Notes on network/generation interval

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1 Censored generation interval distributions

Following Champredon and Dushoff (2015), we can write the number of infection occuring at time t caused by infectors who were themselves infected at time s as

$$i_s(t) = K(t-s)i(s)S(t) \tag{1}$$

Writing the kernel as the product of the intrinsic genreation distributions and $\mathcal{R}_0$, we get

$$i_s(t) = \mathcal{R}_0 g(t-s)i(s)S(t) \tag{2}$$

The censored generation interval distributions is what is often measured and we have to account for all infections that happen before time t. Note that number of infection occuring at time s caused by infectors who were themselves infected at time $s - \tau$ is given by

$$i_{s-\tau}(s) = \mathcal{R}_0 i(s-\tau)g(\tau)S(s) \tag{3}$$

Normalizing this gives the backward generation interval distributions of the cohorts at time s but we are interested in all infections that are τ time steps apart before time t:

$$\mathcal{R}_0 \int_\tau^t i(s-\tau)g(\tau)S(s)ds. \tag{4}$$

Then, the censored generation interval is given by

$$c_t(\tau) = \frac{\int_\tau^t i(s-\tau)g(\tau)S(s)ds}{\int_0^t \int_x^t i(s-x)g(x)S(s)dsdx}. \tag{5}$$

We note that the denominater is basically cumulative incidence up to time t divided by $\mathcal{R}_0$. The stragihtforward intuition behind this is that we are normalizing by all incidence before time t. Mathematically, we have the following:

$$\mathcal{R}_0 \int_0^t \int_x^t i(s-x)g(x)S(s)dsdx = \int_0^t \mathcal{R}_0 S(s) \int_0^s i(s-x)g(x)dxds$$
$$= \int_0^t i(s)ds \tag{6}$$

Then,

$$c_t(\tau) = \frac{\mathcal{R}_0 \int_\tau^t i(s-\tau)S(s)ds}{\int_0^t i(s)ds}g(\tau) \tag{7}$$

During an early outbreak, incidence grows exponentially $(i(t) = i(0)\exp(rt))$ and proportion susceptible doesn't change very much. So we can write this as

$$c_t(\tau) = \mathcal{R}g(\tau)\exp(-r\tau)\frac{\int_\tau^t \exp(rs)ds}{\int_0^t \exp(rs)ds}, \tag{8}$$

where $\mathcal{R} = \mathcal{R}_0 S$. Does this mean anything?

We're also interested in $c_\infty(\tau)$. This is something people can study after the outbreak.

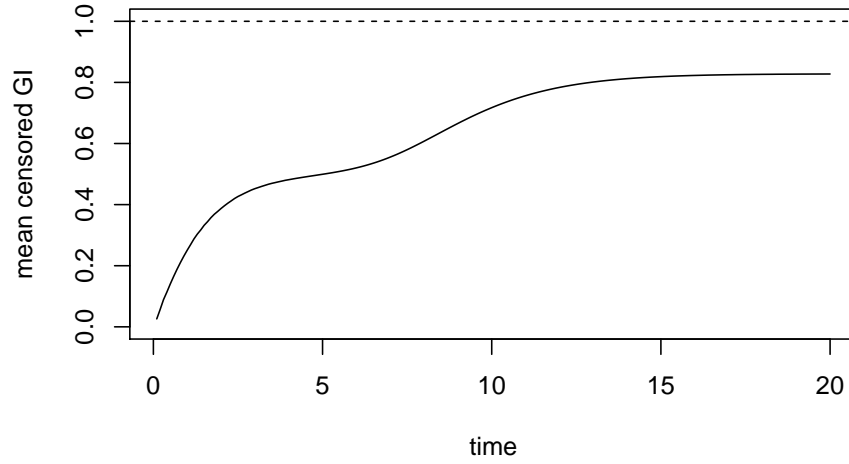$$c_\infty(\tau) = \frac{\mathcal{R}_0 \int_\tau^\infty i(s-\tau)S(s)ds}{\int_0^\infty i(s)ds}g(\tau) \tag{9}$$

1.1 SIR model
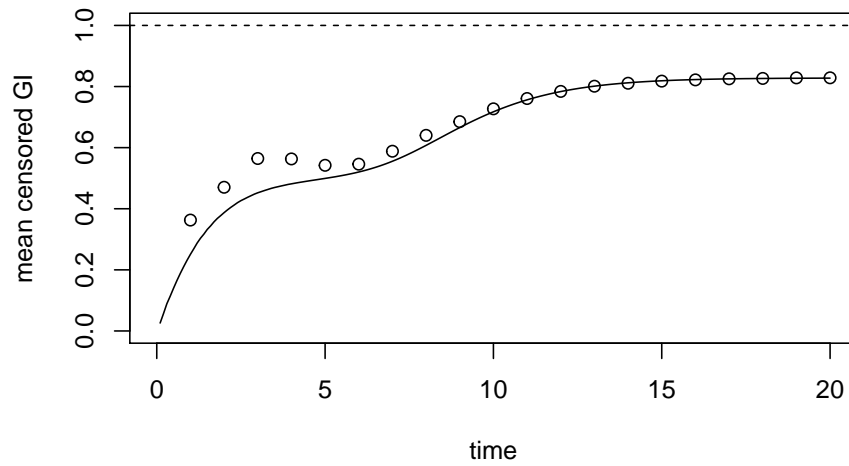
First, let's start with the simplest case: SIR model.

$$\begin{aligned}\frac{dS}{dt} &= -\frac{\beta}{N}SI \\ \frac{dI}{dt} &= \frac{\beta}{N}SI - \gamma I \\ \frac{dR}{dt} &= \gamma I\end{aligned} \tag{10}$$

For this model, intrinsic generation interval distributions follows an exponential distribution with rate parameter γ.

Let's see how mean censored GI changes

Let's compare this curve with what we observe on a homogeneous network $(\beta = 2, \gamma = 1, N = 4000, I(0) = 1)$:

Other than noise due to stochasticity in the beginning, this looks pretty good. I think this brings an important point. People will often analyze epidemic data after it's over and what not and try to make inference but this tells us that using the contact tracing data based on the entire epidemic underestimate the mean generation intervals distribution and hence $\mathcal{R}_0$!

2 Adding network

Notice that we can also write

$$c_t(\tau) = \frac{\int_\tau^t i(s-\tau)S(s)ds}{\int_0^t i(s)ds} K(\tau) \tag{11}$$

So my guess is that we can replace this with an approximated network kernel to obtain censored GI distributions on a network and use observed incidence? This might be a good place to go back to the tree network.

2.1 Recovering GI distributions

In the exponential phase, we observe the following:

$$c_t(\tau) = K(\tau)\exp(-r\tau)\frac{\int_\tau^t \exp(rs)ds}{\int_0^t \exp(rs)ds}, \tag{12}$$

Since we can measure everything, except the kernel, we should be able to recover parts of kernel (before time t) by solving this numerically.

2.1.1 Homogeneous network

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tmax <- 6
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References

Champredon, D. and J. Dushoff (2015). Intrinsic and realized generation intervals in infectious-disease transmission. *Proceedings. Biological sciences 282*(1821), 20152026–20152026.