Homework 4

S520

Due at the beginning of class, Thursday 11th February

Trosset question numbers refer to the hardcover textbook. Show all working and give R code where appropriate.

1. Let X be a random variable with PDF

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the CDF of X, $F(y)$, for all y.

   (b) Find y such that $F(y) = 0.5$. Is this larger than, smaller than, or the same as EX?

2. (a) Suppose that buses go past my stop exactly 30 minutes apart. I arrive at the stop at a completely random time during the day. What is the expected length of time I will have to wait for a bus?

   (b) Suppose that buses go past my father's stop at exactly ten minutes past the hour and thirty minutes past the hour (e.g. 9:10, 9:30) every hour. My father arrives at his stop at a completely random time during the day. What is the expected length of time he will have to wait for a bus?

3. Trosset exercise 5.6.4. (Part (e) is worth one point of extra credit.)

4. Trosset exercise 5.6.7 (use R and give code)

5. Trosset exercise 5.6.8

6. Let X_1, X_2, X_3, and X_4 be independent standard normal random variables. Let $\bar{X}$ be the mean of X_1 to X_4:

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4}{4}.$$

   Note that because $X_1, \ldots, X_4$ are random, $\bar{X}$ is also a ~~standard~~ normal random variable.

   (a) Using R, find $P(X_1 > 1.96)$.

   (b) Using the binomial in conjunction with your answer in (a), find the probability that at least two of the random variables X_1, X_2, X_3, X_4 are greater than 1.96.

   (c) Find $P(\bar{X} > 1.96)$.