APM466 Assignment 1

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February 14, 2022

Fundamental Questions - 25 points

1. (a) Governments issue bonds to raise funds while not creating inflation which occurs when governments print more money.

   (b) Suppose that the price of the monthly Amazon Prime membership costs \$10 and the price of three-month membership costs \$27, then the price of the annual membership should cost less than $\$27*4 = \$108$. Otherwise, the customers will buy the three-month membership and extend it every three months. The annual membership is good for Amazon as they are guaranteed to have the customer for a year and good for the consumer as well since he/she can use Amazon prime for a year at a lower price.

   (c) Quantitative easing is a monetary policy where a central bank purchases long-term securities from the open market in exchange for cash, which leads to an increase in the money supply and thus boosts the economy. In March 2020, the US Fed announced the quantitative easing plan of over \$700 billion in response to the financial fallout caused by the COVID-19.

2. Here are the selected 10 bonds: *"CAN 1.50 May 22"*, *"CAN 0.25 Nov 22"*, *"CAN 0.25 May 23"*, *"CAN 0.50 Nov 23"*, *"CAN 2.25 Mar 24"*, *"CAN 1.50 Sept 24"*, *"CAN 1.25 Mar 25"*, *"CAN 0.50 Sept 25"*, *"CAN 0.25 Mar 26"*, *"CAN 1.00 Sept 26"*. The first bond matures within the 6 months from January, and every subsequent bond matures in at latest 6 months from the previous one.

   Once I have selected the 10 bonds based on their maturity dates, I took their issue dates and coupon rates into consideration to make sure that they are not so dispersed. In fact, all of them were issued between October 2018 and August 2021, so I can say that they were issued around the same time period. Next, they have similar coupon rates which are between 0.25 and 2.25. To summarize, the selected 10 bonds are approximately equidistant, were issued around the same time, and have similar coupon rates.

3. If there are so many features in the data, compute the covariance matrix of the stochastic processes, and find the eigenvalues and eigenvectors. Then, by ordering the eigenvectors in decreasing order based on the magnitude of their corresponding eigenvalues and choosing the top few, we can reduce the dimensionality of the dataset while not losing much information about the data. The larger the magnitude of the eigenvalue is, the larger the variance is, and thus the more information it contains.

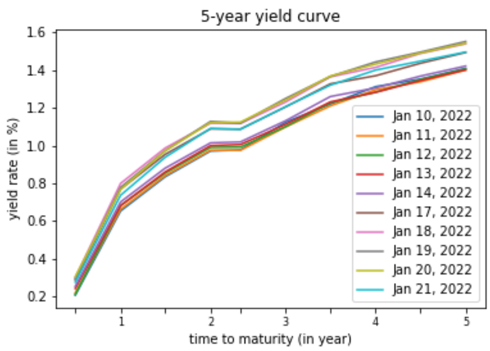
Empirical Questions - 75 points

4. (a) Let n indicate the number of coupon payments remaining. The last coupon payment is made on the maturity date, so the first selected bond has one coupon payment remaining, and thus n runs from 1 to 10. Here is the formula to compute the yield rate:

$$DP_n = \sum_{i=1}^{n} C_n(1+\frac{ytm_n}{2})^{-i} + F(1+\frac{ytm_n}{2})^{-n}$$

where C_n indicates the coupon payment and F denotes the face value.

To calculate the yield rate, the subpackage *optimize* from *Python* library *scipy* has been used to use the interpolation methods. The ridder method is used which guarantees the convergence and converges at a fast rate. Set $f(ytm_n) = \sum_{i=1}^{n} C(1+ytm_n)^{-i} + F(1+ytmr_n)^{-n} - DP$. In this method, we need two inputs, say a and b with one assumption - $f(a) \cdot f(b) < 0$. The following observations have been used: (i) if $DP < F$, then $ytm_n > coupon\ rate$. (ii) if $DP = F$, then $ytm_n = coupon\ rate$. (iii) if $DP > F$, then $ytm_n < coupon\ rate$. (iv) the interest rates are always in between 0 and 1. Therefore, if $DP < F$, set $a = coupon\ rate$ and $b = 1$. If $DP > F$, set $a = 0$ and $b = coupon\ rate$.

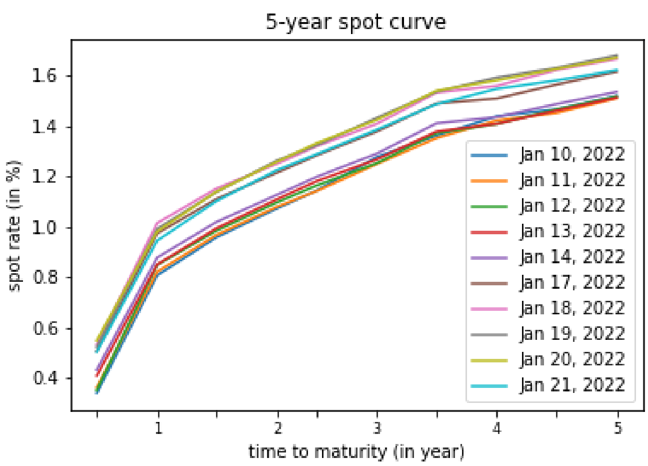


First, we can observe that the yield rate increases as its maturity dates are further. Also, as discussed in Q1(b), we can see that the long-term part of the curve slightly flattened. Moreover, as the day passes, the yield rate tends to be higher than the day before.

(b)

Algorithm 1 Calculate spot rate using bootstrapping method

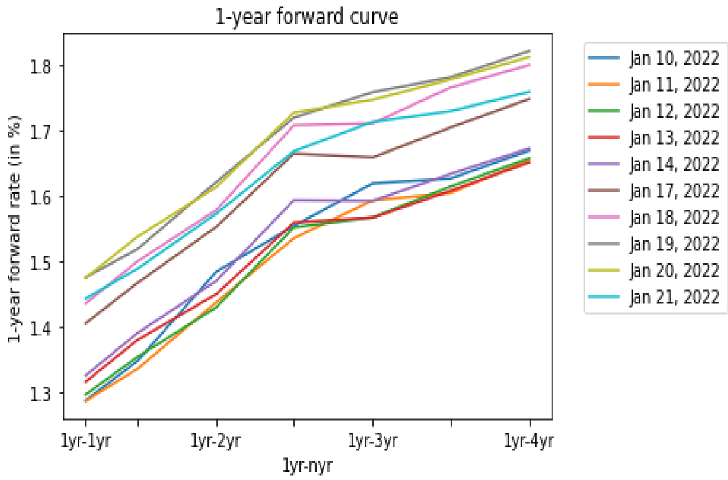
```
for bond in selected_bonds do
    n ← number of coupon payments remaining of bond
    c ← a coupon rate of bond
    C_n ← 100 * c/2
    for day in dates do
        DP ← dirty price of bond at day
        updated_PV ← DP
        i ← 1
        while i < n do
            r_i ← interest rate of i-th period at day
            t_i ← number of days until the i-th coupon payment at day (in years)
            updated_PV ← updated_PV − C_n e^(−r_i·t_i)
            i ← i + 1
        end while
        notional ← 100 * (1 + coupon_rate/2)
        t_n ← number of days until the n-th coupon payment at day (in years)
        spot_rate_df.loc[bond, day] ← −ln(updated_PV/notional) / t_n
    end for
end for
```



(c)

Algorithm 2 Calculate 1-year forward rate

```
for bond in selected_bonds do
    t_n ← number of days until the n-th coupon payment at day (in years)
    for day in dates do
        t_1 ← 1 year
        r_1 ← 1-year spot rate
        r_n ← n-year spot rate
        forward_df.loc[bond, day] ← (r_n * t_n − r_1 * t_1)/(t_n − t_1)
    end for
end for
```



5.

	X_1	X_2	X_3	X_4	X_5
X_1	0.00162126	0.0009987	0.00085001	0.00067166	0.00081897
X_2	0.0009987	0.00071091	0.00067109	0.000542	0.00059184
X_3	0.00085001	0.00067109	0.00073446	0.00059545	0.00058656
X_4	0.00067166	0.000542	0.00059545	0.00055313	0.00051467
X_5	0.00081897	0.00059184	0.00058656	0.00051467	0.00053757

Table 1: Covariance matrix for the time series of daily log-returns of yield rates

	X_1	X_2	X_3	X_4
X_1	0.000496	0.00054673	0.00043765	0.00044421
X_2	0.00054673	0.0007122	0.00055748	0.00050441
X_3	0.00043765	0.00055748	0.00053019	0.00045471
X_4	0.00044421	0.00050441	0.00045471	0.00047574

Table 2: Covariance matrix for the time series of daily log-returns of forward rates

All the possible pairs of variables are positively correlated.

6.

λ	3.73385153e-03	3.50892557e-04	5.01056458e-05	1.53898738e-05	7.08898961e-06
$\bar{v}$	$\begin{pmatrix} -0.62296554 \\ -0.4315786 \\ -0.41356717 \\ -0.34247692 \\ -0.37057469 \end{pmatrix}$	$\begin{pmatrix} 0.69591947 \\ 0.00134837 \\ -0.45329686 \\ -0.52784256 \\ -0.17775953 \end{pmatrix}$	$\begin{pmatrix} 0.10282834 \\ -0.32149117 \\ -0.64382525 \\ 0.5485164 \\ 0.41314472 \end{pmatrix}$	$\begin{pmatrix} -0.33031245 \\ 0.79339478 \\ -0.42251851 \\ -0.15279743 \\ 0.24402758 \end{pmatrix}$	$\begin{pmatrix} -0.08905077 \\ -0.28444023 \\ 0.17450303 \\ -0.52903165 \\ 0.77513736 \end{pmatrix}$

Table 3: Eigenvalues and eigenvectors of the covariance matrix for daily log-returns of yield rates

λ	2.04158415e-03	1.93220481e-05	7.45206519e-05	7.86996452e-05
$\bar{v}$	$\begin{pmatrix} 0.47265139 \\ 0.57295801 \\ 0.48658051 \\ 0.45995563 \end{pmatrix}$	$\begin{pmatrix} 0.57578515 \\ -0.4182961 \\ 0.44617246 \\ -0.54261402 \end{pmatrix}$	$\begin{pmatrix} -0.60322813 \\ 0.34101766 \\ 0.5840337 \\ -0.42276166 \end{pmatrix}$	$\begin{pmatrix} -0.284935 \\ -0.61680989 \\ 0.47230729 \\ 0.56150104 \end{pmatrix}$

Table 4: Eigenvalues and eigenvectors of the covariance matrix for daily log-returns of forward rates

The largest eigenvalue tells that the largest variance in the dataset can be explained in the direction of its corresponding eigenvector. In particular, $\frac{\lambda_1}{\sum_i \lambda_i}$ of the information is carried by its corresponding eigenvector.

References and GitHub Link to Code

References

1. Steepening and Flattening Yield Curves as Indicators

Available: `https://www.thebalance.com/steepening-and-flattening-yield-curve-416920`

2. Explaining Quantitative Easing (QE)

Available: `https://www.thebalance.com/what-is-quantitative-easing-definition-and-explanation-3305881`

3. Quantitative Easing (QE)

Available: `https://www.investopedia.com/terms/q/quantitative-easing.asp`

GitHub Link to Code

`https://github.com/parkyunk/Mathematical-Finance`