Homework 5

Computation Theory (Term II/2020–21)

built on 2021/03/03 at 14:53:52 ***due:*** *fri mar 12 @ 11:59pm*

Ground Rules:

- This assignment contains only written problems. You should first attempt the problems by yourself and start working in groups after a few days of thinking.
- Typeset your solution (a good excuse to learn LaTeX) or write legibly. You're handing in your work electronically. We only accept PDF files. Name the file whatever you want as long as it's a single PDF.
- Exercise common sense when collaborating with others or looking things up online. Even if you work together on a problem, the writeup should be your own. This is the only way I know for you to master this kind of subject.

Task 1: Reject TM (2 points)

In this problem, let us define

$$\mathsf{REJECT}_{\mathsf{TM}} = \{\langle M, x\rangle \mid M \text{ is a TM that rejects input } x\}.$$

Show directly (i.e., without resorting to reduction) that $\mathsf{REJECT}_{\mathsf{TM}}$ is undecidable.

Task 2: Accept vs. Reject (2 points)

Recall $\mathsf{REJECT}_{\mathsf{TM}}$ from the previous problem and that

$$\mathsf{ACCEPT}_{\mathsf{TM}} = \{\langle M, x\rangle \mid M \text{ is a TM that accepts input } x\}.$$

Prove that (i) $\mathsf{ACCEPT}_{\mathsf{TM}} \le \mathsf{REJECT}_{\mathsf{TM}}$ and (ii) $\mathsf{REJECT}_{\mathsf{TM}} \le \mathsf{ACCEPT}_{\mathsf{TM}}$.

Task 3: Reverse on TM (2 points)

We will define

$$T = \left\{\langle M\rangle \mid M \text{ is a TM that accepts } \boldsymbol{rev}(w) \text{ whenever it accepts } w\right\},$$

where $\boldsymbol{rev}(w)$ is the reverse of the string w. Show that T is undecidable.

Task 4: Undecidability (6 points)

Consider the following languages, which contain Turing-machine descriptions:

$$\begin{aligned}
\mathsf{TOTAL} &= \{\langle M\rangle \mid M \text{ is a Turing machine that halts on every input}\}\\
\mathsf{FINITE} &= \{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) \text{ is a finite set}\}\\
\mathsf{REGULAR} &= \{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) \text{ is regular}\}
\end{aligned}$$

For each of these languages, show that it is undecidable. You may refer to other theorems/problems you have seen in class—but you cannot reduce among themselves.

Task 5: Total Is No Harder Than Finite (2 points)

Using the language definition above, prove that

$$\text{TOTAL} \leq_T \text{FINITE}.$$

Task 6: Finite Is No Harder Than Total (2 points)

Using the language definition above, prove that

$$\text{FINITE} \leq_T \text{TOTAL}.$$

Task 7: Extra: Undecidability of Nontrivial Properties (0 points)

Let P be a language consisting of Turing-machine descriptions such that

i. P is nontrivial—it contains some, but not all, TM descriptions.

ii. P represents a property of the TM language: whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P \iff \langle M_2 \rangle \in P$, where M_1 and M_2 are any TMs.

Prove that P is undecidable.