Simulation-based Inclusion Checking Algorithms for ω-Languages

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Presentation

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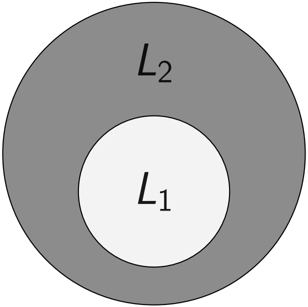
The Language Inclusion Problem

Definition (Language Inclusion Problem)

Let L_1 and L_2 be two languages. The **language inclusion problem** consists in deciding whether $L_1 \subseteq L_2$ holds or not.



Characteristics

- Whether the problem is computable or not depends on the class of the languages
- Also if it turns out to be computable, it is usually an hard problem

Applications

- Model checking
- Compilers construction
- Automata-based Verification

ω-languages

Definition (ω-language)

An ω-**language** L is a set of strings of *infinite length* over some alphabet Σ.

Examples of words of **infinite** length:

$$abbb\cdots = ab^\omega$$

$$babbaababab\cdots = babba(ba)^\omega$$

Büchi automata

A **Büchi automaton** is a tuple $\mathcal{B} = \langle Q, \delta, \{i\}, F\rangle$

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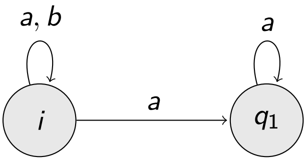

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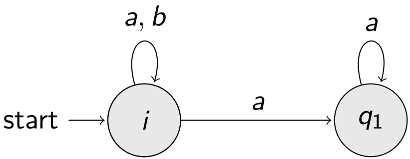

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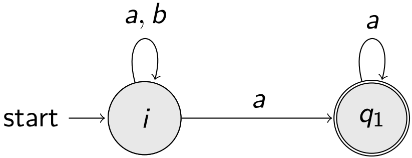

Traces

A **trace** over the word $a_1 a_2 a_3 \ldots$:

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots$$

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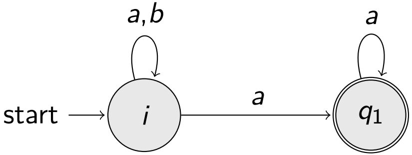

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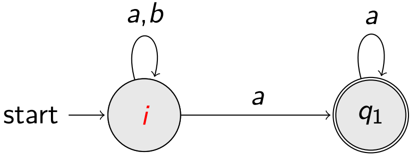

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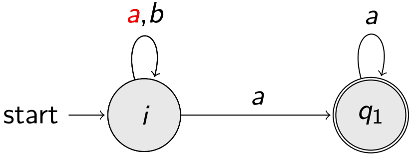

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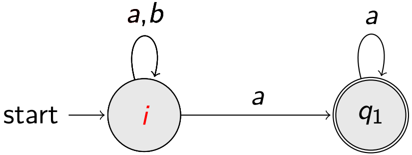

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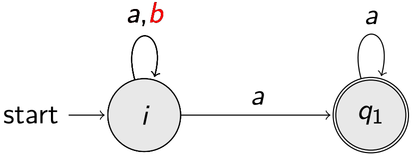

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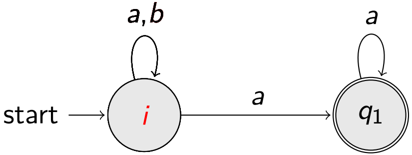

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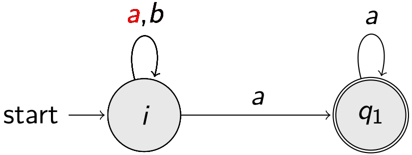

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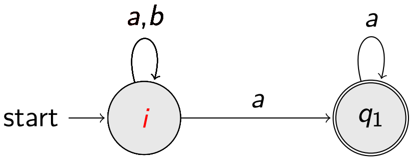

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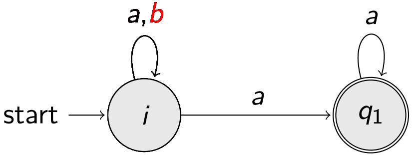

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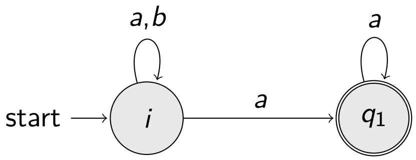

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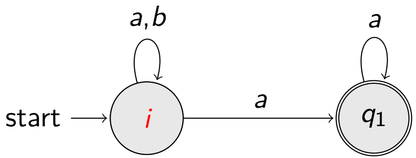

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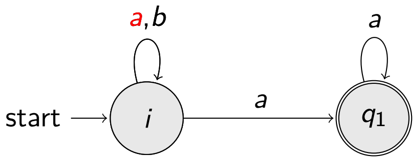

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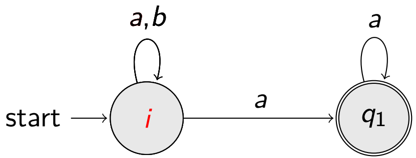

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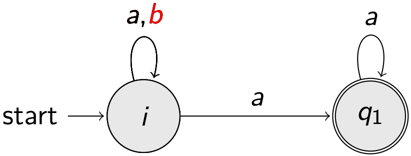

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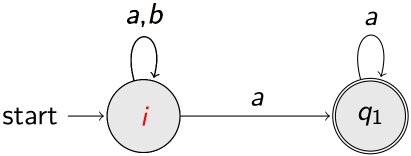

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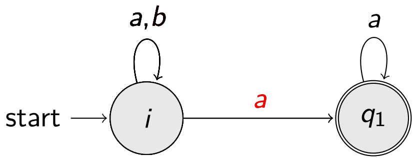

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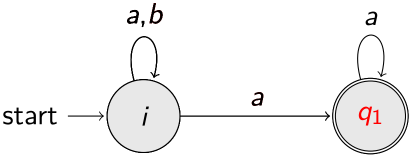

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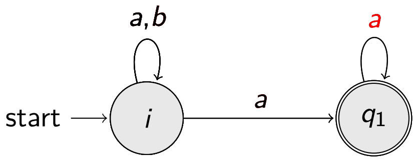

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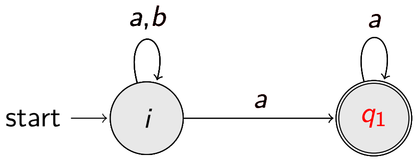

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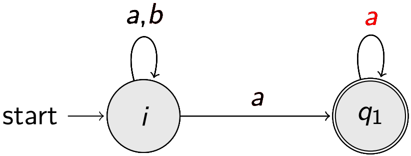

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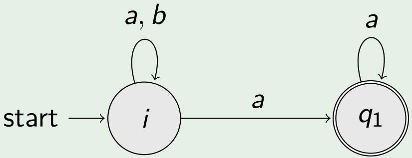
The language of a Büchi automaton

The *language recognized by a Büchi automaton* $\mathcal{B}$ is:

$$\mathcal{L}(\mathcal{B}) = \{w \mid \text{there is an initial and fair trace over } w\}$$

Example



$$\mathcal{L}(\mathcal{B}) = \{a^\omega, ba^\omega, aba^\omega, bba^\omega, \ldots\} = (a+b)^* a^\omega$$

$\omega-$regular languages

Definition (ω-regular language)

The class of languages recognized by Büchi automata is called ω-**regular languages**.

Applications

- Model checking
- Type systems

Deciding the Language Inclusion

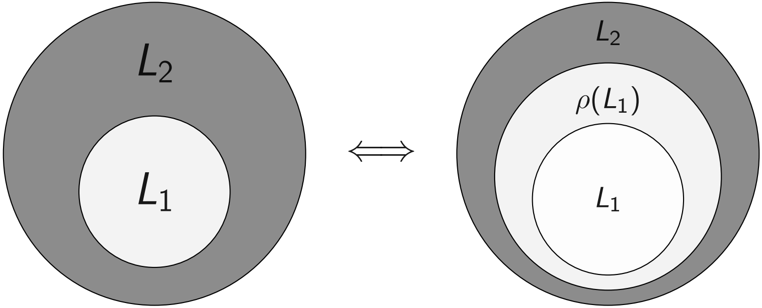
- Languages are **not finite**, we can't just compare them

Deciding the Language Inclusion

- Languages are **not finite**, we can't just compare them
- **Abstract Interpretation:**
  - Static program analysis
  - Giving up precision for computability

Deciding the Language Inclusion

We started from the "Doveri-Ganty" framework for checking the language inclusion, which relies on *Abstract Interpretation* techniques.



Details

A **ultimately periodic** word:

$$abc(de)^\omega$$

We define:

$$I_L \triangleq \{(u,v) \mid uv^\omega \in L\}$$

Then, one key observation is:

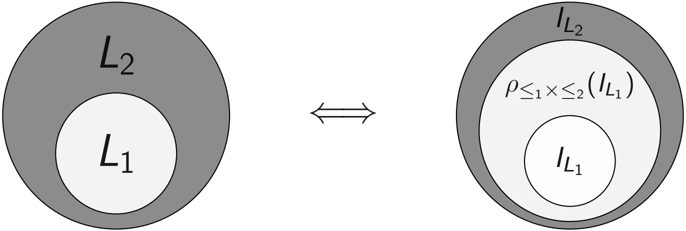
$$L_1 \subseteq L_2 \Longleftrightarrow I_{L_1} \subseteq I_{L_2}$$

Let $\leq_1, \leq_2$ be two **preorders** on words.

$$\rho_{\leq_1 \times \leq_2}(I_L) \triangleq \{(s,t) \mid \exists (u,v) \in I_L, u \leq_1 s \ \wedge \ v \leq_2 t\}$$

Let $\leq_1, \leq_2$ be two preorders on words that meet a list of requirements related to **computability** and **completeness**.

$$L_1 \subseteq L_2 \iff \rho_{\leq_1 \times \leq_2}(I_{L_1}) \subseteq I_{L_2}$$

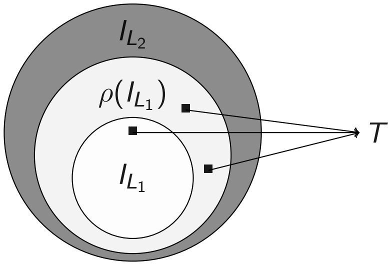


Observation: usually when abstracting one object we gain decidability, but here the abstraction goes from one infinite set (I_{L_1}) to another infinite set... Why?

Gaining decidability

We can extract from the abstraction $\rho_{\leq_1 \times \leq_2}(I_{L_1})$ a **finite** set, say T, such that:

$$L_1 \subseteq L_2 \iff \forall (u, v) \in T, uv^\omega \in L_2$$



Algorithm to solve $L_1 \subseteq L_2$

- They give `BAInc`, algorithm to solve $L_1 \subseteq L_2$
  1. Computes T
  2. Checks if $\forall (u, v) \in T, uv^\omega \in L_2$
- `BAInc` is parametrized by $\leq_1, \leq_2$

$$\begin{tikzcd}
\leq_{\mathcal{B}}^{2} \arrow[r] \arrow[d] & \leq_{\mathcal{B}}^{1} \arrow[r] \arrow[d] & \leq_{\mathcal{B}}^{r} \arrow[d] \\
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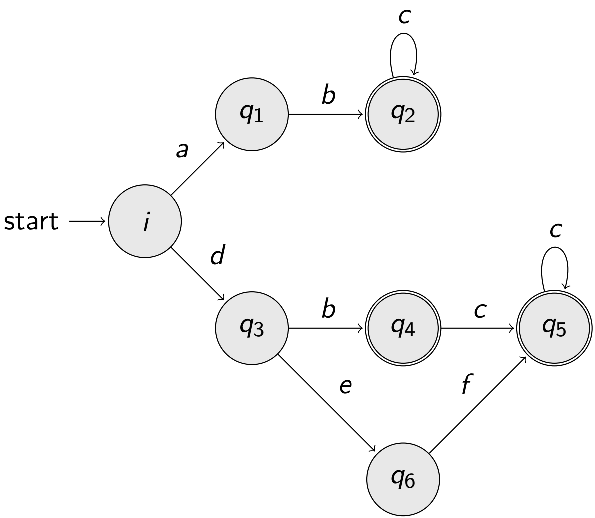
\end{tikzcd}
$$

My task: to define new preorders $\leq_1, \leq_2$

Simulations

- Behavioural relations
- Intuitively, one state is simulated by another if the second can match all the moves of the first
- Fundamental in Process Calculi
- There are many known algorithms to compute simulations

The Game of Simulation



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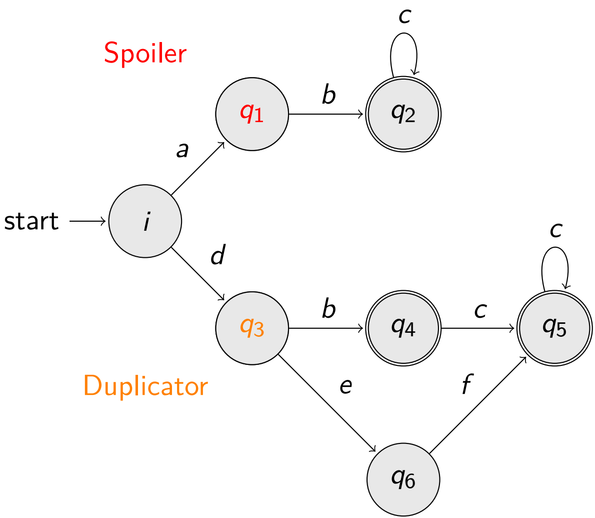

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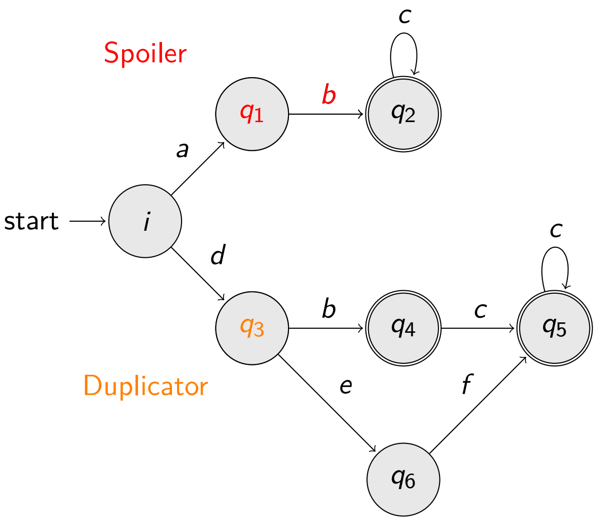

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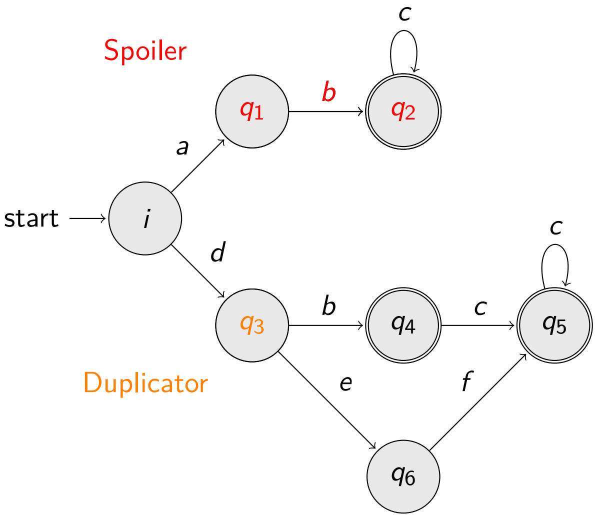

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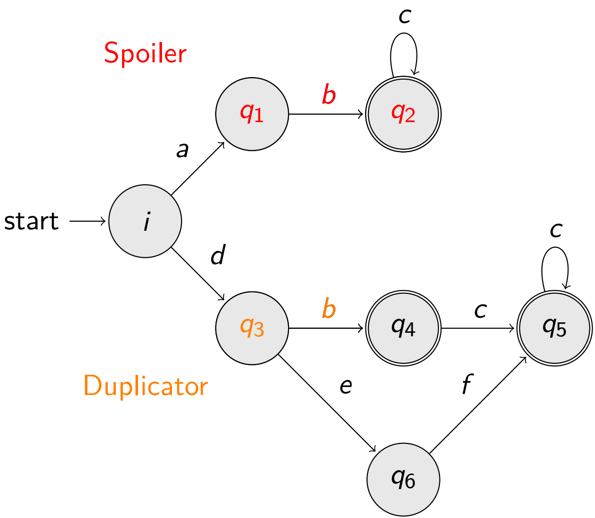

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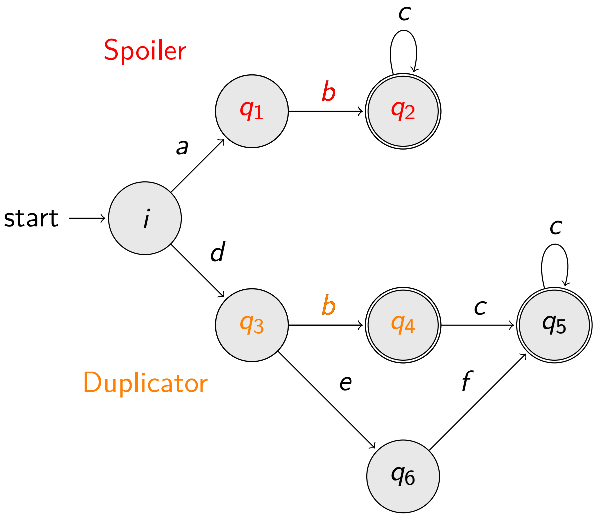

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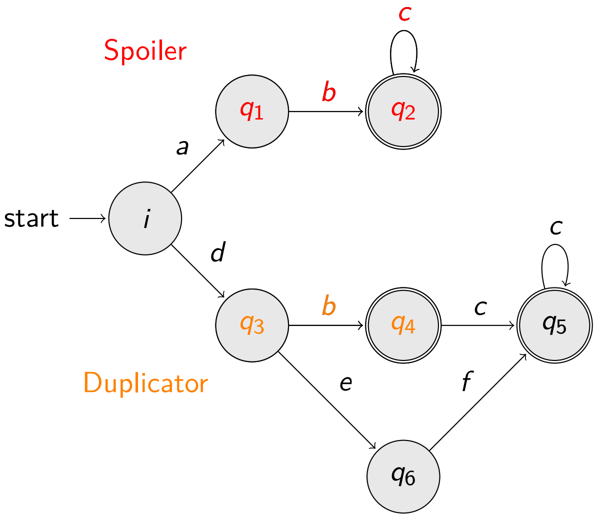
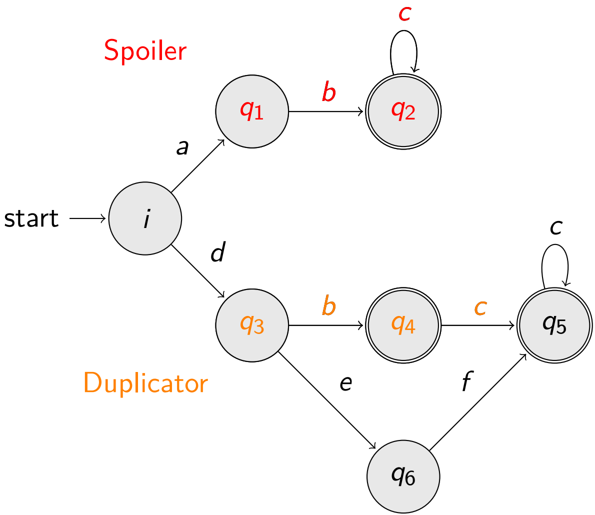


The Game of Simulation



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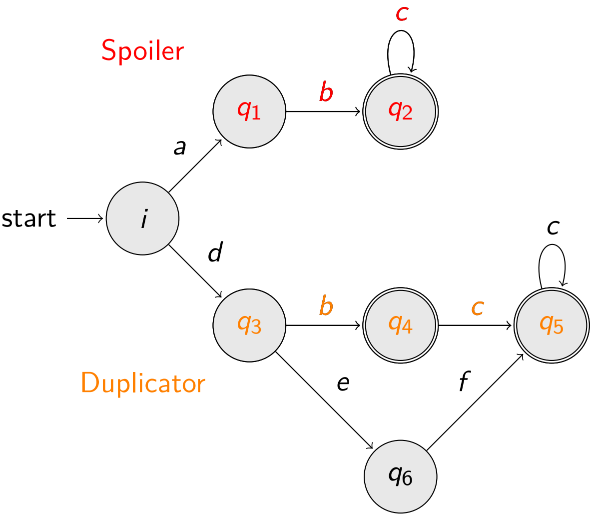

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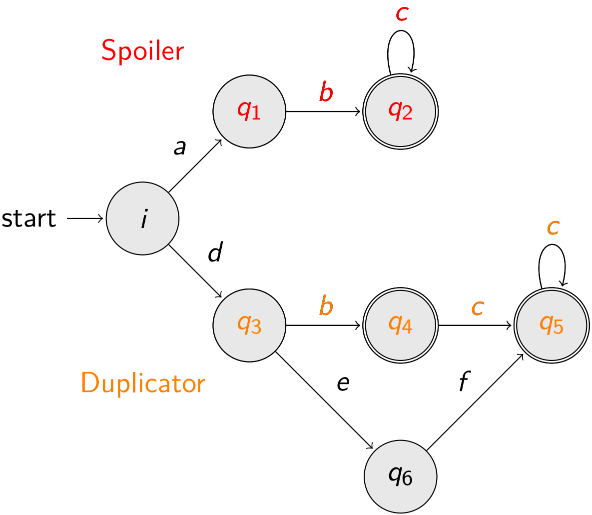

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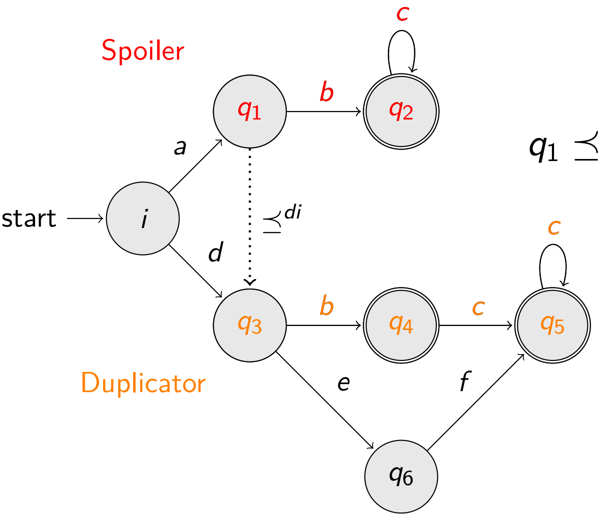
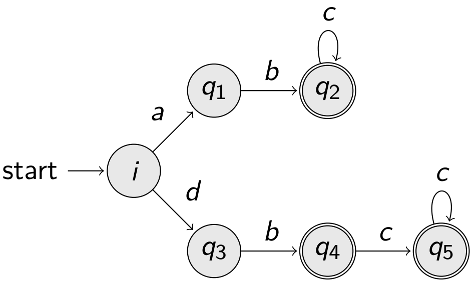


$q_1 \preceq^{di} q_3$

My work

I started from:

$$u \sqsubseteq^r_{\mathcal{B}} v \iff \textbf{for each} \text{ state } p \text{ such that } i \overset{u}{\rightsquigarrow} p, \textbf{ exists} \text{ a state } q \text{ such that } i \overset{v}{\rightsquigarrow} q \text{ and } p \preceq^{di} q$$



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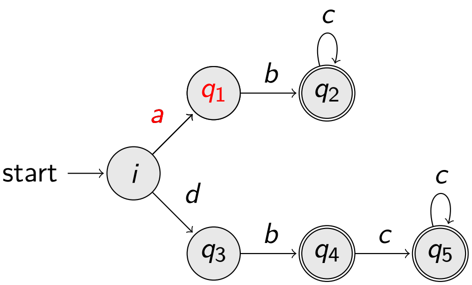

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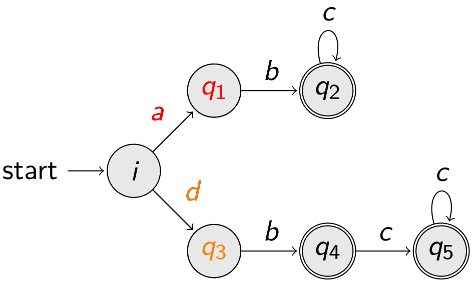

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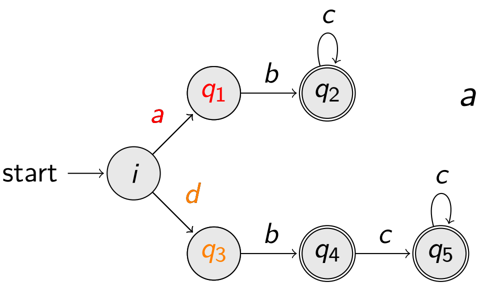
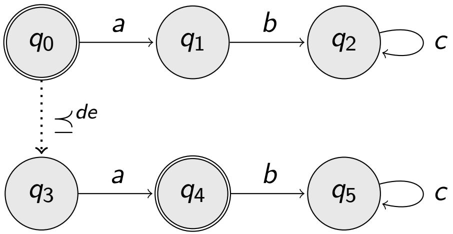


$$a \sqsubseteq^r_{\mathcal{B}} d$$

New preorders

Generalization using **different simulations**:

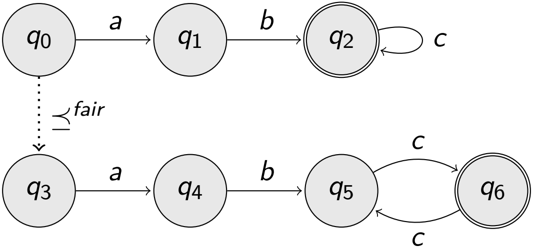
- $\sqsubseteq_{\mathcal{B}}^{de,r}$



New preorders

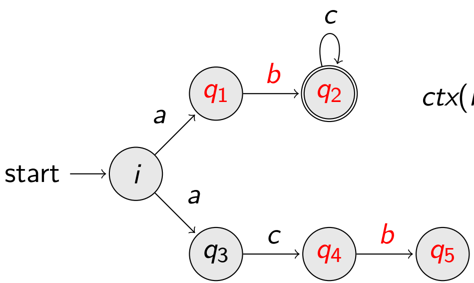
Generalization using **different simulations**:

- $\sqsubseteq_{\mathcal{B}}^{de,r}$
- $\sqsubseteq_{\mathcal{B}}^{fair,r}$



New preorders

The **context** of a word:



$$ctx(b) = \{(q_1, q_2), (q_4, q_5)\}$$

Generalization using **pairs** of states:

- $\sqsubseteq^1_{\mathcal{B}}$
- $\sqsubseteq^2_{\mathcal{B}}$

My work

- Proved a list of requirements related to **computability** and **completeness**
  1. computability
  2. right-monotonicity ($u \leq v \Longrightarrow uw \leq vw$)
  3. being a well-quasiorder (for each infinite sequence $\{x_i\}_{i \in \mathbb{N}}$, $\exists i, j : i < j \ \wedge \ x_i \leq x_j$)
  4. $\rho_{\leq_1 \times \leq_2}(I_{L_2}) = I_{L_2}$
- Identified which pairs are suitable for the framework

$$\sqsubseteq^1_{\mathcal{B}}, \sqsubseteq^2_{\mathcal{B}}$$

$$\sqsubseteq^r_{\mathcal{B}}, \sqsubseteq^2_{\mathcal{B}}$$

$$\sqsubseteq^{de,r}_{\mathcal{B}}, \sqsubseteq^2_{\mathcal{B}}$$

$$\sqsubseteq^{fair,r}_{\mathcal{B}}, \sqsubseteq^2_{\mathcal{B}}$$

Other considered simulations

- K-lookahead simulations
- Trace inclusions
- "K-delayed" simulations

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- K-lookahead simulations
- Trace inclusions
- "K-delayed" simulations

Problems related to **transitivity** and **completeness**.

Taxonomy of the preorders

$$
\begin{tikzcd}
 & \leq^2_{\mathcal{B}} \arrow[r] \arrow[dl] & \leq^1_{\mathcal{B}} \arrow[r] \arrow[dl] & \leq^r_{\mathcal{B}} \arrow[dl] \arrow[d] \arrow[dr] & \\
\sqsubseteq^2_{\mathcal{B}} \arrow[r] \arrow[dr] & \sqsubseteq^1_{\mathcal{B}} \arrow[r] \arrow[dr] & \sqsubseteq^r_{\mathcal{B}} \arrow[r] \arrow[dr] & \sqsubseteq^{de,r}_{\mathcal{B}} \arrow[r] \arrow[d] & \sqsubseteq^{fair,r}_{\mathcal{B}} \arrow[dl] \\
 & \leq^2_L \arrow[r] & \leq^1_L \arrow[r] & \leq^r_L &
\end{tikzcd}
$$

Why bother?

Simulations and the language inclusion problem:

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- **2017**: Mayr, R. and Clemente, L. *Efficient reduction of nondeterministic automata with application to language inclusion testing.*

What's next

Thanks for your attention