An overview of :
A theoretical view on the T-web statistical description of the cosmic web

Emma Aycoberry, Alexandre Barthelemy, Sandrine Codis

Astronomy & Astrophysics
October 6, 2023

Resources:
github.com/parsa-ghafour/Conferences_and_Seminars

Introduction:

- Cosmic web
- Void
- Filament(s)
- Wall
- Node

Vast regions with very low cosmic mean densities

Have roughly similar major and minor axes in cross-section

Have a significantly greater major axis than minor axis in cross-section

Highly concentrated zones where walls meet and intersect

T-web classification of the cosmic web:

- Gravitational potential
- Density field
- Variance of the contrast of the density field
- Normalize the derivatives of the gravitational potential

$$\Phi\ ,\rho$$

$$\delta = \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$$

$$\sigma_0^2 = \langle\delta\rangle^2$$

Pogosyan et al. 2009

$$\phi_{ij} = \frac{1}{\sigma_0}\nabla_i\nabla_j\Phi$$

T-web classification of the cosmic web:

- Standard deviation at different redshifts and (Gaussian) smoothing scales.

$$\sigma_0^2 = \langle \delta \rangle^2$$

$$\boldsymbol{R = cons}$$
$$\boldsymbol{z \uparrow \sim \sigma_0 \downarrow}$$

$$\boldsymbol{z = cons}$$
$$\boldsymbol{R \uparrow \sim \sigma_0 \downarrow}$$

(Cui et al. 2017, 2019)

z / R (Mpc/h)	5	15	25	35	45	55	65
0	0.71 (0.68)	0.26 (0.26)	0.15 (0.15)	0.10 (0.10)	0.074 (0.072)	0.057 (0.055)	0.045 (0.043)
0.5	0.54 (0.52)	0.20 (0.20)	0.12 (0.12)	0.078 (0.077)	0.57 (0.056)	0.044 (0.042)	0.34 (0.033)
1	0.42 (0.41)	0.16 (0.16)	0.091 (0.091)	0.061 (0.061)	0.045 (0.044)	0.034 (0.033)	0.027 (0.026)
2	0.29 (0.28)	0.11 (0.11)	0.063 (0.062)	0.042 (0.042)	0.031 (0.030)	0.024 (0.023)	0.019 (0.018)
3	0.21 (0.22)	0.082 (0.082)	0.047 (0.047)	0.032 (0.031)	0.023 (0.023)	0.018 (0.017)	0.014 (0.014)

T-web classification of the cosmic web:

- Tidal tensor
- Tidal shear tensor
- Diagonalize
- Joint probability distribution

$$\begin{cases} T_{ab} = J_{ab} - {}^{1}/_{3} J^{m}{}_{m} \eta_{ab} \\ J_{ab} = \dfrac{\partial^2 \Phi_{ab}}{\partial x^a \partial x^b} \end{cases}$$

Forero-Romero et al. (2009)

Hahn et al. (2007a)

$$\begin{cases} T_{ij} = \dfrac{\partial^2 \Phi_{ij}}{\partial x^i \partial x^j} \\ \phi_{ij} = \dfrac{1}{\sigma_0} \nabla_i \nabla_j \Phi \end{cases}$$

$$\boxed{\{\lambda_i\}_{i=1,2,3}}$$

$$\boxed{\mathcal{P}(\lambda_1, \lambda_2, \lambda_3)}$$

T-web classification of the cosmic web:

- Probability of void
- Probability of wall
- Probability of filament
- Probability of knot

$$P_{\text{void}} = \int \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \, \mathcal{P}(\lambda_1, \lambda_2, \lambda_3) Boole(\lambda_1 < \lambda_2 < \lambda_3 < \lambda_{\text{th}})$$

$$P_{\text{wall}} = \int \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \, \mathcal{P}(\lambda_1, \lambda_2, \lambda_3) Boole(\lambda_1 < \lambda_2 < \lambda_{\text{th}} < \lambda_3)$$

$$P_{\text{filament}} = \int \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \, \mathcal{P}(\lambda_1, \lambda_2, \lambda_3) Boole(\lambda_1 < \lambda_{\text{th}} < \lambda_2 < \lambda_3)$$

$$P_{\text{knot}} = \int \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \, \mathcal{P}(\lambda_1, \lambda_2, \lambda_3) Boole(\lambda_{\text{th}} < \lambda_1 < \lambda_2 < \lambda_3)$$

$$\underbrace{Boole}_{Boolean}\underbrace{(\lambda_1 < \lambda_2 < \lambda_3 < \lambda_{\text{th}})}_{condition}$$

$$condition \begin{cases} 1, & satisfied \\ 0, & otherwise \end{cases}$$

T-web classification of the cosmic web:

- Chosen threshold
- Applied smoothing
- Matter density power spectrum

Free parameter

$$\begin{cases} \Lambda_{\rm th} = 0.01 \\ \lambda_{\rm th} = (\Lambda_{\rm th} = 0.01)/\sigma(z) \end{cases}$$

$$\sigma^2(z) = 4\pi \int \mathrm{d}k\, k^2 P(k,z) W(kR)^2$$

$$W_{\rm G}(kR) = \exp\left(-\frac{1}{2}k^2R^2\right)$$

Evolution regimes:

- Linear regime:
  - Gaussian expansion
  - Doroshkevich formula
  - Uncorrelated variables
  - Rotation invariant

$$\mathcal{P}_{\mathcal{G}}(X) = (2\pi)^{-N/2}|C|^{-1/2}\exp\left(-\frac{1}{2}XC^{-1}X\right)$$

$$\mathcal{P}_D(\lambda_1,\lambda_2,\lambda_3) = \frac{675\sqrt{5}e^{\frac{3}{4}(\lambda_1+\lambda_2+\lambda_3)^2-\frac{15}{4}(\lambda_1^2+\lambda_2^3+\lambda_3^2)}}{8\pi}(\lambda_3-\lambda_2)(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)$$

(Doroshkevich 1970)

$\{I_k\}_{1\geq k\geq 3}$:

$$\begin{aligned}
I_1 &= Tr(\phi_{ij}) = \phi_{11}+\phi_{22}+\phi_{33} = \lambda_1+\lambda_2+\lambda_3 = \nu,\\
I_2 &= \phi_{11}\phi_{22}+\phi_{22}\phi_{33}+\phi_{11}\phi_{33}-\phi_{12}^2-\phi_{23}^2-\phi_{13}^2 = \lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_1\lambda_3,\\
I_3 &= det|\phi_{ij}| = \phi_{11}\phi_{22}\phi_{33}+2\phi_{12}\phi_{23}\phi_{13}-\phi_{11}\phi_{23}^2-\phi_{22}\phi_{13}^2-\phi_{33}\phi_{12}^2 = \lambda_1\lambda_2\lambda_3
\end{aligned}$$

Pogosyan et al. 2009

$\{J_k\}_{1\geq k\geq 3}$:

$$J_1 = I_1, \qquad J_2 = I_1^2 - 3I_2, \qquad J_3 = I_1^3 - \frac{9}{2}I_1I_2 + \frac{27}{2}I_3$$

Evolution regimes:

- Linear regime:
  Gaussian random filed
  Probabilities for the different environments

Pogosyan et al. 2009

$$\mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3) = \frac{25\sqrt{5}}{12\pi} \exp\left[-\frac{1}{2}J_1^2 - \frac{5}{2}J_2\right]$$

Pogosyan et al. (2009); Gay et al. (2012)

$$P_{\rm void} = \int_0^\infty dJ_2 \int_{-\infty}^{-2\sqrt{J_2}+3\lambda_{\rm th}} dJ_1 \int_{-J_2^{3/2}}^{J_2^{3/2}} dJ_3\, \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3) + \int_0^\infty dJ_2 \int_{-2\sqrt{J_2}+3\lambda_{\rm th}}^{-\sqrt{J_2}+3\lambda_{\rm th}} dJ_1 \int_{-J_2^{3/2}}^{-\frac{1}{2}(J_1-3\lambda_{\rm th})^3+\frac{3}{2}(J_1-3\lambda_{\rm th})J_2} dJ_3 \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3).$$

$$P_{\rm wall} = \int_0^\infty dJ_2 \int_{-2\sqrt{J_2}+3\lambda_{\rm th}}^{\sqrt{J_2}+3\lambda_{\rm th}} dJ_1 \int_{-\frac{1}{2}(J_1-3\lambda_{\rm th})^3+\frac{3}{2}(J_1-3\lambda_{\rm th})J_2}^{J_2^{3/2}} dJ_3 \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3),$$

$$P_{\rm filament} = \int_0^\infty dJ_2 \int_{-\sqrt{J_2}+3\lambda_{\rm th}}^{2\sqrt{J_2}+3\lambda_{\rm th}} dJ_1 \int_{-J_2^{3/2}}^{-\frac{1}{2}(J_1-3\lambda_{\rm th})^3+\frac{3}{2}(J_1-3\lambda_{\rm th})J_2} dJ_3 \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3),$$

$$P_{\rm knot} = \int_0^\infty dJ_2 \int_{\sqrt{J_2}+3\lambda_{\rm th}}^{2\sqrt{J_2}+3\lambda_{\rm th}} dJ_1 \int_{-\frac{1}{2}(J_1-3\lambda_{\rm th})^3+\frac{3}{2}(J_1-3\lambda_{\rm th})J_2}^{J_2^{3/2}} dJ_3 \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3) + \int_0^\infty dJ_2 \int_{2\sqrt{J_2}+3\lambda_{\rm th}}^{\infty} dJ_1 \int_{-J_2^{3/2}}^{J_2^{3/2}} dJ_3 \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3)$$

Evolution regimes:

- Linear regime:
  3D integrals can be reduced to 1D:
  two degrees of freedom can be analytically integrated out

$$P_{\mathrm{void}} = \int_0^\infty \frac{25\sqrt{5}}{48\pi} e^{-\frac{5J_2}{2}} \Bigg[-\sqrt{2\pi}(2J_2^{3/2} - 9J_2\lambda_{\mathrm{th}} + 9\left(3\lambda_{\mathrm{th}}{}^3 + \lambda_{\mathrm{th}}\right))\mathrm{erf}\left(\frac{3\lambda_{\mathrm{th}} - 2\sqrt{J_2}}{\sqrt{2}}\right)$$

$$+ \sqrt{2\pi}\left(2J_2^{3/2} - 9J_2\lambda_{\mathrm{th}} + 9\left(3\lambda_{\mathrm{th}}{}^3 + \lambda_{\mathrm{th}}\right)\right)\mathrm{erf}\left(\frac{3\lambda_{\mathrm{th}} - \sqrt{J_2}}{\sqrt{2}}\right) + 4\sqrt{2\pi}J_2^{3/2}\mathrm{erfc}\left(\frac{2\sqrt{J_2} - 3\lambda_{\mathrm{th}}}{\sqrt{2}}\right)$$

$$- 2e^{-\frac{1}{2}\left(2\sqrt{J_2} - 3\lambda_{\mathrm{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\mathrm{th}} + J_2 + 9\lambda_{\mathrm{th}}{}^2 + 2\right) + e^{-\frac{1}{2}\left(\sqrt{J_2} - 3\lambda_{\mathrm{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\mathrm{th}} - 4J_2 + 18\lambda_{\mathrm{th}}{}^2 + 4\right)\Bigg] \mathrm{d}J_2$$

$$P_{\mathrm{wall}} = \int_0^\infty \frac{25\sqrt{5}}{48\pi} e^{-\frac{5J_2}{2}} \Bigg[-\sqrt{2\pi}(2J_2^{3/2} + 9J_2\lambda_{\mathrm{th}} - 9\left(3\lambda_{\mathrm{th}}{}^3 + \lambda_{\mathrm{th}}\right))\mathrm{erf}\left(\frac{3\lambda_{\mathrm{th}} - 2\sqrt{J_2}}{\sqrt{2}}\right)$$

$$+ \sqrt{2\pi}\left(2J_2^{3/2} + 9J_2\lambda_{\mathrm{th}} - 9\left(3\lambda_{\mathrm{th}}{}^3 + \lambda_{\mathrm{th}}\right)\right)\mathrm{erf}\left(\frac{\sqrt{J_2} + 3\lambda_{\mathrm{th}}}{\sqrt{2}}\right) + 2e^{-\frac{1}{2}\left(2\sqrt{J_2} - 3\lambda_{\mathrm{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\mathrm{th}} + J_2 + 9\lambda_{\mathrm{th}}{}^2 + 2\right)$$

$$+ 2e^{-\frac{1}{2}\left(\sqrt{J_2} + 3\lambda_{\mathrm{th}}\right)^2}\left(3\sqrt{J_2}\lambda_{\mathrm{th}} + 2J_2 - 9\lambda_{\mathrm{th}}{}^2 - 2\right)\Bigg] \mathrm{d}J_2,$$

Evolution regimes:

- Linear regime:

  3D integrals can be reduced to 1D:

  two degrees of freedom can be analytically integrated out

$$P_{\text{filament}} = \int_0^\infty -\frac{25\sqrt{5}}{48\pi} e^{-\frac{5J_2}{2}} \Bigg[-\sqrt{2\pi}\left(2J_2^{3/2} - 9J_2\lambda_{\text{th}} + 9\left(3\lambda_{\text{th}}{}^3 + \lambda_{\text{th}}\right)\right)\text{erf}\left(\frac{2\sqrt{J_2}+3\lambda_{\text{th}}}{\sqrt{2}}\right)$$
$$+ \sqrt{2\pi}\left(2J_2^{3/2} - 9J_2\lambda_{\text{th}} + 9\left(3\lambda_{\text{th}}{}^3 + \lambda_{\text{th}}\right)\right)\text{erf}\left(\frac{3\lambda_{\text{th}} - \sqrt{J_2}}{\sqrt{2}}\right) - 2e^{-\frac{1}{2}\left(2\sqrt{J_2}+3\lambda_{\text{th}}\right)^2}\left(-6\sqrt{J_2}\lambda_{\text{th}} + J_2 + 9\lambda_{\text{th}}{}^2 + 2\right)$$
$$+ e^{-\frac{1}{2}\left(\sqrt{J_2}-3\lambda_{\text{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\text{th}} - 4J_2 + 18\lambda_{\text{th}}{}^2 + 4\right)\Bigg] \mathrm{d}J_2,$$

$$P_{\text{knot}} = \int_0^\infty \frac{25\sqrt{5}}{48\pi} e^{-\frac{5J_2}{2}} \Bigg[-\sqrt{2\pi}\left(2J_2^{3/2} + 9J_2\lambda_{\text{th}} - 9\left(3\lambda_{\text{th}}{}^3 + \lambda_{\text{th}}\right)\right)\text{erf}\left(\frac{\sqrt{J_2}+3\lambda_{\text{th}}}{\sqrt{2}}\right)$$
$$+ \sqrt{2\pi}\left(2J_2^{3/2} + 9J_2\lambda_{\text{th}} - 9\left(3\lambda_{\text{th}}{}^3 + \lambda_{\text{th}}\right)\right)\text{erf}\left(\frac{2\sqrt{J_2}+3\lambda_{\text{th}}}{\sqrt{2}}\right) + 4\sqrt{2\pi}J_2^{3/2}\text{erfc}\left(\frac{2\sqrt{J_2}+3\lambda_{\text{th}}}{\sqrt{2}}\right)$$
$$- 2e^{-\frac{1}{2}\left(2\sqrt{J_2}+3\lambda_{\text{th}}\right)^2}\left(-6\sqrt{J_2}\lambda_{\text{th}} + J_2 + 9\lambda_{\text{th}}{}^2 + 2\right) + e^{-\frac{1}{2}\left(\sqrt{J_2}+3\lambda_{\text{th}}\right)^2}\left(-6\sqrt{J_2}\lambda_{\text{th}} - 4J_2 + 18\lambda_{\text{th}}{}^2 + 4\right)\Bigg] \mathrm{d}J_2$$

Evolution regimes:

- Linear regime:

  In each probability:

  Integration interval

  Error function

  Complementary error function

$$\boxed{\mathrm{erf}(z) = 2\int_0^z e^{-t^2}\,dt/\sqrt{\pi}}$$

$$\boxed{\mathrm{erfc}(z) = 1 - \mathrm{erf}(z)}$$

$$P_{\mathrm{void}} = \int_0^\infty \frac{25\sqrt{5}}{48\pi} e^{-\frac{5J_2}{2}} \Bigg[-\sqrt{2\pi}(2J_2^{3/2} - 9J_2\lambda_{\mathrm{th}} + 9\left(3{\lambda_{\mathrm{th}}}^3 + \lambda_{\mathrm{th}}\right))\mathrm{erf}\left(\frac{3\lambda_{\mathrm{th}} - 2\sqrt{J_2}}{\sqrt{2}}\right)$$
$$+ \sqrt{2\pi}\left(2J_2^{3/2} - 9J_2\lambda_{\mathrm{th}} + 9\left(3{\lambda_{\mathrm{th}}}^3 + \lambda_{\mathrm{th}}\right)\right)\mathrm{erf}\left(\frac{3\lambda_{\mathrm{th}} - \sqrt{J_2}}{\sqrt{2}}\right) + 4\sqrt{2\pi}J_2^{3/2}\mathrm{erfc}\left(\frac{2\sqrt{J_2} - 3\lambda_{\mathrm{th}}}{\sqrt{2}}\right)$$
$$- 2e^{-\frac{1}{2}\left(2\sqrt{J_2} - 3\lambda_{\mathrm{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\mathrm{th}} + J_2 + 9{\lambda_{\mathrm{th}}}^2 + 2\right) + e^{-\frac{1}{2}\left(\sqrt{J_2} - 3\lambda_{\mathrm{th}}\right)^2}\left(6\sqrt{J_2}\lambda_{\mathrm{th}} - 4J_2 + 18{\lambda_{\mathrm{th}}}^2 + 4\right)\Bigg]\mathrm{d}J_2$$

Evolution regimes:

- Non-Linear regime:
  - Low redshift/small scales
  - Non-gaussian corrections
  - Gram-Charlier expansion
  - Gaussian kernel
  - Hermite tensors

$$\mathcal{P}(X) = \mathcal{P}_{\mathcal{G}}(X)\left[1 + \sum_{n=3}^{\infty}\frac{1}{n!}Tr[\langle X^n\rangle_{GC}.h_n(X)]\right]$$

$$\mathcal{P}_{\mathcal{G}}(X) = (2\pi)^{-N/2}|C|^{-1/2}\exp\left(-\frac{1}{2}XC^{-1}X\right)$$

$$\boxed{h_n(X) = (-1)^n\mathcal{P}_{\mathcal{G}}{}^{-1}(X)\partial^n\mathcal{P}_{\mathcal{G}}(X)/\partial X^n}$$

$$\boxed{\langle X^n\rangle_{GC} = \langle h_n(X)\rangle}$$

Pogosyan et al. (2009); Gay et al. (2012); Codis et al. (2013).

Evolution regimes:

- Non-Linear regime:
  - Rotation invariant variables
  - Hermite polynomials
  - Laguerre polynomials
  - Normalization coefficient
  - Orthogonal polynomials

$$H_n$$

$$L_l^{(\alpha)}(x)$$

$$c_{lm}$$

$$F_{lm}$$

$$\mathcal{P}(J_1, J_2, J_3) = \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3)\Bigg[1 + \sum_{n=3}^{\infty}\sum_{k,l}^{k+2l=n} \frac{(-1)^l 5^l \times 3}{k!(3+2l)!!}\langle J_1^k J_2^l \rangle_{GC} H_k(J_1) L_l^{(3/2)}\left(\frac{5}{2}J_2\right)$$
$$+ \sum_{n=3}^{\infty}\sum_{k}^{k+3=n} \frac{25}{k! \times 21}\langle J_1^k J_3 \rangle_{GC} H_k(J_1) J_3 + \sum_{n=5}^{\infty}\sum_{k,l,m=1}^{k+2l+3m=n} \frac{c_{lm}}{k!}\langle J_1^k J_2^l J_3^m \rangle_{GC} H_k(J_1) F_{lm}(J_2, J_3)\Bigg]$$

Evolution regimes:

- Non-Linear regime:
  First corrective term
  Gram-Charlier cumulants

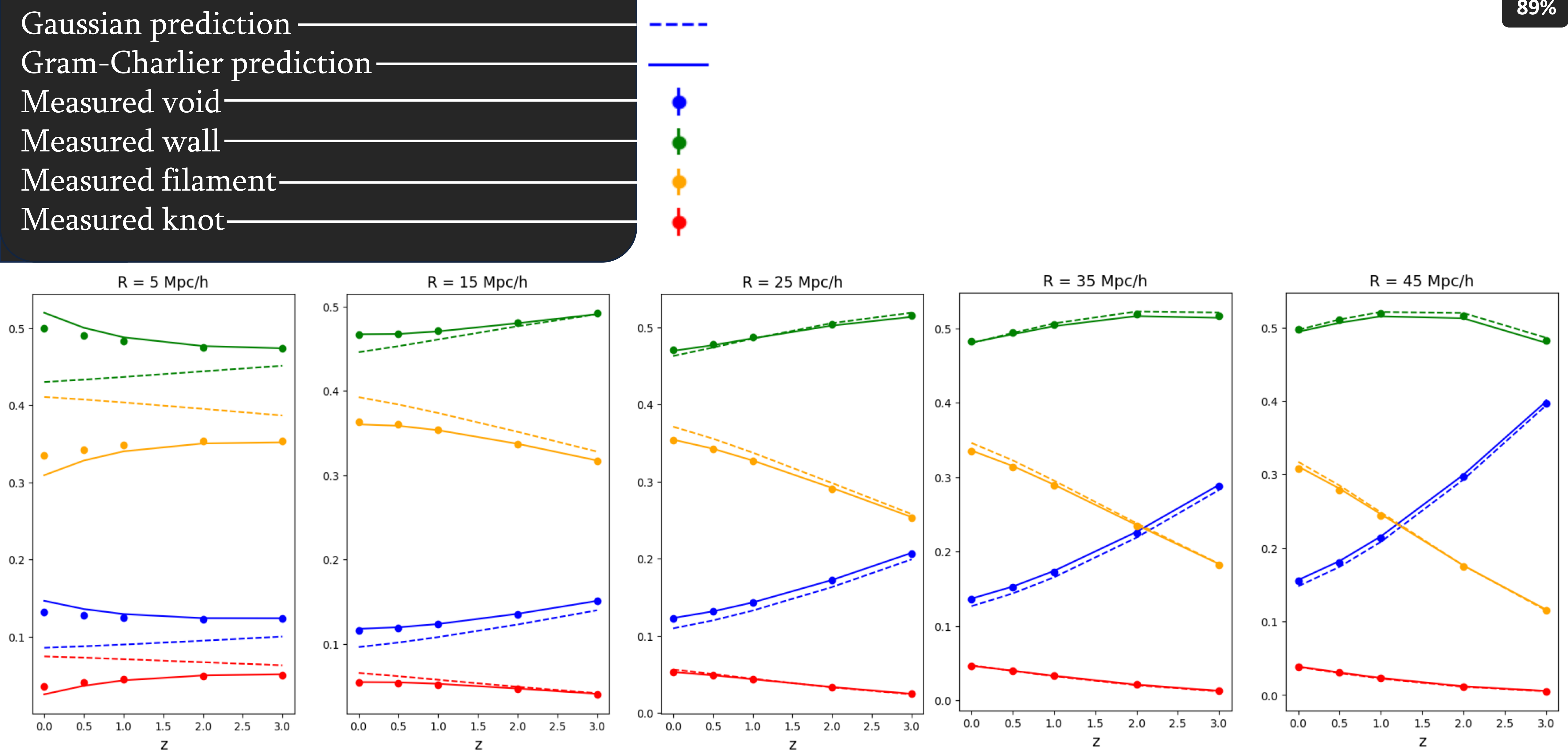
$$\mathcal{P}(J_1, J_2, J_3) = \mathcal{P}_{\mathcal{G}}(J_1, J_2, J_3)\left[1 + \frac{1}{6}\langle J_1^3\rangle_{GC} H_3(J_1) - \langle J_1 J_2\rangle_{GC} H_1(J_1) L_1^{(3/2)}\left(\frac{5}{2}J_2\right) + \frac{25}{21}\langle J_3\rangle_{GC} J_3\right] + o(\sigma_0^2)$$

$$\langle J_1^3\rangle_{GC} = \langle H_3(J_1)\rangle$$

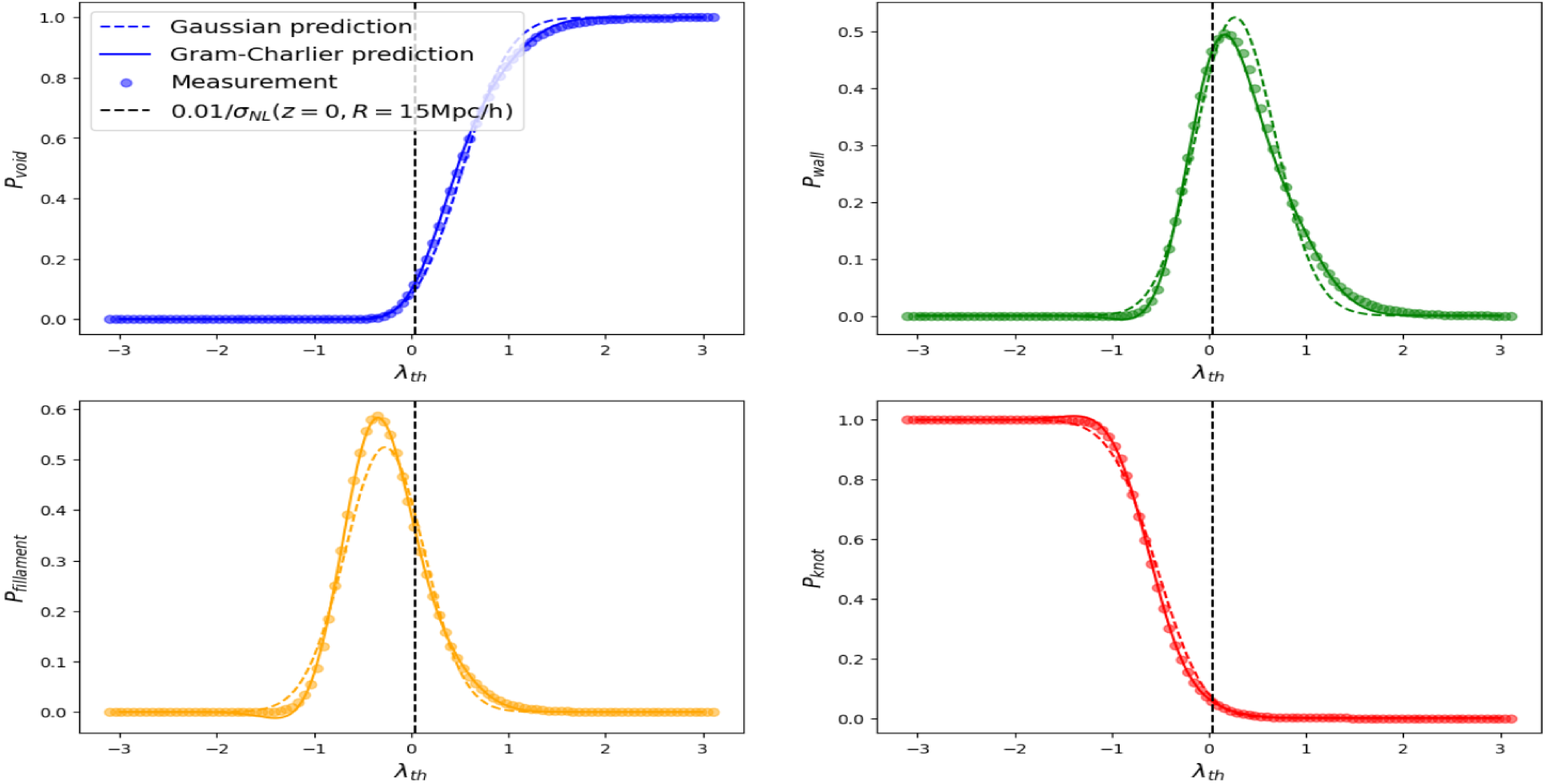
$$\langle J_1 J_2\rangle_{GC} = -\frac{2}{5}\left\langle H_1(J_1) L_1^{(3/2)}\left(\frac{5}{2}J_2\right)\right\rangle$$

$$\langle J_3\rangle_{GC} = \langle J_3\rangle$$

Results:


Gaussian prediction
Gram-Charlier prediction
Measured void
Measured wall
Measured filament
Measured knot
R = 5 Mpc/h
R = 15 Mpc/h
R = 25 Mpc/h
R = 35 Mpc/h
R = 45 Mpc/h
z

Results:


Gaussian prediction
Gram-Charlier prediction
Measurement
$0.01/\sigma_{NL}(z=0, R=15\mathrm{Mpc/h})$
P_{void}
P_{wall}
$P_{filament}$
P_{knot}
λ_{th}

Results:

Voids (dark blue)
Walls (blue)
Filaments (green)
Nodes (red)