

# Notes

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## 1 TSIR theory

- The TSIR model was first introduced by Bjornstad et al. (2002). The model relies on five basic assumptions: (1) the true number of cases can be inferred from the observed number of cases and underreporting rate, (2) reconstructed susceptible dynamics is deterministic, (3) spatial coupling process is a stochastic process that is independent of number of cases (4) transmission rate varies across time, and (5) zero observations do not provide any information for the parameter estimation. Unlike recent versions of the model, the original framework included a stochastic migration term. All other parameters are estimated deterministically using linear regression except for the migration term, which is estimated by minimizing the Kullback-Leibler distance between the distribution of observed and simulated cases.

## 2 Susceptible reconstruction

- The susceptible reconstruction method used in the TSIR framework was first introduced by Finkenstadt and Grenfell (2000). Essentially, it is assumed that the susceptible,  $Z_t$ , with respect to the mean susceptible is a random deviate with mean 0. After some amount of algebra, we obtain:

$$\sum_{i=1}^t B_{t-d} = -Z_0 + \sum_{i=1}^t \rho_i C_i + \sum_{i=1}^t u_i + Z_t.$$

## 3 Applications of the TSIR model

### References

Bjornstad, O. N., B. F. Finkenstadt, and B. T. Grenfell (2002). Dynamics of measles epidemics: estimating scaling of transmission rates using a time series sir model. *Ecological Monographs* 72(2), 169–184.

Finkenstadt, B. F. and B. T. Grenfell (2000). Time series modelling of childhood diseases: a dynamical systems approach. *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 49(2), 187–205.