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INVESTMENT MANAGEMENT UNDER TAXATION

Billions of shares of stock, or fractions of ownership in a business, are traded on the stock market every day. Over half of all adults in the United States own stocks and 1.2 billion people worldwide invest in the stock market. Many people invest in stocks to increase their wealth and to increase their earnings beyond their salary. If the business that you own stock in does well, then your stock value will increase and you will make money.

An individual who owns stock can sell their shares, or a fraction of their shares, to get cash that can be used for a down payment on a home, to buy a new car, or for any other purchase. However, when you sell stock, you have to pay both a transaction fee and tax on the money you gain. If you own many different stocks, you have to decide what stocks and how much to sell to make sure you have enough cash for your purchase. In this problem, we'll use linear optimization to decide which shares of stock and how many you need to sell in order to have enough cash to make your purchase, and to maintain a strong portfolio of stocks.

PROBLEM 1.1 - FORMULATING THE PROBLEM (1/1 point)

Suppose that, last year, you purchased **150 shares of eight different stocks** (for a total of 1200 shares). The spreadsheet Investment.ods for LibreOffice or OpenOffice, and Investment.xlsx for Microsoft Excel, lists the stocks that you purchased, the price you purchased them for last year, the current price, and the price estimate for next year.

If you sell any shares, you have to pay a **transaction cost** of 1% of the amount transacted.

In addition, you must pay a **capital-gains tax** at the rate of 30% on any capital gains at the time of the sale. For example, suppose that you sell 100 shares of a stock today at \$50 per share, which you originally purchased for \$30 per share. You would receive \$5,000. However, you would have to pay capital-gains taxes of:

$$0.30 \times (\$5,000 - \$3,000) = \$600,$$

and you would have to pay:

$$0.01 \times \$5,000 = \$50$$

in transaction costs. Therefore, by selling 100 shares of this stock, you would have a net cashflow of

$\$5,000 - \$600 - \$50 = \$4,350.$

Note that none of the stocks decreased in value since the time of purchase, so we don't have to deal with capital losses.

You would like to sell enough shares of stock today to **generate $10,000** to use as part of a down payment on a new home. You need to decide how many shares of which stocks to sell in order to generate $10,000, after taxes and transaction costs, while maximizing the estimated value of your stock portfolio next year. Let's formulate this as a linear optimization problem.

How many decision variables should your model have?

8 ✔ **Answer:** 8

EXPLANATION

We need one decision variable for each stock, representing the number of shares to sell of that stock. Since we have 8 stocks, there are 8 decision variables.

You have used 1 of 3 submissions

PROBLEM 1.2 - FORMULATING THE PROBLEM (2/2 points)

We'll assume for this problem that you can't sell more shares of stock than you own, and you can't buy additional shares. What is the maximum value your decision variables can be?

150 ✔ **Answer:** 150

EXPLANATION

You can't sell more shares that you own, and since you own 150 shares of each stock, the decision variables can't be any larger than 150.

What is the minimum value your decision variables can be?

0 ✔ **Answer:** 0

EXPLANATION

Since you can't buy additional shares (giving the decision variables negative values) the minimum value the decision variables can be is 0.

You have used 1 of 3 submissions

PROBLEM 1.3 - FORMULATING THE PROBLEM (1/1 point)

Your objective is to maximize the estimated value of your stock portfolio next year. To do this, you should sum the estimated value of each stock next year. Suppose you sell x shares of your stock in Microsoft. What is the estimated value of your Microsoft stock next year?

- (●) $\$34.55 \times (150 - x)$ ✔
- ( ) $\$34.55 \times x$
- ( ) $\$2.05 \times (150 - x)$
- ( ) $\$2.05 \times x$

EXPLANATION

We expect Microsoft stock to be worth $34.55 next year. Since you will have (150 - x) shares next year (after selling x shares), the estimated value of your stock is $34.55(150 - x).

You have used 1 of 1 submissions

PROBLEM 1.4 - FORMULATING THE PROBLEM (1/1 point)

You need to make sure you get $10,000 in cash from selling your stocks, after taxes and transaction costs. How much would you get in cash, after taxes and transaction costs, if you sell 50 shares of your Intel stock?

1154.71 ✔ **Answer:** 1154.71

EXPLANATION

The Intel stock is currently worth $23.67. If you sell 50 shares, then you will receive $1183.50. You will have to pay taxes of thirty percent times the current profit minus the amount you paid for it, or 0.30($1183.50 - $1127.00) = $16.95, and transaction costs of 1 percent times the total amount, or 0.01($1183.50) = $11.84. So you will get in cash, after taxes and transaction costs, $1183.50 - $16.95 - $11.84 = $1154.71.

In LibreOffice (or in the spreadsheet software you are using), formulate and solve this optimization problem. Make sure to include a constraint for the amount of cash you generate, and upper and lower bounds for the values of your decision variables.

You have used 2 of 3 submissions

PROBLEM 2.1 - ANALYZING THE SOLUTION (2/2 points)

In the optimal solution, which stocks do you sell at least one share of? Select all that apply.

- [x] Yahoo! ✔
- [ ] General Electric
- [x] Microsoft ✔
- [ ] Bank of America
- [ ] JPMorgan Chase
- [ ] Cisco Systems, Inc
- [x] Intel ✔
- [ ] Pfizer

✔

EXPLANATION

In the optimal solution, the decision variables with non-zero values are those corresponding to Yahoo!, Microsoft, and Intel.

You can set up this optimization problem as follows. Suppose that you add a column to the end of your table that indicates the number of shares to sell (the decision variables). The decision variables will therefore be in cells G6:G13.

Then the objective formula is: SUMPRODUCT(F6:F13;(C6:C13-G6:G13))

And the cash constraint is given by the following formula:

SUMPRODUCT(G6:G13;E6:E13) - 0.3*(SUMPRODUCT(G6:G13;E6:E13) - SUMPRODUCT(G6:G13;D6:D13)) - 0.01*SUMPRODUCT(G6:G13;E6:E13) >= 10000

Don't forget to also bound all of your decision variables to be less than or equal to 150, and to be non-negative.

You have used 2 of 5 submissions

PROBLEM 2.2 - ANALYZING THE SOLUTION (2/2 points)

What is the objective value of the optimal solution (the estimated value of your portfolio of stocks next year)?

26771.88 ✔ **Answer:** 26773.6627

EXPLANATION

The objective value after solving the problem is $26773.66.

You have used 1 of 5 submissions

PROBLEM 2.3 - ANALYZING THE SOLUTION (2/2 points)

How many shares of stock in total should you sell to make sure you have enough cash, according to the optimal solution? (Assume that you can sell fractional shares.)

367.72 ✔ **Answer:** 367.723

EXPLANATION

According to the optimal solution, you should sell 150 shares of Microsoft, 150 shares of Intel, and 67.723 shares of Yahoo!. Since you can only sell whole shares, you should sell 68 shares of Yahoo!, for a total of 368 shares.

You have used 1 of 5 submissions

PROBLEM 3.1 - ADJUSTING THE FORMULATION (1/1 point)

As an invester, you like having a portfolio of eight different stocks because it diversifies your investment. If one or two stocks do poorly this year, you won't worry as much because you have many other stocks. In the optimal solution for this problem, you sold all of your shares of some stocks, but you would like to keep at least half of the shares of each of your stocks.

Adjust the formulation so that you sell no more than 75 shares of each stock, and solve it again.

In the optimal solution, you sell at least one share of which of your stocks? Select all that apply.

- [x] Yahoo! ✔
- [x] General Electric ✔
- [x] Microsoft ✔
- [ ] Bank of America
- [ ] JPMorgan Chase
- [x] Cisco Systems, Inc ✔
- [x] Intel ✔
- [x] Pfizer ✔

✔

EXPLANATION

The decision variables with non-zero values in the optimal solution are those corresponding to Yahoo!, General Electric, Microsoft, Cisco Systems, Inc, Intel, and Pfizer. To reach this solution, just change the upper bounds for the decision variables from 150 to 75 in the Solver.

You have used 1 of 3 submissions

PROBLEM 3.2 - ADJUSTING THE FORMULATION (1/1 point)

What is the objective value of the optimal solution now?

26468.54 ✔ **Answer:** 26468.54116

EXPLANATION

The objective value found in Solver is $26,468.54.

You have used 1 of 3 submissions

PROBLEM 3.3 - ADJUSTING THE FORMULATION (1/1 point)

Even though your investment is worth a bit less next year by diversifying, you expect this diversification to help you long term.

However, you notice that you expect the Yahoo! stock to decrease in value next year. So, while you would like to sell no more than 75 shares of your other stocks, you would like to sell exactly 100 shares of your Yahoo! stock. Adjust your formulation in LibreOffice again, and re-solve to get the new optimal solution.

You now sell at least one share of how many different stocks?

5 ✔ **Answer:** 5

EXPLANATION

You sell at least one share of Yahoo!, General Electric, Microsoft, Intel, and Pfizer, for a total of 5 different stocks.

You should remove the upper bound of 75 for Yahoo!, and add an equality constraint stating that you should sell exactly 100 shares of your Yahoo! stock.

You have used 1 of 3 submissions

PROBLEM 3.4 - ADJUSTING THE FORMULATION (1/1 point)

What is your estimated portfolio value next year?

26507.52 ✔ **Answer:** 26507.52535316

EXPLANATION

The new objective value, or estimated portfolio value next year, is $26,507.53.

You have used 1 of 3 submissions

This problem showed how we can easily use linear optimization to solve a simple portfolio optimization problem. However, there are many ways that we can extend this problem to make it more realistic. We could use regression to predict the future stock prices, and incorporate regression models into the optimization problem. You'll see how to do this next week.

Portfolio optimization is a very advanced and sophisticated field of optimization. In reality, it is often a **multi-stage nonlinear optimization problem**. For more information, see http://en.wikipedia.org/wiki/Portfolio_optimization.

Please remember not to ask for or post complete answers to homework questions in this discussion forum.