Chapter 6

A Model of Growth

Thinking with models, even simple models, can help us reach deeper understandings that enable us to better explain data, forecast the future with greater reliability, and to see the potential ramifications of decisions, whether those be individual choices or national policies. Our next models, some simples model of economic growth will provide all three of those benefits. The models will explain growth (at least partly) show why growth may differ across countries and change over time, and point towards particular policy actions.

The growth models that I present have three parts: labor, physical capital, and output. Labor combined with physical capital (machines) produce output. Exactly how much output gets produced depends on the current state of technology. In the model, output will be a single commodity that can either be consumed or invested. If invested, the output gets turned into more physical capital. Note the tradeoff implicit in this construction: the more output turned into physical capital, the more productive labor will be. That's great. However, any output that's invested cannot be consumed.

Clearly, countries that invest more should perform better in the long run than countries that invest little. In the short run though, more investment means less consumption. Some countries, and some individuals, choose to be like Aesop's grasshoppers and save little. Others countries are more like ants and re-invest substantial portions of their output. In the fable, the ants can only store food, so there's no investment, only inventory for the winter, but the essential insight's the same: those who save do better in the long run.

In the models, I assume a fixed investment or savings *rate*. In reality, the amount invested depends on the decisions of individuals, businesses, and governments. How much gets invested depends on several factors: the income level and its distribution, beliefs about future growth, and tax rates to name just three. More sophisticated growth models include such variables. In those models, individuals choose levels of investment using *model optimal rules.* Here, we're assuming a *fixed rule*. We do so for two reasons. First, as always, models with fixed rules are easier to analyze, and second, by varying the fixed rules we'll gain some insights into what a model optimal rule will look like.

I begin by creating a stark growth model that produces several insights. It shows how long term well being depends on saving. From the fable, we know that the ants did better than the grasshopper, so we know to save. But, we don't know how much to save. The model gives explicit relationships between savings rates and output. Interestingly, the model produces growth at a declining rate. This finding provides insight into why underdeveloped countries can grow fast early but then turn sluggish. Notice that the non linear trend also was true for the Markov Model. Linearity tends to become the exception not the rule as models have more moving parts.

Finally, given that it produces declining growth, the model implies the necessity of innovation. Absent innovation, growth stops (at least according to the model). Our first, most basic model has no role for innovation. It takes the level of technology to be fixed, that's a serious shortcoming given that without innovation growth stops. Therefore, once we have a solid understanding of the basic model, we'll add in innovation.

These models will be described in terms of an economy using terms like labor, physical capital, investment and depreciation. That said, like all of our models, the basic structure of this model can be applied more broadly. Economies are not the only things that grow. Businesses grow, and so do people. As we acquire more skills, we too grow in our intellectual capacity. We become better at carrying out tasks. We will attempt to apply our model of growing economies to people growing intellectually. We'll find that it doesn't quite fit. That's good. Because seeing how and why it doesn't fit will produce insights into differences between economies increasing in productive capacity and individuals gaining breadth and depth of expertise.

A Simple Growth Model

We start with a model of growth that's a simplification of a model formulated by Robert Solow. Imagine a society that subsists entirely on coconuts that grow high in trees. Suppose that in addition to providing flesh and milk, the coconuts can be made into machines that enable workers to harvest coconuts at a faster rate, in other words, the more coconuts that are turned into picking machines, the more coconuts that can be picked.

Now we have a crucial assumption: how many more? We could assume a linear increase, so that doubling the number of machines would double the rate. That's mathematically convenient but it's probably not very realistic. The first machine probably increases the rate of output more than the second, and the second probably increases the rate of output more than the third. Economists say that production exhibits *decreasing returns to scale*. Don't get too caught up in mastering the economic jargon. Focus on the fundamental insight: the machines increase productivity but the rate of increase in productivity decreases as more machines are added.

To capture this in a mathematical model, we need a function for output as a function of

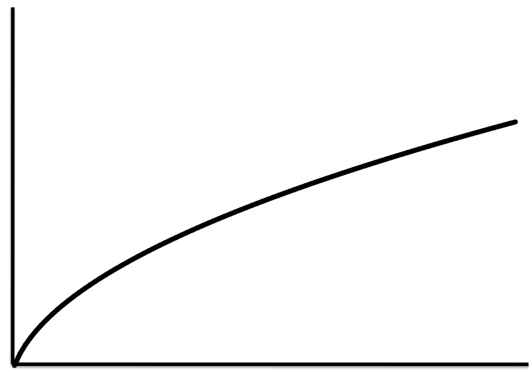

Figure 6.1: A Concave Function

the number of machines: *Output* $= F(M)$ where F increases in M, i.e. the more machines the more output, but whose rate of increase declines. Mathematicians refer to such functions as *concave.* Concave functions capture diminishing returns to scale.

We'll use the most common concave function: the square root function We will assume that the total output per worker equals the square root of the number of machines. If we assume that our imaginary economy has one hundred workers, then total output will equal one hundred times the square root of the number of machines. This will be our *output function.*

$$\textbf{Output} : F(M) \; = \; 100\sqrt{M}$$

Let's first see how this simple model can produce growth. Suppose that initially there exists just one machine. This means that total output equals 100. If the people in our community eat all one hundred coconuts, then the next day they will again be able to harvest one hundred coconuts. As a result, the economy will exhibit no growth. But suppose that they put one coconut aside each day, eating only ninety-nine. Suppose further that if they do this for a year, that the 365 coconuts can be used to build a second machine. This would mean that at the start of the next year, they would produce $100\sqrt{2} = 141$ coconuts. By sacrificing one coconut per day for a year, they can enjoy 41 more coconuts per day the next year. That's fantastic growth, a rate of forty-one percent.

Let's add some formal accounting. Let O equal our daily output of coconuts. Note that this equals $F(M)$. The coconuts can be eaten, E, or they can be invested to build machines, I. We can then write the following equation:

$$O = E + I$$

However, since this is a yearly decision, we've got to be a little more careful and note the year so that we can keep track of growth. If we denote years by t, we then have an equation for each year:

$$O_t = E_t + I_t$$

The coconuts invested, I_t become new machines the next year. If we let M_t be the machines that exist in year t, then the number that exist in year $t+1$ will be given by the following equation

$$M_{t+1} = M_t + I_t$$

Upon reflection, this equation feels too optimistic. Machines break down. They need repairs. The number of coconuts needed for repairs should be proportional to the number of machines. Therefore, we need to emend our equation for machines to take into account this wear and tear. We do so as follows. The parameter d characterizes the proportion of machines that are damaged during the year. Economists call this *depreciation.* The new number of machines at time $t + 1$ will equal the number at time t plus investment minus depreciation.

$$M_{t+1} = M_t + I_t - dM_t$$

We now have our full model. We will call this our **basic growth model**. It has a mere three parts.

Basic Growth Model

Output Function: *Output is increasing and concave in machines*

$$(1)\ O_t \ = \ 100\sqrt{M_t}$$

Consume or Invest Decision: *Output can be consumed or invested in future machines*

$$(2)\ O_t \ = \ E_t + I_t$$

Investment and Depreciation: *The number of machines increases through investment and decreases through depreciation*

$$(3)\ M_{t+1} \ = \ M_t + I_t - dM_t$$

These three equations describe a simple growth process. The first says that output increases in the number of machines. The second says that the outcput can either be consumed or used to make machines. And the third says that the new number of machines equals the old number, plus those built and minus those damaged.

An Example

Let's work through an example to see how this model produces growth. We'll assume that our society already has one hundred machines and that they allocate twenty percent of their total output of coconuts to investment. Let's also assume that ten percent of the machines get destroyed each year ($d = 0.1$). Using our formulae, we obtain the following:

Year 1: $M_1 = 100$, $O_1 = 100\sqrt{100} = 1000$. $I_1 = 200$, $E_1 = 800$.

We can use the third equation to solve for the number of machines in the next year: $M_2 = M_1 + I_1 - (0.1) \cdot M_1$. Plugging in the appropriate values, we get $M_2 = 100 + 200 - 10 = 290$. We can then compute the values for all of the variables for year two.

Year 2: $M_2 = 290$, $O_2 = 100\sqrt{290} = 1702$. $I_1 = 341$, $E_1 = 1362$.

We can do the same calculation for year three. First, we solve for the number of machines: $M_3 = M_2 + I_2 - (0.1) \cdot M_2$. Plugging in the values, we get $M_3 = 290 + 340 - 29 = 601$. We then obtain the following values for year three.

Year 3: $M_3 = 601$, $O_3 = 100\sqrt{601} = 2453$. $I_1 = 491$, $E_1 = 1962$.

If we look at the consumption path, we see that our society produced 1000 bananas the first year, 1702 the second year, and 2453, the third year. That's impressive growth.

The Irony of Growth

We might now ask whether this growth will continue forever. That's a great question. One reason that we have models is to work through logic. It turns out that it's not very hard to answer the question of whether growth continues forever. The answer is no. First, let's see the intuition. To continue to have growth, the society needs to build more and more machines. However, the added benefit of having more of machines decreases. Therefore, at some point, the society gets very little benefit from adding another machine. That would be okay if machines didn't break down, but, as the number of machines gets large, the cost of upkeep becomes prohibitive. We can see this graphically. The graph below shows output

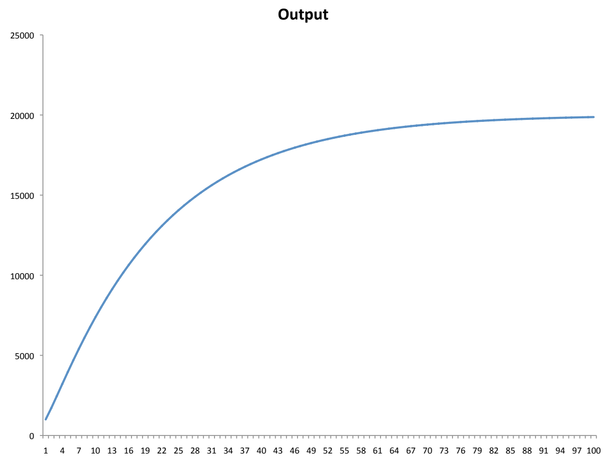

Figure 6.2: Growth From Our Model for 100 years

for the next hundred years using our equations and starting with one hundred machines.

Notice that output reaches a maximum level and that growth rates decline. The graph of output opens up two puzzles. First, why does growth stop? And second, where does it stop? To see why growth stops, notice that the investment in new machines is a constant times the output. We've been assuming that it's twenty percent. Economists call this the *savings rate*. Let's suppose that we fixed the savings rate. So long as the investment in new machines (the savings rate times output) exceeds the rate of machines lost to depreciation (the depreciation rate times the number of machines) then the number of machines will grow and so will output. Two observations explain why that can only happen for so long.

Observation 1 *The cost of depreciation increases* **linearly** *in the number of machines*

Observation 2 *Output increases at a declining rate in the number of machines.*

If the loss due to depreciation increases linearly and output increases at a declining rate, then at some point growth stops for an obvious reason: *the cost of replacing depreciated* *machines becomes prohibitive.* To see the logic consider an extreme case. Suppose that at some time T that our society has one million machines: $M_T = 1,000,000$. Using the third equation, we get that output will be 100,000 coconuts per day: $C_T = 100\sqrt{1,000,000} = 100,000$. However, guess how many of those coconuts must be used to repair the machines? Well, it's d times one million, or $100,000$. In other words, *all of the coconuts produced go to maintaining the machines.*

We now understand why growth stops, but we don't know where it stops. That's an amount that's easy to solve for as well. It's the number of machines that produces a level of output so that the the amount invested in new machines to equals the amount lost do to damage. Economists refer to this level of output as the *long run equilibrium.* It is the level of output at which growth stops.

Long Run Equilibrium: *Machines Lost to Depreciation = New Machines Produced*

The amount invested equals twenty percent of output and the amount lost to depreciation equals ten percent of the current machines. Let M^* denote the number of machines at which growth stops. Output equals $100\sqrt{M^*}$, so twenty percent of output equals $20\sqrt{M^*}$. Loss to damage equals $0.1M^*$. To solve for the long run equilibrium level of output, we first set these two quantities equal. This will give us the number of machines.

$$20\sqrt{M^*} = 0.1M^*$$

This equation can be rewritten as $\sqrt{M^*} = 200$. It has the solution, $M^* = 40,000$. At that number of machines, output equals $100 * \sqrt{40000} = 20,000$. Loss to depreciation equals 4000 machines, which also equals the number of new machines. Everything balances. The amount at which it balances depends on the rate of investment.

To see how equilibrium output depends on investment, let r denote the percentage rate of investment. In our example, $r = 0.2$. As above let M^* denote the number of machines at which growth stops. Output equals $100\sqrt{M^*}$, so the number of new machines equals $100r\sqrt{M^*}$. To simplify the calculations, let $R = 100r$ denote the percentage investment rate. We then have that the number of new machines equals $R\sqrt{M^*}$. Again as before loss to depreciation equals $0.1M^*$. We can solve for the long run equilibrium number of machines as follows:

$$R\sqrt{M^*} = 0.1M^*$$

Rearranging terms gives $M = 100R^2$. Here's something we might not have anticipated.[1] The equilibrium number of machines increases with the square of the savings rate. Long run output equals $100 * \sqrt{100R^2}$, which can be simplified to $1000R$. Thus, output increases *linearly* in the savings rate: a country that saves twice as much will have twice as much output. That's not something that we might have expected. It occurs because the number of machines in equilibrium increases with the square of the savings rate and output depends on the square root of the number of machines.

Our last result would seem to imply that countries should save as much as possible. That's not true, because the more a country invests, the less it consumes. Consumption equals non invested output. In our model, long run equilibrium consumption equals total output, $1000R$, times the percentage consume $(1 - R)$. That's equal to $100,000R - 1000R^2$. Therefore, consumption will be maximized at $R = 50$, a fifty percent savings rate.[2] Therefore, a country that wants to maximize output would choose to invest half of it's output.

Does this mean that investing half of output is the model optimal rule? Yes, if the objective is to maximize total output in the long run, but no if people care relatively more

[1]Notice that for $R = 20$, this equation gives that $M = 40,000$.

[2]We find $R = 50$ by taking the derivative of $100,000R - 1000R^2$ and setting it equal to zero. The derivative equals: $100,000 - 2000R$.

about the short term, which we often do. In that case, the optimal rule would call for less saving. Notice that in the long run equilibrium there's no growth. One reason why we construct models is to work through the logic of a situation. It's rather ironic that our first attempt to build a growth model failed so fundamentally.

The Irony Of The Growth Model *The basic growth model eventually results in a fixed level of output, i.e. no growth.*

One could see this as a weakness of the model. To the contrary, the growth model identifies one form of growth – namely growth that derives from leveraging a technology to it's limits. Implicit in this model was an assumption that we had a single technology for turning coconuts into machines. That's a pretty strong assumption. It leaves no room for innovation. With innovation, the number of coconuts needed to produce a machine could fall or the amount of output could increase. Let's assume the latter, that innovation increases the rate at which we can harvest coconuts. When we add that assumption, we obtain the Solow growth model, a model that can produce continual growth.

The Solow Growth Model

The Solow Growth model differs from our simple growth model in three ways. First, instead of assuming 100 workers, it applies to any number of workers, which is denoted by L. Second, it assumes that both labor and machines, which it calls capital and denotes by K, have diminishing returns to scale, and finally, it assumes that there is a technological sophistication parameter that changes over time $A(t)$. This represents changes in technology due to innovation. Here's the functional form.

$$\textbf{Output} : F(K, L, A(t)) \;=\; A(t)L^{\beta}K^{1-\beta} \quad \text{with } \beta \in (0, 1)$$

Though this looks a little complicated, it's actually intuitive. As technology advances, $A(t)$ grows larger, and output increase. The amount of labor, L, is raised to a power $\beta \in (0,1)$. This creates diminishing returns to labor. Note that when $\beta = \frac{1}{2}$, we just have the square root function. In our earlier, simpler model, we assumed a fixed amount of labor so we didn't have to concern ourselves with whether there existed diminishing returns. One last thing to notice is that the exponents for L and K sum to one. This is so that if both labor and capital are doubled, output will also double. Economists refer to this as *constant returns to scale*. At a crude level it makes sense, if we doubled the size of the economy without any changes in technology, then output should double.

In the Solow Growth Model, changes in $A(t)$ enable growth to continue. That insight may seem obvious. And, at one level it is. If technology makes labor and capital twice as productive, then output doubles. One might be tempted to ask whether we need a model for such a "deep" insight (one might even be shocked to learn that Solow won a Nobel Prize). But let's withhold judgement for a moment. The Solow model, by forcing us to think through all of the logic, reveals a second contribution to growth: *the improved technology creates the possibility of investing in more capital to produce even more growth.*. Thus, an output doubling change in technology results in more than a doubling of output!!!

Let's see how that works in our simpler model. Recall that with 100 workers, we had a long run equilibrium of $40,000$ machines and our economy produced $20,000$ coconuts. Let's suppose that someone has an idea an figures out a way to make the machines twice as productive. The new outcome function equals:

$$\textbf{Output} : F(M) \; = 2 \cdot \; 100\sqrt{M}$$

Let's solve for the new long run equilibrium. Remember all we have to do is set the loss due to depreciation equal to the number of new machines produced through investment.

Output equals $200\sqrt{M^*}$, so twenty percent of output equals $40\sqrt{M^*}$. Loss to depreciation equals $0.1M^*$. To solve for the final level of output, we first set these two quantities equal. This will give us the number of machines.

$$40\sqrt{M^*} = 0.1M^*$$

This equation can be rewritten as $\sqrt{M^*} = 400$. It has the solution, $M^* = 160,000$. At that number of machines, output equals $200 * \sqrt{160000} = 80,000$. Notice that output has not doubled. Output has quadrupled! The quadrupling occurs because both capital and labor are more productive and therefore it makes sense to have more machines. We will refer to this as the *innovation multiplier.*

The Innovation Multiplier *Innovation, changes in $A(t)$ increase output both by directly making labor and capital more productive and by creating incentives to invest in more capital.*

Note that this quadrupling doesn't occur immediately. There are two reasons that it takes time. The first is *replacement time.* The old coconut machines all must be replaced with new ones in order to benefit from the technology. The parameter $A(t)$ remains at its previous level for old machines. Think of it this way: your computer doesn't get faster, when a new faster computer chip gets developed. You only get a faster computer when you buy one! The second reason is that *capital accumulation takes time.* Increasing the number of coconut machines from $40,000$ to $160,000$ won't occur overnight. It'll take some time. And during that time, output will continually increase, i.e. the economy will grow. So, from the Solow Growth model we see the importance of innovation in producing sustained growth. This model provides a logical framework from which we can contemplate all sorts of interesting features of the world, including the potential for backward countries to catch

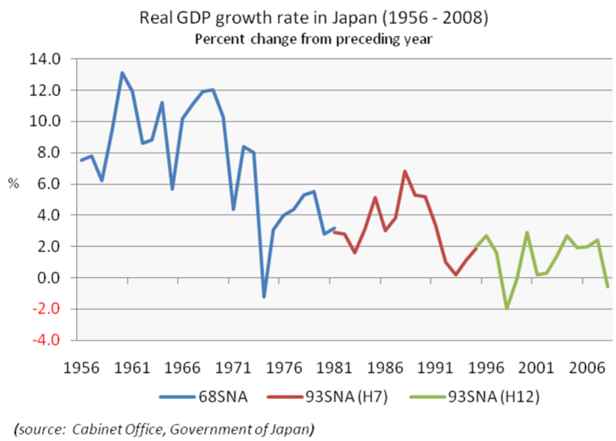

Figure 6.3: Growth in Real GDP in Post War Japan

up, the limits of extraction, the lags between technological innovation and growth, and the role of government in promoting growth,

The Catchup Model of Growth

Both our simple model and the Solow Growth model imply that an economy that has little capital (not enough coconut machines) can grow for a long time without any new technological innovations. An economy than keeps reinvesting its output into capital can grow and grow. However, the model also tell us that it will grow at a decreasing rate. Japan provides an example of such a country. Following world war II, Japan invested heavily in existing technologies. And, just as the model predicts, Japan produced incredible growth. This led some economic forecasters to predict that Japan's economy would overtake the United States in size. Those were *linear projections*.

Our growth models suggest that we should not expect linearity. We should expect declining growth, unless the economy also produces a series of innovation. The graph of real

GDP growth in Japan shows that growth did decrease, and Japan did not overtake the US as the world's largest economy despite the proclamations in numerous books and magazine articles that it would. Those articles were written by linear extrapolators who did not think with multiple models.

I don't pretend to know if the thinking that led Nikita Khrushchev at the height of the cold war to vow that "we will bury you" was linear or merely delusional. But if it was linear, it has reasonable support. From 1953-1960, the Soviet Union GDP grew at nearly 4% per capita, whereas the United States GDP grew at a mere 0.5% over that same period.[3] However, those trends did not continue. Part of the reason that those trends reversed stems from innovation. This model leaves out a lot of detail. And, we should be careful not to say that it "explains" the pattern of Japanese GDP growth or the fall of the Soviet Union. Instead, we should think of this model as providing a lens through which we can look at GDP growth across countries. To show that the model explains the Japanese miracle and subsequent demise, we'd need to show that levels of output, investment, and consumption fit the pattern the model implies. Nevertheless, we can infer from the Solow Growth model that when forecasting economic futures of countries, we need to look at levels of capital and rates of innovation.

One last point on growth by increased capitalization. One insight that follows directly from the model is that the less capital that a country has, the faster it can grow by investing in new technologies. Gerschenkron (1952) referred to this as the *advantage of backwardness*. The more backward a country is, the less capital it has invested in non state of the art technologies. This allows a country to immediately jump to the technological frontier. In the context of our model, they can have a larger $A(t)$.[4]

[3] Hanson, Phillip (2003) *The Rise and Fall of the Soviet Economy An Economic History of the USSR 1945-1991* Longman, New York.

[4] Gerschenkron, A. (1952). Economic backwardness in historical perspective. In *The Progress of Underdeveloped Areas*, ed. B.F. Hoselitz. Chicago: University of Chicago Press.

The Limits of Extraction

As a thought experiment, let's suppose that you become the dictator of a country with very little physical capital, e.g., no coconut machines. The Solow Growth Model shows how by importing existing technology, you could spur growth. As an oh by the way, the Soviet Union followed this strategy quite literally, dismantling, relocating, and rebuilding entire German factories. If as dictator, you cream a little off the top each year, then you'll reduce the growth rate because output that could have gone to capital lines your pocket. Thus dictators who extract will rule over countries that do not produce consistent growth.

If in addition, as dictator you decide to limit freedom and the potential for people to accrue wealth, then you may also destroy the roots of innovation. And, as we've seen from the model, continued growth requires innovation, unless of course, you keep copying the innovations of other countries. But, even that requires incentives to destroy existing technologies and replace them with new ones. This process, what the Schumpeter called *creative destruction*, where new technologies wipe out old ones, lies at the core of growth. The car replaced the horse, and that change destroyed hundred of supporting industries at the same time that it created hundreds more. Farriers disappeared. Gas stations rose up.

These two insights: that extraction will limit growth and that innovation will drive it, lie at the core of Acemoglu and Robinson's massive study of why nations prosper and fail. In looking at economic growth across countries for over two thousand years, they find that countries that fail extract resources and hinder the processes of creative destruction. Prosperous countries have strong central governments that promote pluralism, this has three effects: the strong center creates stability – an implicit assumption in these growth models: if you build a factory, you get to keep it!. Pluralism prevents capture by a power elite and limits extraction. And, pluralism plus stability promotes innovation and creative destruction[5]

[5]Acemoglu, D, and Robinson, J. (2012) *Why Nations Fail?* Harvard University Press

People know that if they have new ideas, that they'll be able to implement them even if this means destruction and a loss of jobs. Craiglist, a single idea implemented by a handful of people, contributed mightily to the demise of newspapers in the United States. In a less pluralistic society, the newspaper industry might have prevented Craigslist from posting classified ads in order to save jobs. They could not. Newspapers failed. The economy marches on.

Lags Between Innovation and Growth

If we look at growth in real GDP in a country like the United States, we see that it's anything but regular (see figure 6). Manifold causes produces changes in growth rates, including wars, changes in oil prices, political unrest, natural disasters, and government activities. Here, we want to just focus on what our model tell us about these flucuations. The Solow Growth Model implies that technological shocks have two effects on growth: the *direct effect*: the fact that the economy is now more productive, and the *innovation multiplier*: the fact that, the economy will start investing in more capital. The direct effect occurs immediately. The indirect effect though, takes some time. If we think of new innovations arising all the time, and they do. The United States Patent and Trademark Office approved around a quarter of a million patents in 2010, and to date has issued more than eight million patents. Not all of these improve productivity, but many do. These create a lot of direct and spillover effects on growth.

In our models, the spillover effects result in more accumulation of machines. But, in reality, the new machines will be different. When capitalists invest in trains they also wipe out a lot of jobs (creative destruction). Often the decreases in jobs in the short run, as existing industries disappear, can be large than the number of new jobs created. As a result, as W. Brian Arthur elaborates in his book *The Nature of Technology* and in other writings,

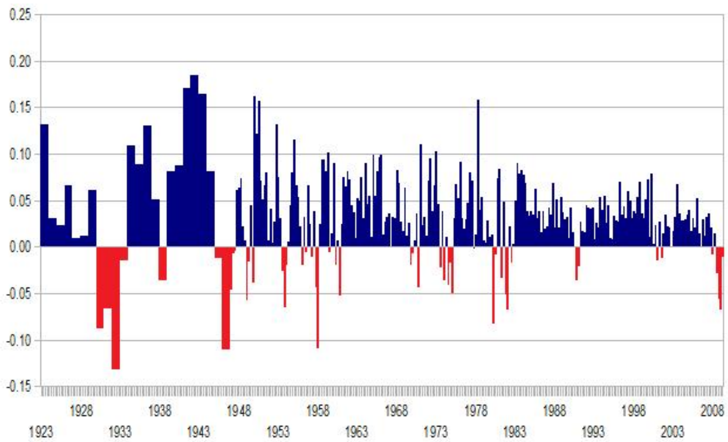

Figure 6.4: Historical Growth in Real GDP in the United States

the lags between technology and its effects on growth can be enormous. Trains were invented in the early 1800's but the gilded age until the latter part of that century, a gap of over fifty years. The Internet boom took place three decades after the Internet was up and running.[6]

Endogenous Growth

Though models of growth that we've learned provide a framework for thinking about growth and demonstrate the importance of innovation in creating sustained growth, they do *not* explain how innovation occurs. Our simple growth model resulted in no long run growth – hence, *The Irony of the Growth Model*. The Solow Growth model assumes that innovations just arise through the function $A(t)$. Ideally, a growth model would make the rate of technological change *endogenous*, by that I mean, produced from within the model.

In the late1980's economists began writing endogenous growth models. These models take several forms. An early model by Paul Romer assumes that growth arises from increased variety of goods. As the economy grows, so does variety. Variety in turn increases growth. Later models explicitly take into account investments in R&D. Even more elaborate models,

[6]In 1968, DARPA officially created the ARPANET, the boom occurred in the 1990's.

such as those proposed by Weitzman explicitly model the generation and recombination of ideas. These models can help us to understand how the nature of output can influence future growth rates. Though the models disagree in their particulars, all suggest that an economy that only produces coconut and coconut picking machines won't grow as fast as an economy that consists of multiple goods and multiple types of labor. Diversity in the product market and the labor market result in more ideas that can bump into one another producing ever more growth.

Summary

Possessing a variety of models enables you to make sense of the world. Some models, such as the growth models that we've covered here, were constructed for a particular purpose – to understand what creates economic growth. These models have highlighted two causes of growth. The first, we might call *growth from investment*. Even our simple growth model showed how by investing in machines, an economy can grow for a long time. Eventually though, growth ceases, and the economy attains a long run equilibrium Enter the second cause of growth: *growth from innovation*. Growth from innovation produces direct effects in increased productivity and a spillover effect in that it creates incentives for even more investment in physical capital.

With this simple model, we can get some understanding of why countries that grow fast often slow down (lack of innovation), why dictators who extract resources cause their countries to fail, why having low levels of physical capital (being backwards) can have its advantages, why growth lags innovation, and why not all output is equal. That's a lot of payoff from a rather simple framework. The growth model proved very fertile – producing unexpected and useful insights.

So far, those insights have entirely been in the context of the macro economy. Yet, this same growth model applies at the level of almost any organization. Suppose that you own a landscaping firm. As you produce more output and reap greater profits, and you can invest in more physical capital like trucks and tractors. You can also hire more employees, increasing the L in our formula for output. That formula, which is known as a Cobb-Douglass production function, can be used to determine optimal numbers of employees given levels of physical capital.

We can even use the model to model ourselves as thinkers. We can let M the number of machines (or K in the general model) denote the amount of material that we have learned. We can then let L denote the amount of effort we put in a project. In the simple model, we're, in effect, assuming that we always put in 100% effort (though pretty to think, probably not true!). Damage (or depreciation) corresponds to "use it or lose it." If you haven't used high school chemistry in four years, you're unlikely to be able to solve chemistry problems. Investment in new machines corresponds to learning new tools. So, suppose that you have a job where you add to M at a fast clip and only lose a little bit of M to depreciation. Your output will grow!

Now, we can start asking some more nuanced questions. Does this mean that each of us has a long run equilibrium level of intellectual output? Maybe. Would it really make sense to have the amount of material learned be a percentage of output? Perhaps not. Does it make sense to have output as a concave function of amount of material we know? Perhaps. I'll be frank, do I think our growth models are a great way to model intellectual growth? No. But I think there's a far more important question. Do I think that my thinking about intellectual growth by contrasting it with economic growth? Absolutely! What about the growth of a plant? Or the spread of an idea? I think the same holds. There are better models of each, but one route to developing those better models is by thinking with existing models and seeing where their assumptions fail.

Models have limits. We wouldn't want to use this growth model to explain how the tiny acorn turns into a mighty oak, or how fauna reach a fixed size. People, wolves, and even giant tortoises stop growing not because physical capital depreciates but because of genetic instructions. That said, their may well be some analog of physical capital for fauna – the size of our digestive systems. As we invest more in our digestive systems, we acquire the ability to gather more energy and to grow faster. So perhaps our growth model does provide some insights into biological growth.