ICCS310: Assignment 4

Possawat Sanorkam
possawat2017@hotmail.com
February 23, 2021

1: Eh? They Have The Same Cardinality?

Prove the following statements using rigorous mathematical reasoning:

(1) $|[0, \frac{1}{2})| = |[0, 1)|$

Proof:

(2) $|[0, 1)| = |(-1, 1)|$

Proof:

(3) $|[0, 1)| = |\mathbb{R}|$

Proof:

2: The Power Set of A

(1) Prove that $|2^A| = |\{0, 1\}^A|$.

Proof:

(2) Prove that $|A| < |\{0, 1\}^A|$ and conclude that $|A| < |2^A|$.

Proof:

3: Hamming Code

Consider applying the Hamming coding scheme to send 8 bits of data. This will require 4 parity bits, so an encoded code word in this scheme is 12 bits long.

(1) If the data bits are $d_1, d_2, d_3 ... d_8$, what is β_2 in terms of d_i's?

Answer:

(2) Encode the following 8-bit data: 01101010.

Answer:

(3) Assuming that at most a single single bit flip, decide the following codewords (indicate also whether there was any error):

Answer:

(i) 010011111000

(ii) 011101010010

Possawat Sanorkam
possawat2017@hotmail.com

4: Same Number of 0s and 1s

Consider the language $L = \{w \in \{0,1\}^* |$ w contains an equal number of 0s and 1s $\}$. Show that L is (Turing) decidable by providing a TM that decides it (a medium-level detail is preferred).

Proof:

5: Infinite DFA

Show that the following language is (Turing) decideable:

$$\text{IDFA} = \{\langle M\rangle |\ \text{M is a DFA and L(M) is an infinite language}\ \}.$$

Proof:

6: Lucky 9

(1) Let $L_1 \subseteq \Sigma^*$ be defined as

$$L_1 = \begin{cases} \varnothing & \text{if } 2^{74207281} - 1 \text{ is prime} \\ \{99\} & \text{if } 2^{74207281} - 1 \text{ is not prime} \end{cases}$$

Prove that L_1 is (Turing) decidable.

Proof:

(2) Let $L_2 \subseteq \Sigma^*$ be defined as

$w \in L_2 \iff w$ appears somewhere (not necessarily consecutively) in the decimal expansion of π

Prove that L_2 is (Turing) decidable.

Proof:

7: β-reduction

(1)

Solution:

(2)

Solution:

8: Fibonacci

Using the functions we have developed (e.g., pred, if_then_else, mult, add, etc.), write down an explicit λ-term fib such that $fib\bar{n} =_\beta f_(n)$. *Solution*:

Possawat Sanorkam
possawat2017@hotmail.com

9: Power Of 2

Implement a λ-term for the $pow(n) = 2^n$ *Solution*: