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Adaptive Learning on Time Series: Theory and Practice

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Introduction

- In a discrete time series, we consider ordered and irreversible processes: with time $t \in \mathbb{N}$, write the vector of potential explanatory variables $x_t \in \mathbb{X}$ and the vector of dependent variables $y_t \in \mathbb{Y}$, where $\dim(\mathbb{X}) \gg \dim(\mathbb{Y})$. At time t, we have access to an information set representing all data available up to t, written as $\Phi_t := \{(x_\tau, y_\tau) : \tau \in [t]\}$. We are interested in producing a conditional forecast of future values of the dependent variable k periods ahead, written as
$$y_{t+k|t} := \mathbb{E}[y_{t+k}|\Phi_t] = f_k(\Phi_t; \theta_t; h_t)$$
The right hand side is a general notation for such a forecast — θ_t stands for the contemporary parameter and h_t stands for the contemporary functional form.
- 20^{th} century problems: we design a finite model space based on intuition. How to estimate $\theta_t(h_t)$ and make forecasts? How to capture time-varying parameters? Solution: MLE estimation. SARIMA models. Windowed time series. Information criteria (AIC, BIC, etc.) for model selections. [1,3,5]
- 21^{st} century problems: Is there a better way to determine $h_t \in H$? What if $|H| > \infty$? What if $h_t \neq h_{t-1}$? How to efficiently capture the right model at the right time? How to effectively penalise the complexity under small data size? [4,5,6,7,9,10]

Traditional algorithm (SARIMA + AIC) [3: pp.48-50]

1. Determine Seasonality and Stationarity by the content of the data (e.g. intuition and ADF Test).
2. Find the maximum lags by ACF and PACF. Usually, total number of explanatory variables are bounded by $\sqrt{T}$ to ensure efficiency and discourage over-fitting.
3. Pin down ARMA lags by finitely searching the upper-bounded model.

Simulated example (similar to [7])

- Consider an SAR process $(1 - \gamma L^7)(1 - \rho L) = \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$
- Without specific knowledge, a traditional algorithm is likely to output $AR(p)$ where $p \approx 8$. Inefficient estimation and poorer forecasts.
- With appropriate training, we are able to find better forecasts by finding a functional form closer to the underlining process.

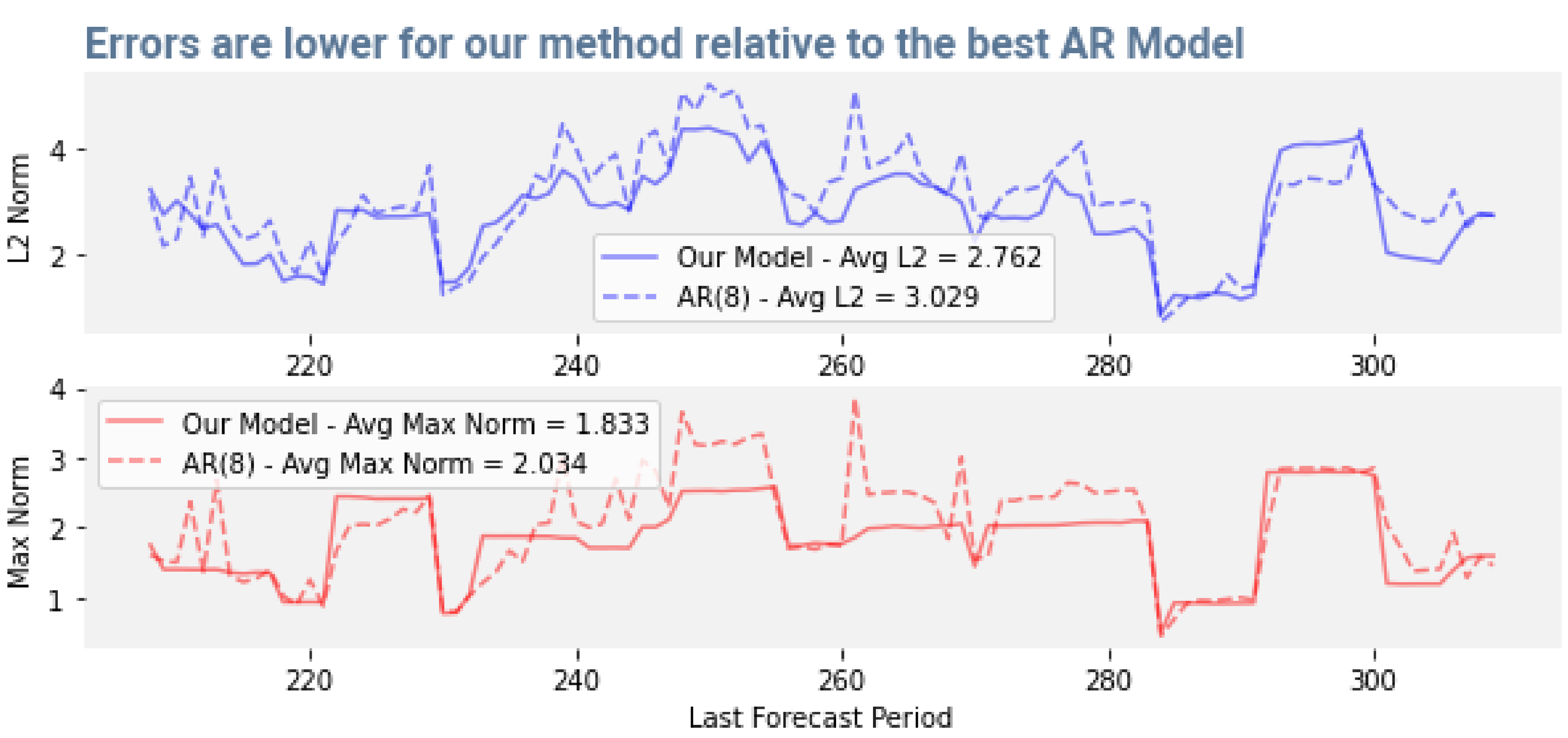

Figure: Our proposed model performs better than an AR(8) model chosen by the AIC

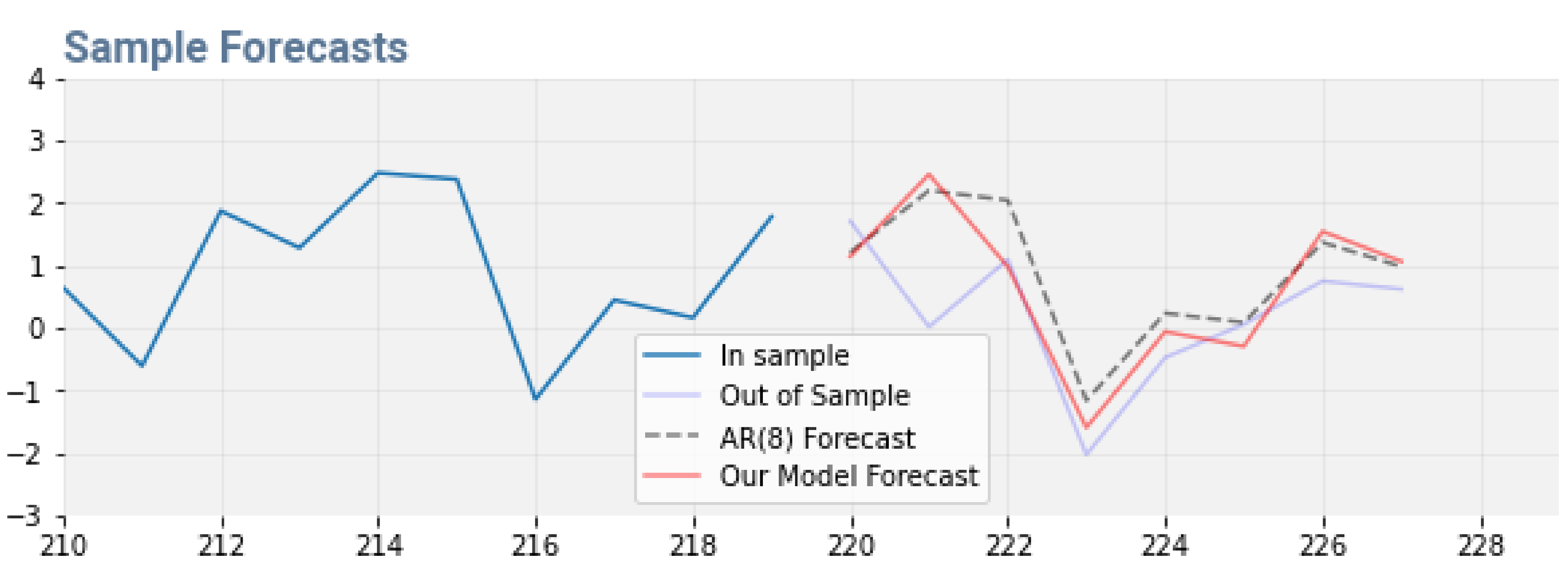

Figure: Forecasts from AR(8) and our proposed Model

Motivating example: Yield curve, SP500, and VIX curve [11]

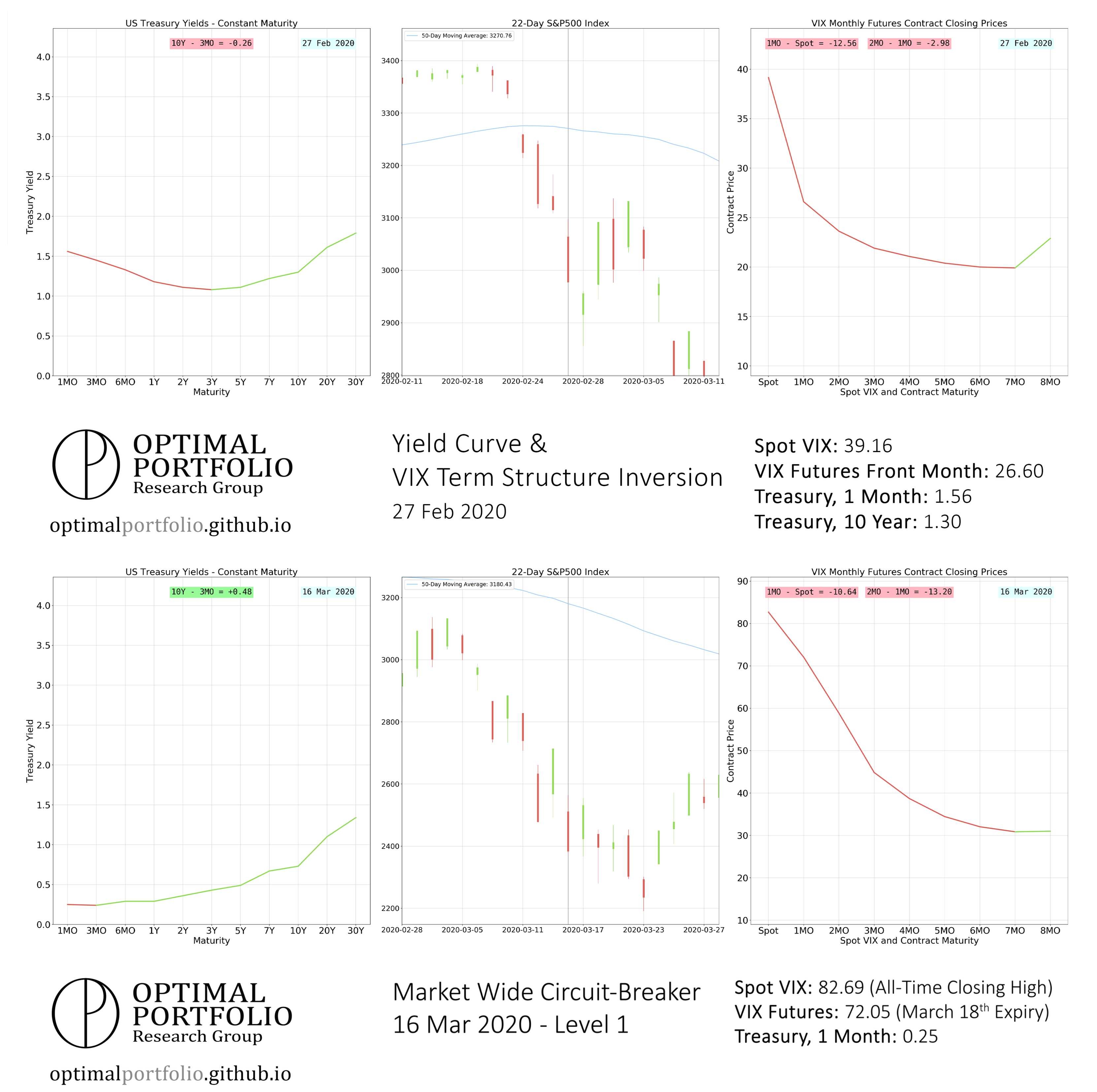

Figure: The February 2020 US Treasury yield curve inversion, coupled with a negatively sloping VIX futures term structure, preceded the pandemic-induced market crash of March 2020 (above). During the crisis, the VIX term structure exhibited extreme inversion, then accompanied by a drop in the overall level of Treasury yields (below). There is widespread existing literature on the predictive power of the VIX and interest rate term structure [2,7,8,9], and we can observe from preliminary data analysis that a relationship likely exists between the shape of the VIX futures term structure and the US Treasury yield curve that offers potential to forecast significant macroeconomic regime changes. However, this relationship may not remain static over time; parameters may change, and the functional form of this relationship may vary over different windowed periods. This motivates the adoption of adaptive learning techniques to best specify h_t.

Adaptive learning: theory [11]

An adaptive learning on a time series dataset is a learning method in which there is a learning function l, a set of models H, and estimation techniques $\Lambda_h : \Theta(h) \mapsto \theta(h) \quad \forall h \in H$ such that for all time $t \in T^{\text{validation}} \cup T^{\text{test}}$, we are able to obtain the following:

- $\forall h \in H$, an estimated statistic $\theta_t(h)$
- $\forall h \in H$, an estimated forecast $y_{t+k|t}(h)$
- an optimal model $h^* \in H$ with an induced optimal forecast $y_{t+k|t}(h^*)$

Adaptive learning: algorithm [10: p.18]

1. Produce $\tilde{H}_t \subset H$ subject to the computational capacity and statistical restrictions (e.g. flat or boundary MLE due to stationarity).
2. Obtain $\theta_t(h_t)$ for all $h_t \in \tilde{H}_t$ and subsequently $h_t^* = \arg\min_{h \in \tilde{H}_t} \ell(\Phi_t, h, H \setminus \{h\})$

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