

# Homework 6

S520

**Upload your answers as a PDF or HTML file through the Assignments tab on Canvas by 4 pm, Thursday 3rd March.**

Trosset question numbers refer to the hardcover textbook. Draw all graphs in R and include all R code. You may work with others, but you must write up your homework independently — you should not have whole sentences in common with other students or other sources.

1. Trosset exercise 7.7.1 parts (f), (g), and (h).
2. Trosset exercise 7.7.4.
3. Trosset exercise 7.7.6.
4. Trosset exercise 8.4.4.
5. Let  $X$  be a discrete random variable with probability mass function

$$P(X = x) = \begin{cases} 0.3 & x = -2 \\ 0.6 & x = -1 \\ 0.1 & x = 12 \\ 0 & \text{otherwise.} \end{cases}$$

Let  $X_1, \dots, X_n$  be an iid sequence of random variables with the same distribution as  $X$ . Let  $\bar{X}$  be the sample mean (of  $X_1, \dots, X_n$ .)

- (a) Find  $EX$ .
- (b) Find  $\text{Var}(X)$ .
- (c) What is the expected value of  $\bar{X}$ ?
- (d) What is the variance of  $\bar{X}$ ? (Note: This will depend on  $n$ .)
- (e) Suppose  $n = 100$ . Use the R function `pnorm()` to find the approximate probability that  $\bar{X}$  is greater than 0.5.

(Question 6 is on the next page.)

6. I want to find out the average number of people per household in the U.S. I survey a simple random sample of U.S. households and obtain the results displayed in the following table.

Household size	Number of households
1	27
2	34
3	16
4	13
5	6
6	3
7	1

- (a) Lacking any other information, our best estimate for the population mean household size is the sample mean. What is the sample mean of our data?
- (b) What is our estimate for the standard deviation of household sizes?
- (c) What is the estimated standard error of the sample mean? (That is, based on our answer to (b), what is our estimate for the standard deviation of the distribution of the sample mean?)
- (d) Our error is the difference between the sample mean and the population mean. Using the normal distribution, find the approximate probability that the absolute value of the error in a survey of this form and size is less than 0.5.
- (e) Can we be reasonably sure that the average household size for all U.S. households is between 2 and 3?