ICCS310: Assignment 3

Possawat Sanorkam
possawat2017@hotmail.com
February 23, 2021

1: NFA vs. DFA Expressiveness

---

(1) For every $k \geq 1$, there is an NFA with $k+1$ states that recognizes C_k.

Proof: We want to directly show that there is an NFA with $k+1$ states that recognizes C_k.

Suppose there is $G_k = (Q, \Sigma, \delta, q_0, F)$ and each G_k contains $Q = \{s_0, s_1, ..., s_k\}$ with each state showing how many of the last k bits that G_k has seen for every $k \geq 1$. Then, let $\delta(s_0, b) = s_0$, $\delta(s_0, a) = \{s_0, s_1\}$, $\delta(s_{i-1}, a) = s_i$ and $\delta(s_{i-1}, b) = s_i$ for $2 \leq i \leq k$. So, let $q_0 = s_0$ and $F = \{S_k\}$. G_k starts at s_0, and it may process any character until a is found. Once, a is found, fork the processes into two and we will get one process starts on s_0 and s_1 at the same time. Just keep changing state from s_1 to s_k on any character after a is found and G_k can accepts the string if and only if there are exactly $k-1$ characters following a. The process dies immediately when the number of string exceeds k after the a we found at j position where $0 \leq j \leq n$ where n is the length of string. Hence, G_k exists for all $k \geq 1$.

Therefore, for every $k \geq 1$, there is an NFA with $k+1$ states that recognizes C_k. $\square$

(2) If M is a DFA that correctly recognizes C_k, then M has at least 2^k states.

Proof: Consider $\Sigma = \{a, b\}$, we have that there are 2 possible characters, which is a or b. Then, let $x, y \in \Sigma^*$ where $|x| = |y| = k$ and $x_i \neq y_i$ for some $0 \leq i \leq k$. If $x_i = a$, then $y_i = b$, vice versa. So, we let $z = b^{k-1}$. Then, z distinguishes x and y as exactly one of xz and yz has the k^{th} character from the end as a.

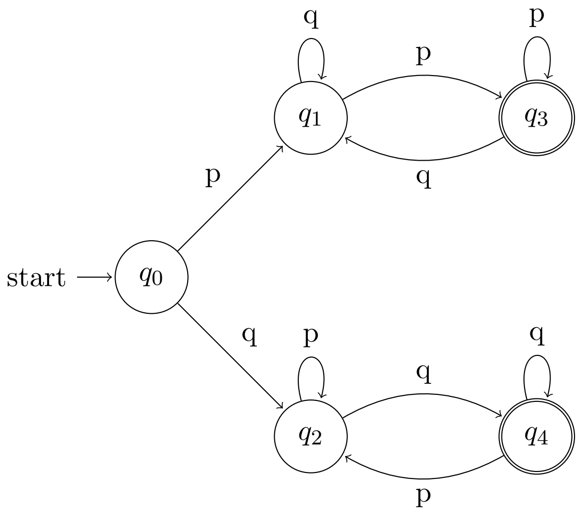
Since there are 2^k characters of length k, which are all mutually distinguishable by the above argument, any DFA for the language must have at least 2^k states. $\square$

Possawat Sanorkam
possawat2017@hotmail.com

2: Regular or Not

(1) $L_1 = \{xyx^R | x, y \in \Sigma^*, x \neq \varepsilon\}$ is regular

Proof:



S_0 represents the state where first character is not known.
S_1 represents the state where first character is p.
S_2 represents the state where first character is q.
S_3 represents the state where last character is p, accepted.
S_4 represents the state where last character is q, accepted.

The idea is that we do not care what is the given y, we only care what character starts first and that character must be the ending character since the reverse of px is xp and qx is xq where $x \in \Sigma^*$.

(2) $L_2 = \{xx^R | x \in \Sigma^*, x \neq \varepsilon\}$ is not regular

Proof: Assume for the sake of contradiction that L is regular. Then according to pumping lemma there exist an integer n such that for every string w where $|w| \geq n$, we can break w into three strings $w = xyz$ such that:

- (1) $xy^iz \in L_2$ for every $i \geq 0$;

- (2) $|y| > 0$; and

- (3) $|xy| \leq n$.

Consider $w = pqq^sp$. Let $|xy| \leq n$ and $|y| = i$. Then, $x = pq$, $y = q^i$, and $z = p$. This implies that $xyyz \notin L_2$. So, we found a contradiction to the lemma's assertion that $xyyz$ must be in L_2! Hence, L_2 is not regular. $\square$

3: Nonregular

(1) $L = \{10^{n^2} | n \geq 0\}$

Proof: Assume for the sake of contradiction that L is regular. Then according to pumping lemma there exist an integer n such that for every string w where $|w| \geq n$, we can break w into three strings $w = xyz$ such that:

- (1) $xy^iz \in L$ for every $i \geq 0$;

- (2) $|y| > 0$; and

- (3) $|xy| \leq n$.

Consider $w = 10^s$ where $s = n^2$. Let $|xy| \leq n$, $k > 0$ and $|y| = i$. Then, $x = 10000$, $y = 0^i$, and $z = 0^k$. So, y itself consists only of 0s. This means, $xyyz$ does not have the number of 0s equal to n^2.

$$|xyyz| = |xz| + 2|y| = (n^2 - i + 1) + 2i = n^2 + i$$

where $n^2 + i < n^2 + n < (n+1)^2$ and $n^2 + i > n^2$. Hence, $n^2 < n^2 + i < (n+1)^2$.

Then, $n^2 + k$ is not a perfect square. This implies that $xyyz \notin L$. So, we found a contradiction to the lemma's assertion that $xyyz$ must be in L! Hence, L is not regular. □

(2) $E = \{0^ix | i \geq 0, x \in \{0,1\}^*, \text{and } |x| \leq i\}$

Proof: Assume for the sake of contradiction that E is regular. Then according to pumping lemma there exist an integer n such that for every string w where $|w| \geq n$, we can break w into three strings $w = xyz$ such that:

- (1) $xy^iz \in E$ for every $i \geq 0$;

- (2) $|y| > 0$; and

- (3) $|xy| \leq n$.

Consider $w = 00^s1^s1$. Let $|xy| \leq n$ and $|y| = s$. Then, $x = 00^s$, $y = 1^s$, and $z = 1$. So, y itself consists only of 1s. This means $|yyz| > |x|$, but $w \in E$ when $|yyz| \leq |x|$. This implies that $xyyz \notin E$. So, we found a contradiction to the lemma's assertion that $xyyz$ must be in E! Hence, E is not regular. □

4: HackerRank Challenge

My username is Possawat2017. All problems solved.