ICCS310: Assignment 5

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1: Reject TM

$$\mathsf{REJECT_{TM}} = \{\langle M, x\rangle | \text{ M is a TM that rejects input}\}$$

Show directly (i.e., without resorting to reduction) that $\mathsf{REJECT_{TM}}$ is undecidable.

Proof:

2: Accept vs. Reject

$$\mathsf{ACCEPT_{TM}} = \{\langle M, x\rangle | \text{ M is a TM that accepts input x}\}$$

(i) Prove that $\mathsf{ACCEPT_{TM}} \leq \mathsf{REJECT_{TM}}$

Proof:

(ii)) Prove that $\mathsf{REJECT_{TM}} \leq \mathsf{ACCEPT_{TM}}$

Proof:

3: Reverse on TM

$$\mathsf{T} = \{\langle M\rangle | \text{ M is a TM that accepts } \mathbf{rev}(w) \text{ whenever it accepts}\}$$

where $\mathbf{rev}(w)$ is the reverse of the string w. Show that T is undecidable.

Proof:

4: Undecidability

(i) Show that

$$\mathsf{TOTAL} = \{\langle M\rangle | \text{ M is a Turing machine that halts on every input}\}$$

is undecidable

Proof:

(ii) Show that

$$\mathsf{FINITE} = \{\langle M\rangle | \text{ M is a Turing machine and } L(M) \text{ is a finite set}\}$$

is undecidable

Proof:

(iii) Show that

$$\mathsf{REGULAR} = \{\langle M\rangle | \text{ M is a Turing machine and } L(M) \text{ is regular}\}$$

where **rev**(w) is the reverse of the string w. Show that T is undecidable.

Proof:

5: Total Is No Harder Than Finite

Prove that

$$\mathsf{TOTAL} \leq_T \mathsf{FINITE}$$

is undecidable

Proof:

6: Finite Is No Harder Than Total

Prove that

$$\mathsf{FINITE} \leq_T \mathsf{TOTAL}$$

is undecidable

Proof:

7: Extra: Undecidability of Nontrivial Properties

Proof: It is non trivial. How to decide on it though?