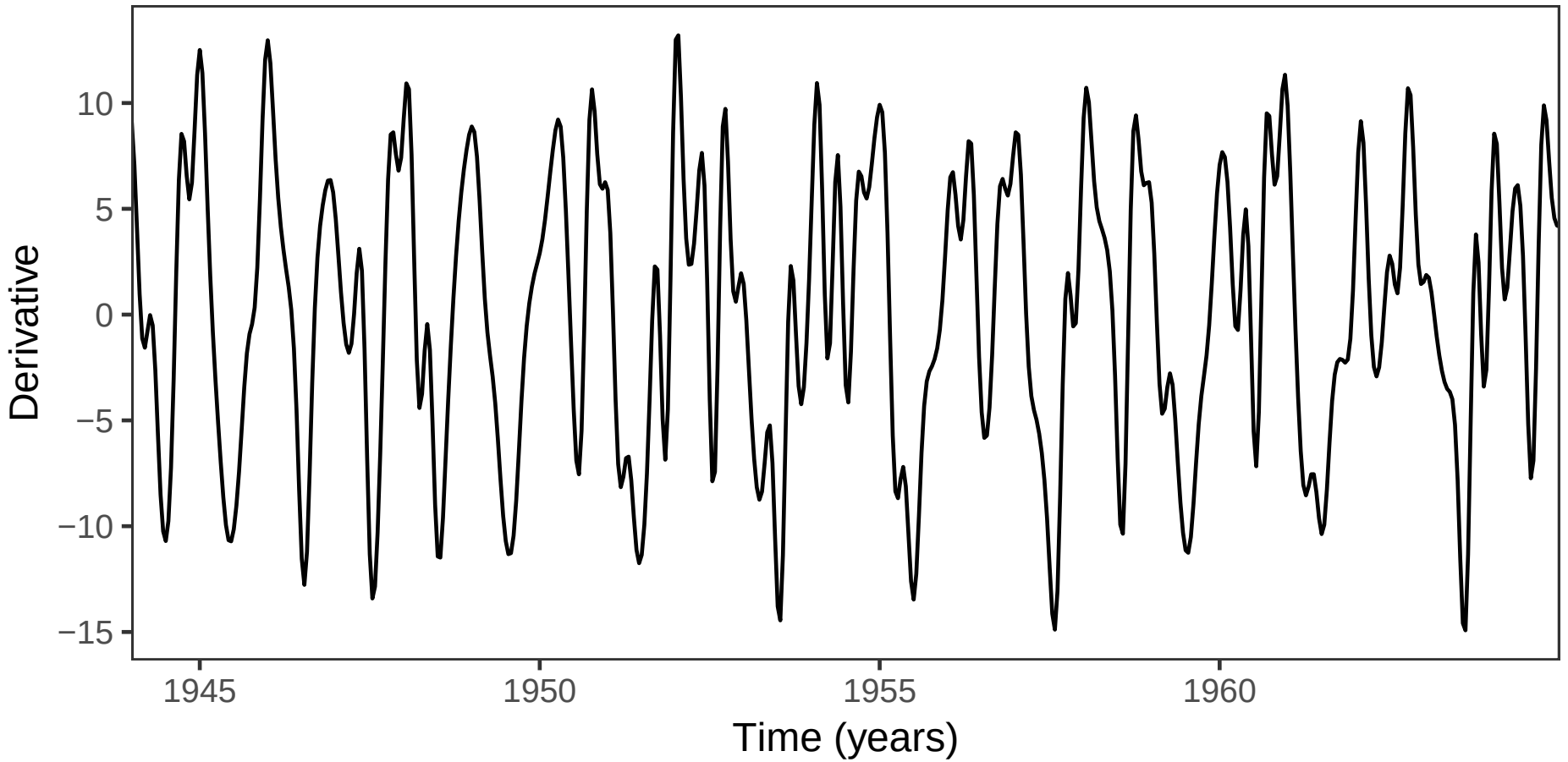
Gradient matching

- Try to match the gradient of the ODE with the estimated gradient



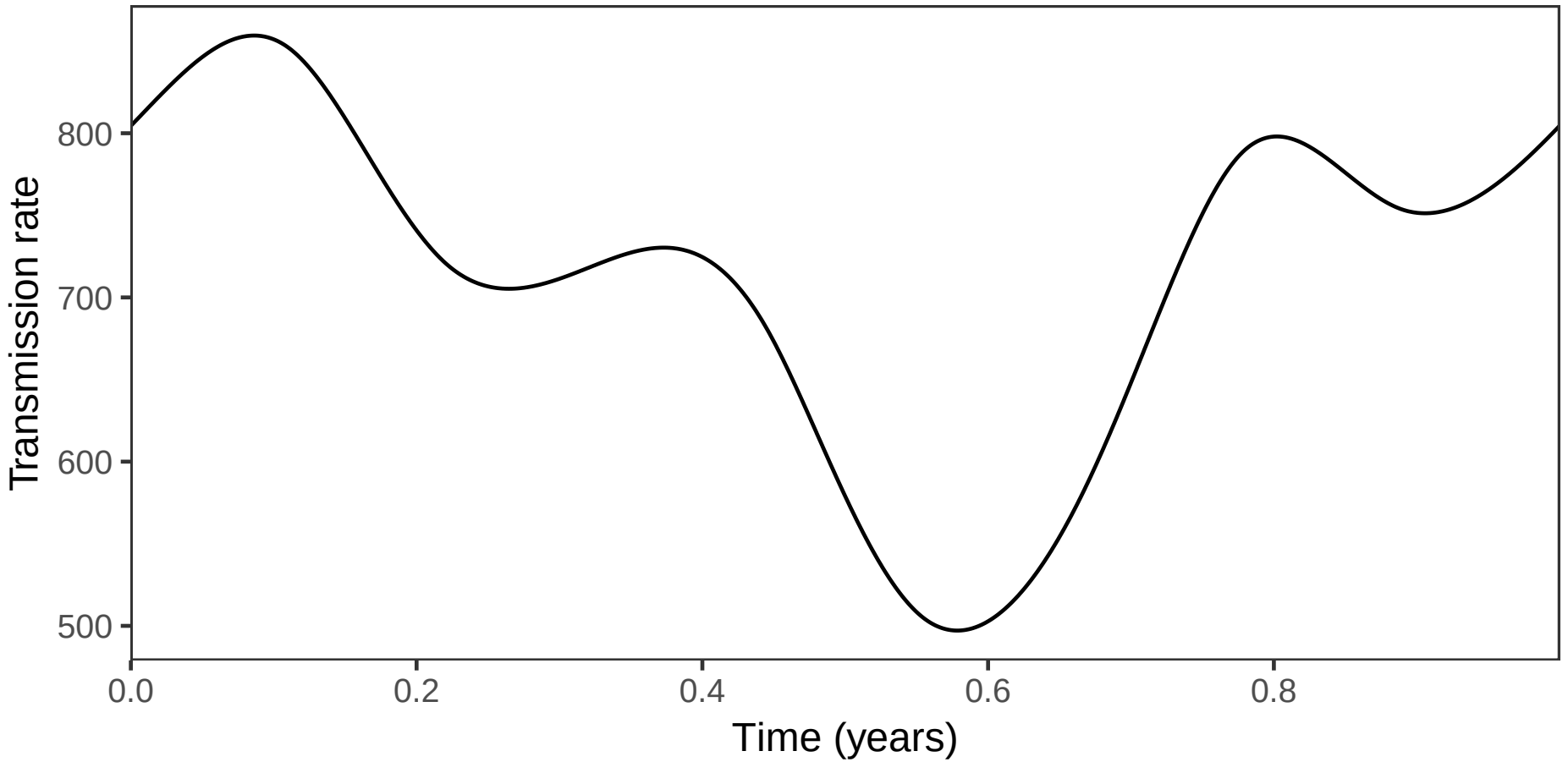
Gradient matching

- Estimate gradient by fitting a smooth curve and taking its derivative



Gradient matching

- Match $d \log I/dt = \beta S - (\gamma + \mu)$ with the estimated derivative using a regression
- Reconstruct S from birth and case reports



References