Temporal and spatial effects on measuring generation intervals through contact tracing

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1 Generation interval

Let $K(t)$ be the infection kernel. Then, the reproduction number is defined as

$$\mathcal{R} = \int_0^\infty K(t).$$

The intrinsic generation interval distributions [CITE] can be written as

$$g(t) = \frac{K(t)}{\mathcal{R}}.$$

We can think of intrinsic generation interval distribution as an intrinsic characteristic of an infector, as suggested by its name. Here, we look at spatio-temporal components associated with generation interval distribution.

2 Contact tracing

Generation and serial intervals are often sampled through contact tracing. For simplicity, suppose contact tracing is performed during an outbreak from the beginning of an epidemic to some time point. Following Champredon and Dushoff (2015), we can write the number of infection occuring at time t caused by infectors who were themselves infected at time s as

$$i_s(t) = K(t-s)i(s)S(t) \tag{1}$$

Writing the kernel as the product of the intrinsic genreation distributions and $\mathcal{R}$, we get

$$i_s(t) = \mathcal{R}g(t-s)i(s)S(t) \tag{2}$$

We define generation intervals measured through contact tracing as the *right-censored generation interval distributions*. Note that number of infection occuring at time s caused by infectors who were themselves infected at time $s-\tau$ is given by

$$i_{s-\tau}(s) = \mathcal{R}i(s-\tau)g(\tau)S(s) \tag{3}$$

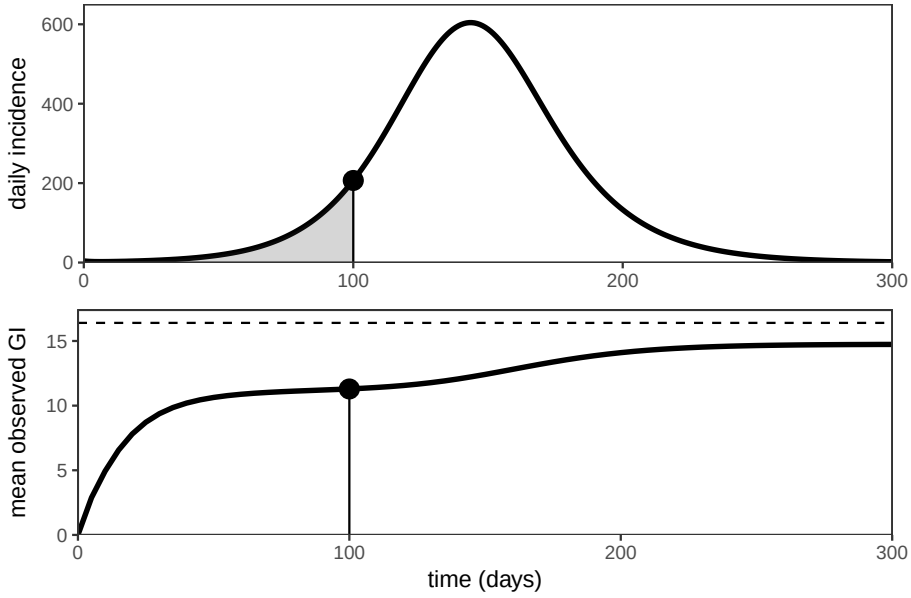

Figure 1: Fill this out.

Note we are interested in total number of secondary infections that are τ time steps apart and occur before time t:

$$\mathcal{R} \int_{\tau}^{t} i(s-\tau) g(\tau) S(s) ds. \tag{4}$$

Then, the censored generation interval at time t is given by

$$c_t(\tau) = \frac{\mathcal{R} \int_{\tau}^{t} i(s-\tau) g(\tau) S(s) ds}{\mathcal{R} \int_{0}^{t} \int_{x}^{t} i(s-x) g(x) S(s) ds dx}. \tag{5}$$

We note that the expression in the denominater is equivalent to cumulative incidence at time t. The intuition behind this is that we are normalizing acrosss all incidence before time t. Then, we have

$$c_t(\tau) = \frac{\mathcal{R} \int_{\tau}^{t} i(s-\tau) g(\tau) S(s) ds}{\int_{0}^{t} i(s) ds}. \tag{6}$$

For convenience, we ignore normalizing constants and write

$$c_t(\tau) \propto g(\tau) \int_{0}^{t} i(s-\tau) S(s) ds. \tag{7}$$

Note that the observed mean generation interval through contact tracing will always be shorter be shorter than intrinsic mean generation interval (Figure 1).

There are two reasons for this. First, due to right censoring, we do not observe generation intervals that are longer than the time at which contact tracing was performed. Second, number of susceptibles decrease over the course of an epidemic and any infector is less likely to infect someone through long generation intervals (Champredon and Dushoff, 2015). Due to the second reason, even if we could contact trace through an entire epidemic, we will still underestimate mean generation interval.

In the following sections, we introduce different ways to recover intrinsic generation interval distributions from the right-censored genration interval samples.

2.1 Non-parametric approach

During an exponential growth period, we can write $i(\tau) \propto \exp(rt)$. Assuming that $S(t) \approx 1$, censored generation interval distribution can be written as follows:

$$g_{\text{obs}}(\tau) = \mathcal{R} g(\tau) \exp(-r\tau), \tag{8}$$

This is the right-censored generation interval distributions that we expect to observe during an early outbreak (growing at rate r). Hence, to recover the intrinsic generation interval distributions, we can take the weighted distribution of the observed distribution:

$$g(\tau) = \frac{1}{\mathcal{R}} g_{\text{exp}}(\tau) \exp(r\tau). \tag{9}$$

Furthermore, $\mathcal{R}$ should be estimated by

$$\mathcal{R} = \int_0^\infty g_{\text{obs}}(\tau) \exp(r\tau) d\tau \tag{10}$$

This contrasts with the well-known Euler-Lotka equation:

$$\mathcal{R} = 1 \Big/ \int_0^\infty g(\tau) \exp(-r\tau) d\tau \tag{11}$$

Using the censored generation interval without correction leads to underestimation of $\mathcal{R}$ as well as mean generation interval.

2.2 Parametric approach

The likelihood is given by

$$\mathcal{R}^{n_e} \cdot \prod g(\tau_e) \cdot \exp\left(-\mathcal{R} \int_0^{c - t_{\text{inf}}} g(s) ds\right)$$

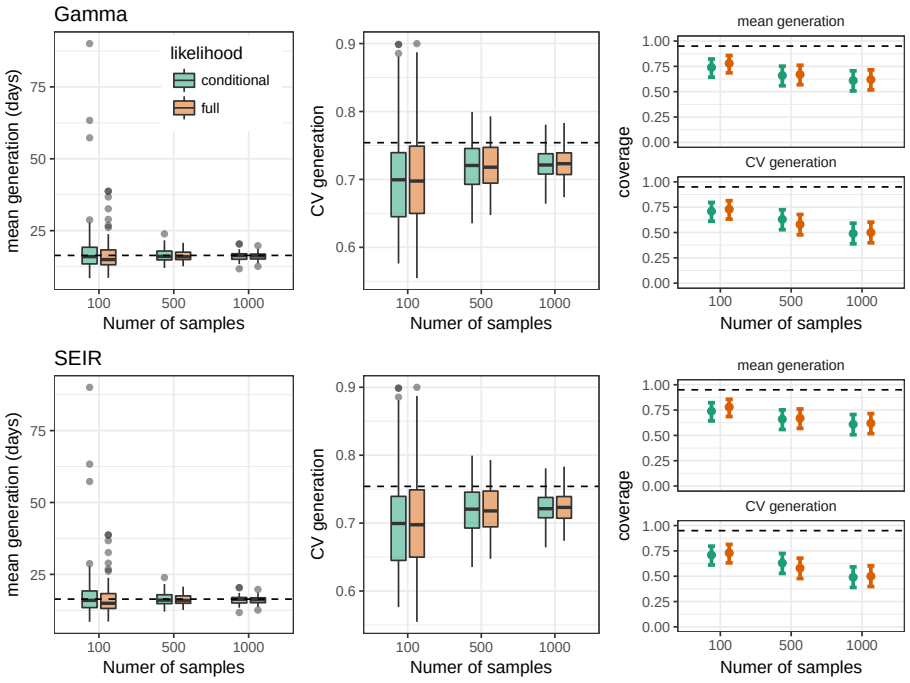

Figure 2: Fill this out as well.

3 Spatial variation - Effective generation interval

Intrinsic generation interval distribution implicitly that an infector can exert all infectious contacts without wasting any throughout the infectious period. In other words, it is conditional on the assumption that a contacted individual has not been contacted before. When the population is limited, we must take the probability that a susceptible individual can be found into account.

Let $\beta(t)$ be infectious contact rate per pair. The probability that a susceptible is still susceptible at time t is given by

$$\exp\left(-\int_0^t \beta(s)ds\right).$$

Then, the effective generation interval distribution of an infected individual is proportional to the product of intrinsic generation interval distribution and this survival probability

$$g_{\text{eff}}(\tau) \propto g(\tau)\exp\left(-\int_0^\tau \beta(s)ds\right).$$

Note that the previous formulation does not take into account presence of other potential infectors. During an outbreak, we can imagine a susceptible individual being exposed to multiple infected individuals. Since effective generation interval is conditional on the assumption that a contacted susceptible individual has not been contacted previously, we have to take this into account... Then, the previous formulation can be taken as an upper bound of the actual effective GI distribution... It is very difficult to formalize this idea but we use a numerical example to demonstrate the idea:

3.1 Numerical example

We can imagine contact structure being embedded in generation or serial intervals sampled through contact tracing. Then, when we apply temporal correction, we expect to obtain the effective generation interval distributions rather than the intrinsic generation interval distribution. In particular, we claim that we must use effective generation interval distribution in order to estimate $\mathcal{R}$.

[Example]

- Call it right censored
- time censored are different from backward but during exponential period, they are
- local correction and full correction
- 1) epidemic is growing, 2) I'm infecting people (local), 3) semi-local effect - sharing contact

- unsolved problem of stats
- relationship between R and beta
- gamma distributed
- fit at individual level (poisson level)

References

Champredon, D. and J. Dushoff (2015). Intrinsic and realized generation intervals in infectious-disease transmission. *282*(1821).