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A model-based view of physics for computational activities in the introductory physics course

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A model-based view of physics provides a framework within which computational activities may be structured so as to present to students an authentic representation of physics as a discipline. The use of the framework in teaching computation at the introductory physics level is illustrated by a case study based on the simultaneous translation and rotation of a disk-shaped spaceship. Student responses to an interactive worksheet are used to support guidelines for the design of computational tasks to enhance the understanding of physical systems through numerical problem solving.

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I. INTRODUCTION

Numerical computations are finding their way into the curricula of more and more introductory physics courses.[1] Inexpensive computing and the advent of free, open-source languages have supported this growth, which has also been fueled by the increasing importance of computation in modern physics research. The possible pedagogical advantages of using computers in physics courses have long been recognized,[2] including the development of numerical problem solving skills[3] and the modeling[4] and visualization[5] of physical systems. It is increasingly recognized that the advantages of computational activities are not realized by simply giving computational problems, just as students do not necessarily learn physics effectively by solving traditional pencil and paper problems.

We believe that a suitable framework for structuring these tasks, and for computational physics education as a whole, is provided by the model-based view of physics and physics education. We present an overview of this view of physics as a teaching framework, and illustrate our teaching approach by a case study featuring a purposely complex computational task. Our aim is to illustrate how students working on a computational physics problem may be focused on not only achieving the correct numerical solution, but also on deepening their understanding of the physical system on which the problem is based.

II. A MODEL-BASED VIEW OF PHYSICS

Current physics education is beginning to draw significant insight from the model-based view of science[6] in which models and modeling take a central role in the formation and communication of new scientific knowledge. The epistemological and methodological questions related to models and modeling are nontrivial and directly touch on philosophical issues concerning the relation of theory to the world as experienced, or as accessed through experiments.[7] The model-based view of physics has traditionally found philosophical underpinnings from the semantic view of scientific theories,[8] which understands models as mathematical structures that form the constituent parts of theories. More recently it has been suggested[9] that this view is inadequate, and the role of models as conceptual instruments which mediate between theory and evidence[10] is finding wider acceptance.[11]

A model-based view of physics is depicted in Fig. 1, which illustrates the relation between the three relevant worlds in the framework: the world of physical theories, the world of models, and the world of real phenomena. Physics may be understood as a collection of physical theories which are the abstract formulations that have been produced and accepted by the community of physicists, according to the agreed rules of physics (such as the conservation laws) and mathematics. These theories can always be expressed in mathematical and linguistic terms. Because they are semantically blind and acontextual, hence lacking physical frames of reference, these theories are not direct descriptions of real world phenomena.[12]

Within this framework, the interpretation of physical theories occurs only through their application to particular natural phenomena. Because there is no direct correspondence between real world phenomena and theory, there needs to exist a methodology of match making between theory and experiment.[13] In order for this relation to be possible, both the theoretical prediction and the experimental result need to be structured in a mutually compatible form.[10] Starting with the theoretical superstructure, the principles and conditions relevant to the class of experiment under question are evoked, resulting in the production of a theoretical model, which may be understood as constituting statements of physical theory applied to an approximated and idealized[14] target physical system.

In contrast, data (numbers) from experimental observations in the natural world need to be transformed using an appropriate statistical framework to produce an experimental

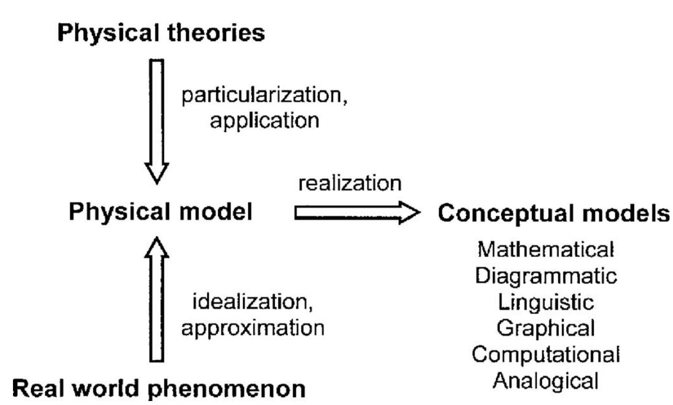


Fig. 1. A model-based view of physics.

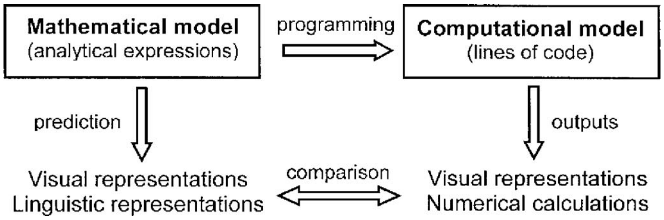


Fig. 2. Conceptual models relevant to computational tasks.

or empirical model (sometimes called an experimental law). The meeting of experiment with theory at the level of the physical model is, hence, a bidirectional process (Fig. 1) and, in practice, often involves the repeated adjustment of both the theoretical formulation and the conditions of the experiment in order to converge to better agreement.[9,10,15]

Physical models therefore develop the applicability of physical theories and determine the ways in which classes of physical phenomena, as observed through experimentation and linked to particular theories, should be understood.[16] A particular physical model may be realized in several conceptual models,[17] which may be formulated as linguistic and mathematical expressions, diagrams, or material artifacts,[18] and are used to communicate or explain aspects of the physical model under consideration. Although it is impossible to visualize physical theory in isolation from its application, physical models provide powerful heuristic pictures[12] of essential aspects of the theory through visual manifestations in different conceptual models.

Conceptual models have particular importance in physics education because they are representative[9] of the physical system, and the conversion between different conceptual models has been shown[19] to play an important role in the learning of physics. Although the structuring of physics education around the notions of modeling is not new,[20–22] there are particular pedagogical advantages in emphasizing the idea that the process of computational physics problem solving occurs at the level of the physical model (see Fig. 1), and that a computational model is one of many possible conceptual models of the physical system of interest. It can then be argued that the inclusion of computational tasks in the introductory physics curriculum is particularly useful if they facilitate conversions between a variety of conceptual models, thus contributing to the students' understanding of physical systems, as illustrated in Fig. 2.

A program intended to provide numerical information about a particular physical system will behave correctly only if the appropriate mathematical model has been used to describe the system. A mathematical model of the idealized system needs to be formulated before being converted into a program, creating a computational model. If a student has not formulated the correct mathematical model, then the conversion of this model into code might lead to a situation where an inappropriate understanding of the physical system is reinforced, especially if a consistently erroneous visualization is produced by the program.

It is also often the case that the behavior of a physical system is difficult for novice students to visualize internally.[23] One of the instructional purposes of the task might be for students to interact with the visual output of their programs in order to make sense of the solution. It is not necessarily the case that students will see past the technical attributes of the visualization (which is often subtly seductive) and consider the behavior of the system being displayed. The effectiveness of these learning activities relies on students thinking[24] about the output of their programs (as visual or numerical conceptual models) and interpreting them in terms of their predictions of the behavior of the system (as linguistic or diagrammatic conceptual models). This cognitive engagement by students is particularly important in cases where the behavior of the physical system is counter-intuitive or difficult to determine analytically. After their program is working, students should be asked to compare their predictions with the outputs of their program as indicated in Fig. 2, account for any differences, and formulate a revised conceptual explanation for the observed behavior of the system. Unless this loop is closed in the design of the task, it is likely that many students will regard the correct running of their program as the only educational goal of the task and not focus on the physical model on which the problem is based.

To illustrate the advantages of appealing to a model-based framework for computational physics instruction, we present a case study of a task that features the simultaneous translation and rotation of a disk-shaped spaceship.

III. CASE STUDY

A. Context

The task was completed by students in the introductory course for physics majors at the University of Cape Town, South Africa. First-year physics students take parallel courses in physics, mathematics, and applied mathematics, and typically a course in chemistry or computer science. Our introductory physics course is based on the Matter & Interactions texts,[25,26] which highlight the relations between microscopic and macroscopic phenomena, and emphasize the analysis of physical systems using a small number of fundamental principles. The modeling of physical systems is a central theme in the course,[26] and computation is introduced as a modeling tool. In our course, in which there are typically about 60 students, classes meet for 45-minute lectures five times a week for two semesters of 12 weeks each. In addition, 24 afternoon laboratory sessions (three hours each) are split between 12 practical activities, 6 theory tutorials, and 6 computation tutorials. A student's grade comprises examinations (50%), quizzes (20%), and laboratory and tutorial work (30%). For this paper we report on data collected in 2007 from 51 students. These students are typical first-language English speakers from good educational backgrounds which include access to both science laboratories and computers, similar to typical American physics first-year students.

B. VPython in the course

The programming language used in the course is VPython,[27] which is free, multi-platform, and open source, and based on the object-oriented Python language. VPython supports vector computations and automatically displays a three-dimensional visualization of defined objects without the need for complex graphics programming. A survey conducted in the first week of the academic year revealed a wide range of programming abilities among the sample cohort. Although all the students were familiar with computers and commercial software, 33 out of 51 claimed to have no programming experience and only 14 claimed some experience, mainly with Java, the programming language most commonly taught in secondary school. Four students believed that they had significant programming experience at school, with one student claiming proficiency in VPython.

Although VPython programs were used by lecturers in the course, the main computational activities formed part of our laboratory program. During these three-hour laboratory sessions students typically worked in pairs with a computer. All computation tutorials were in the form of interactive worksheets where the technical aspects of learning VPython were interspersed with opportunities for students to reflect on the process of coding by considering the mathematical models they were using, making predictions about the behavior of the system, and identifying and interpreting the visual elements in the output. The first few VPython sessions mainly introduced students to the structure and syntax of the language, with exercises focused on modeling the motion of a ball confined in a box[28] and the voyage of a spaceship to the Moon.[28] In a later tutorial students simulated a vertical spring-mass oscillating system and were able to make measurements using real laboratory apparatus and compare their results with their simulations. The content and level of difficulty of the tutorials were designed to meet students' increasing knowledge of both physics concepts and VPython syntax. During the tutorials, students were encouraged to talk with each other and roving tutors facilitated these exchanges and answered questions. The tutors are physics post-graduate students who are proficient in VPython and have completed courses in computational physics. The tutors had prior access to the worksheets and met with the instructor before each tutorial to agree on a common tutoring strategy. Completed worksheets were handed in at the end of each session with printouts of codes. The final tutorial of the first semester, which we consider in detail here, involved the motion of a disk-shaped spaceship.

C. The task

The task was based on the interesting problem explored in detail by Dudley and Serna.[29] The pedagogic advantages of this problem as a computational task are that the system is simple and the mathematics describing it is straightforward, but the solution is intuitively difficult to visualize.[29] A disk-shaped spaceship of radius R and mass M located in deep space has a single thruster positioned at its circumference which supplies a constant tangential force of magnitude F without changing the mass of the spaceship. The analytical solution for the velocity components involves Fresnel integrals, which introduces a mathematical complexity beyond that of a typical first year student.

The task was designed to facilitate opportunities for students to explore the behavior of the system in a variety of ways and to challenge their intuition and understanding of the relevant physics. The worksheet first asks students to consider a simplified version of the problem, where the thruster is oriented at the circumference so as to produce a constant force through the center of mass of the spaceship. The students are asked to sketch and describe the path of the center of mass of the spaceship and identify the physics concepts governing the motion. The more complicated situation is then presented to students in which the thruster is still positioned at the circumference but rotated by 90° so as to provide a constant force at a tangent to the edge of the disk-shaped spaceship. The initial condition is shown in Fig. 3(a); at $t>0$ [Fig. 3(b)] the spaceship is rotated and translated.

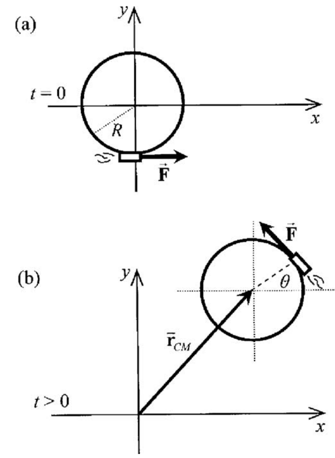


Fig. 3. Position of the spaceship at (a) $t=0$ and (b) $t>0$.

The second part of the worksheet focuses on the mathematical model used to describe the motion of the spaceship. By starting with the momentum principle for translational motion,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \tag{1}$$

which is a strong unifying theme in the course, students are guided to produce the rotational analogue of the momentum principle

$$\tau = FR = I\frac{d\omega}{dt}, \tag{2}$$

which, with the expressions for the momentum

$$\mathbf{p} = M\frac{d\mathbf{r}_{\mathrm{CM}}}{dt}, \tag{3}$$

and angular speed,

$$\omega = \frac{d\theta}{dt}, \tag{4}$$

of the spaceship, forms the core of the mathematical description of the system required in the code.

The next step is to convert the mathematical model to the appropriate computational model. Using the update algorithm[30] for translational motion learned in previous sessions as a starting point, the students are guided to write the lines of code necessary to update the applied force, the linear momentum of the center of mass, position of the center of mass, angular speed, and angular position and to plot the position of the center of mass as a function of time. Students are then asked to sketch, describe, and explain the visual output of their programs and to compare the output with their predictions, accounting for similarities and differences.

Figure 4 shows the computed path (black line) of the position of the center of mass of the spaceship. The path approaches a straight line irrespective of the values of R, M, and F. As the rotation of the spaceship increases, the impulse due to F on the center of mass over each revolution approaches zero, and the spaceship travels at a terminal velocity. There are many other interesting aspects of this problem[29] which were not explored in this exercise, such as the path traced out by the thruster, shown in grey in Fig. 4.

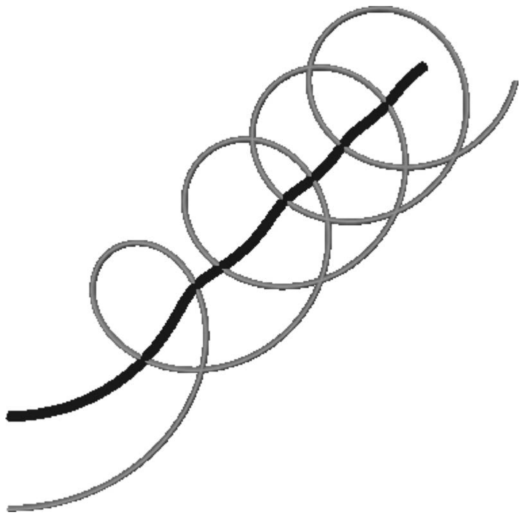


Fig. 4. The path of the position of center of mass of the spaceship (black) and the position of the thruster (grey).

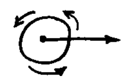
An interpretive analysis[31] was used to sort aspects of the completed worksheets. Student responses to particular questions were clustered according to common features, especially the characteristics of the diagrams, mathematical expressions, and written descriptions of physics concepts. Coding schemes were developed to classify the links, either explicit or implicit, between the students' conceptual models at various stages of the task in particular sketches, descriptions and explanations, and mathematical and computational models (see Fig. 2). Three issues are reported here: the predictions made of the motion of the spaceship, the relation between the students' mathematical models and related computational models, and the nature of their reflection on their computational solution relative to their predictions.

IV. ANALYSIS OF STUDENT RESPONSES

A. Prediction about the behavior of the spaceship

All 51 students were able to express clearly that when the thruster was positioned such that F was directed through the center of mass, the spaceship would accelerate in a straight line without rotation. None of the students were able to predict the correct path of the spaceship when the thruster was rotated by 90° (Fig. 3). Table I summarizes the diagrammatic models provided by the students. Most students $(16+11+6+5=38)$ proposed that the spaceship would both translate and rotate, with the majority of students sketching either looped (16) or spiraled (11) paths. The closed circular path (6) may be viewed as a special case of either one of these categories. The remaining students predicted that the spaceship would either translate with rotation (6), rotate about its center of mass without translating (3), or a combination of the two (5). The predictions of four students could not be classified. The two main categories (looping or spiraling and linear motion) are likely to be the two main ontological classes underpinning the reasoning used by the students.[32] It is also interesting that all the paths predicted by the students were constant (either linear or periodic), except perhaps for the spiral (although the radius is increasing at a constant rate). None of the students considered that an increasing rate of rotation of the disk will result in a path which evolves in shape over time. Although most students (42) were able to provide linguistic conceptual models consistent with their diagrammatic conceptual models, nearly all the students only listed physics concepts such as torque and force without attempting to provide a coherent physical explanation to support their prediction.

Table I. Variation in the students' diagrammatic predictions of the path of the center of mass of the spaceship ($N=51$).

Typical student diagram	Description	Number
	Translates while rotating in repeating loops	16
	Spirals outward (translation and rotation)	11
	Translates in a straight line with no rotation	6
	Translates and rotates in repeating circles	6
	Translates in a straight line while rotating about center of mass	5
	Rotates with no translation	3
Other	Other	4

B. Translation of mathematical model to computational model

The worksheet asked the students to provide the analytical mathematical expressions governing the translation and rotation of the spaceship, and then design and write an initial version of their VPython code before they started to work at a computer. We found that 34 students were able to write a mathematical model similar to the expressions in Eqs. (1)–(4) and, of these, 20 students were able to convert their models into appropriate VPython code. Of the remaining 17 students, 8 wrote the correct lines of code while having incorrect mathematical models, and 9 had both incorrect. By the end of the afternoon, the majority of students (45) had correctly working codes, producing the appropriate output. Students would have received assistance from fellow students and tutors, and the focus of this assistance would have been on their program, and not necessarily on the underlying mathematical model. Therefore, a correctly working code does not imply that a student has understood the underlying physics governing the problem. This understanding may only be inferred from scrutiny of other responses requiring reflection and explanation in written and diagrammatic form.

C. Reflection on numerical solution

The final part of the worksheet asked students to consider the diagrammatic conceptual models of both their prediction and program output, to account for the differences and similarities between them, and to explain what happens in terms of the relevant physical concepts. Half of the respondents either failed to compare their predictions and outputs at all, or offered unsupported statements of agreement or no agreement. A further 14 students offered explanations which made surface-level comparisons of the shapes of their predicted paths and the output of their programs, with no recourse to physical concepts; for example:

> "No, I predicted it would move in a circle, but the actual result is a wavey curve which eventually becomes straight."

> "It does move a bit faster than I predicted. [The spaceship] oscillates going up and down in a squiggly motion."

Although only 11 students attempted to explain what they observed in terms of some physical model, most of these explanations were reasonable; for example:

> "No, it does not agree with our prediction of the spiral shaped path as predicted because we didn't take into account how the constantly increasing momentum would decrease the effect of the force when pointed in the opposite direction."

> "No, this does not agree with our prediction. Our prediction of the angular speed was far too low. We thought that the slow rotation would move the force around the disc a lot slower, thus causing a more rounded trajectory. Because the ω is a lot higher, it means the thruster whips around a lot more often, causing the force to be applied in the same direction."

These students had developed their understanding of the physical model through the explicit comparison of various conceptual models, including the computational model.

Several factors might be responsible for more students not offering reflections on their visual outputs based on a physical model. It is likely that many students were not able to make sense of the new information produced by their program in a way that allowed them to explain the behavior of the system. However, many of these students were able to derive the appropriate mathematical model, thereby suggesting an inability to relate image-based information (the observed path) to the mathematical expressions which describe the motion of the spaceship. In these cases it is unlikely that the students' understanding of the physical behavior of the spaceship were altered by the task. In contrast, it is likely that the students who were able to express plausible physical models to explain the behavior of the spaceship were able to modify their pre-task understanding. Another factor is that our phrasing of the final question combined asking students to compare their predictions with their outputs and explain what is being displayed by the output of their program. This wording resulted in many students comparing only their predictions with the outputs produced by their programs and not offering an explanation to support their comparisons. Fatigue and time pressure might also have played a role. In later tasks, presently under evaluation, we have carefully separated questions asking for a sketch, a description, and an explanation in terms of a physical model. For example, for the shapeship task described here, we may ask, "Draw a careful sketch of the path of the center of mass of the spaceship as provided in your output window. Describe the shape of this path carefully. How does the observed path differ from your prediction?" This question could be followed by, for example, "Refer back to your mathematical model of the shapeship and provide a coherent physical model, in words, of the observed path of the center of mass of the spaceship (as you described earlier). In other words, explain what you observe in terms of the appropriate physics." Preliminary data suggest that such wording results in a significant improvement in the number of students who can reason appropriately about the system in terms of both image-related and mathematical models.

V. DISCUSSION AND CONCLUSION

The issue of central importance when introducing computational problem solving to novice physics students is how the tasks should be structured so as to facilitate meaningful learning of physics. We have suggested that a model-based framework for the design of computational physics tasks provides, at different stages of the task, explicit opportunities to link an authentic view of the nature of physics (physical theories, physical models, and real world phenomena) with a variety of conceptual models including the computational model. We argue further that within the context of computational teaching activities which increasingly make use of visual-heavy output, the highlighting of the role of different types of models at different stages of the task allows students to meaningfully interpret the output of their programs. Our model-based analysis of student responses to the structured worksheet provided some insights as to where the main focus of the students was at various phases of the task discussed in Sec. III, and how the design of the task impacted their understanding of the behavior of the system. The findings suggest further opportunities for learning by explicitly linking different models.

Our task required students to consider the system under study, make predictions about its behavior, and give these predictions in written and diagrammatic form. The pedagogical advantages of this phase are that students first focus on their initial understanding of the behavior of the physical system and formulate their own conceptual models.

As expected, none of the students could predict the path of the center of mass of the spaceship. The next step required the students to develop their mathematical model of the system and write a first version of their program before working at a computer. This phase focused the students on the relation between their mathematical and computational models. Although this phase was useful in highlighting the physics nature of the task, the physical theories underpinning the mathematical and computational models should also be considered, to be returned to later when comparing their predicted diagram and the visual output.

During the subsequent programming phase of the task, students would have edited their code, sometimes with help from peers and tutors, until they were able to produce the correct behavior of the spaceship. As the afternoon progressed, knowledge of the correct output would have permeated through the class, and the focus of many students would have been on getting their own codes to produce the known results. A sense of completion on the part of these students would have naturally followed. In order for there to have been meaningful learning, the observed motion of the center of mass would need to have been considered and explained in terms of a physical model. We attempted to have the students reflect on the process, interpret their outputs in relation to their predictions, and offer an explanation for the observed behavior of the system. Although many students did provide an explanation, some students were not able to progress beyond merely describing what was displayed in the visual output of their programs. The explicit requirement to provide explanations for the differences in the predicted and observed visual models in terms of physics concepts used earlier on the mathematical model may help students to frame their understanding of physical theories.

If one of the goals of including computational activities in the introductory physics course is to enhance opportunities for the development of appropriate understanding of physics concepts, then a model-based approach appears to offer value for the design of computational tasks because it promotes opportunities for students to develop their understanding through working with a variety of conceptual models. It is useful in this regard for students to declare their predictions of the output of their programs in written and diagrammatic form. This requirement provides insight to themselves and others regarding their initial understanding of the system and opportunity for later comparison and reflection. Second, the correct mathematical model of the problem needs to be converted into the language of the computational model. An incorrect mathematical model is likely to lead to an incorrect simulation of the system and the reinforcement of an inappropriate mental model. Therefore, there is pedagogical value in students developing their mathematical models and having them reviewed before programming commences, especially in contexts where novice students might focus solely on getting their programs to work and not prioritize the physics purpose of the task. When asked by students to assist with their computational models, tutors should first focus on the underlying mathematical model and not only provide answers in the syntax of the programming language. Our tutors needed constant reminding in this regard, and we plan to precede future courses with an awareness session with our tutors during which we will discuss our educational goals for the computational activities in terms of the modeling framework discussed here.

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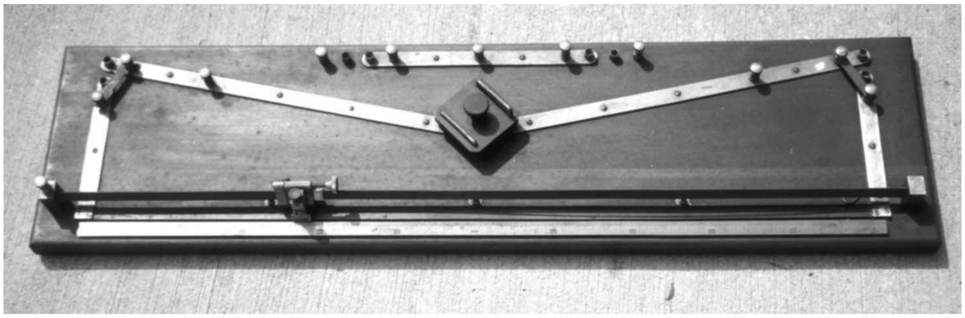

Slide Wire Wheatstone Bridge. The slide wire form of Wheatstone's bridge was introduced by Gustav Kirchhoff (1824–1887). This Leeds and Northrup Reversible Meter Bridge, in the Greenslade collection, is the Rolls Royce of slide wire bridges, and is listed at $115.00 in the 1907 catalogue. The catalogue description says, in part: "Base is of mahogany, strap connections of heavy copper, all joints soldered and scale is engine divided on metal and plated. ... The slider has both coarse and fine adjustments and is designed so that it is impossible to cut or jam the wire. Vernier enables readings to be made to 1/10 mm." The reversing switch in the middle has mercury contacts. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)