ITERATIVE METHODS WITH ADAPTIVE THRESHOLDING FOR SPARSE SIGNAL RECONSTRUCTION

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ABSTRACT

In this paper we have proposed new iterative methods for sparse signal reconstruction. The underlying foundation of these strategies is the Iterative Method with Adaptive Thresholding (IMAT). We have applied these methods to spectral estimation and impulsive noise cancellation as two principal applications. In sparse spectral estimation, rather than conventional parametric methods, we have introduced the implementation of other sparse signal recovery techniques to the subject, such as Basis Pursuit and Greedy methods alongside a modified IMAT technique. For the case of impulsive noise removal, we have proposed a soft version of IMAT, which exhibits robust results for both impulsive and Gaussian noise. Furthermore, a useful table has been presented for setting the parameters of the method for any arbitrary case. Simulation results clearly compare the performance of our methods with conventional methods, stating that our proposed algorithms exhibit robust characteristics in dealing with noisy sparse models.

Keywords— Sparse signal reconstruction, Spectral Estimation, Impulsive noise cancellation, Iterative Method with Adaptive Thresholding

1. INTRODUCTION

There are many applications in signal processing and communication systems where the discrete signal samples are sparse in some domain such as time, frequency, or space i.e., most of the samples are zero, or alternatively their transform in another domain. Although there has been a lot of research in this field, novel methods seem to outperform conventional techniques, regarding both complexity and optimality.

Since the utilizations of sparse signal processing are broadly spanned, this paper is mainly concentrated on two main applications; Spectral estimation and Impulsive noise cancellation.

Spectral estimation has been a principal point of attention in many signal processing societies. The application is basically focused on frequency-sparse signals, from which noisy time domain samples are available. Conventional methods for spectrum analysis are nonparametric and parametric methods. For example the use of a mere FFT is a special case of non-parametric techniques, which are computational cheap, but suffer from the fundamental limitation of low precision in noisy and closely-spaced sparsity conditions [1]. However, by assuming a statistical model with some unknown parameters, parametric methods can get more resolution by estimating the parameters from the data at the cost of more computational complexity. The techniques developed for this branch of science is quite unique; with examples such as MUSIC [2], Prony [3], and Pisarenko [4].

In this paper, we have introduced the presentation of Basis Pursuit (BP) algorithms [5], such as L1 minimizations, as well as the Greedy methods family [6] into the subject. Moreover, the Iterative Method with Adaptive Thresholding (IMAT), as mentioned in [1], is modified and adapted to the specific application.

The implementation of sparse signal processing in impulsive noise cancellation is also a common topic in the literature. The most important fact about impulsive noise is its sparsity in the time domain. Accordingly, sparse signal reconstruction methods can be used to detect these impairments and eliminate them.

In this paper, we have modified IMAT in order to be capable of removing impulsive noise. In order to show the vigor of our new method; namely *soft-IMAT;* we have implemented a number of greedy and Basis Pursuit methods and compared their capability and complexity. Among the greedy methods we focus on Iterative Hard Thresholding (IHT) [7], Orthogonal Matching Pursuit (OMP) [6], and an improved version of the original Recursive Detection-Estimation (RDE) [8]. Since the BP methods are supposed to present the best answer in each case, we consider the Lasso L1-minimization [9] among them.

In the next section, the basic preliminaries of this research, i.e. IMAT, is explained. According to this basis, our proposed adaptations of the iterative algorithm have been implemented throughout this paper.

In section 3, the spectral estimation applications are brought to attention. Deep explanations to our new method and its capabilities are illustrated. Also, as depicted in the results, the prospect of implementing BP and OMP algorithms in the field has been introduced.

In section 4, mentioning the hard-decision setbacks of the original IMAT algorithm, our proposed *soft*-IMAT technique for overcoming a mixture of Gaussian and impulsive noise, has been presented.

Section 5 contains the simulation results for the proposed algorithms beside the conventional methods for each one of the two applications. Clear comparisons have been illustrated stating the advantages of our new techniques.

Finally, in section 6 we have concluded our observations and possible future works have been presented.

2. PROBLEM PRELIMINARIES

The Iterative Method with Adaptive Threshold (IMAT) for sparse signal reconstruction, as introduced in [1], is the prime basis of this research. Unlike its predecessors, this technique utilizes nonlinear thresholds in an iterative manner in order to recover a sparse signal from its noisy samples in another domain. We shall designate the domain where the signal exhibits sparse characteristics, as the sparsity (or basically sparse) domain and the domain where the signal is being sampled, as the information domain. Moreover, the projection of the signal from the sparse domain to the information domain is regarded as the Discrete Transform (DT), which is priorly known. For instance, for a time-sparse signal that is sampled in frequency, the discrete transform is a mere DFT.

The stepwise procedure of the algorithm, which is in fact a modification of Matching Pursuit algorithms, is as explained in Table 1. Fig. 1 also illustrates the block diagram procedure.

Table 1. The general IMAT algorithm

1) Use an all-zero vector as the initial value for the sparse domain signal. (iteration 0)
2) Convert the current estimate of the signal in the sparse domain into the information domain using the known Discrete Transform.
3) Replace the inexact values of the estimated signal with the exact, but still noisy, samples in the information domain.
4) Use IDT to return to the sparse domain.
5) Hard-threshold the signal with an adaptive exponential threshold as mentioned in Fig. 1.
6) Continue steps 2-5 until the stop criterion. (e.g. maximum iteration number or minimum error between estimations) has been met.

According to IMAT, by alternate projections between the information and sparsity domains, alongside the adaptive lowering of the sparsity domain threshold, the unknown sparse coefficients are gradually picked up after several iterations. Furthermore, the inherent noise due to the imperfect sampling process tend to fade away only if IMAT parameters, i.e. α and β, are chosen appropriately.

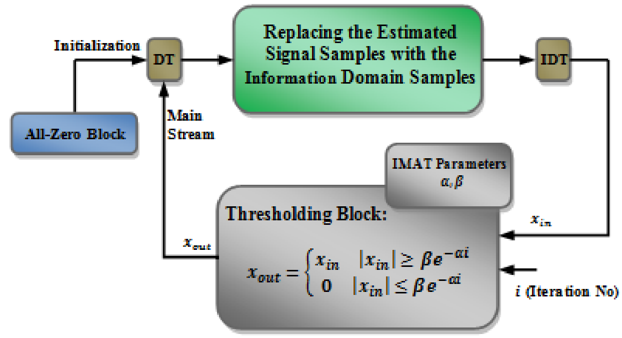

Fig. 1. The Iterative Method with Adaptive Thresholding (IMAT) detecting the number, location and values of sparsity.

3. SPECTRAL ESTIMATION

The goal of Spectral estimation is to estimate the power spectrum of a random signal. Spectral estimation methods are widely applied to speech and sonar signals. These signals are usually sparse in the frequency domain, thus exploiting sparsity feature the estimation of signal's spectrum would be more accurate especially in the presence of noise.

3.1. Proposed Novel-IMAT

In this subsection a modified version of IMAT is proposed, which shows a better performance than the general algorithm introduced in the section 1. In IMAT, decreasing the threshold will improve the precision of the algorithm as the steps proceed, but after a certain step the threshold will be less than many noise samples, thus disabling the algorithm accuracy. To solve this problem we fix the threshold after a certain stage forth. This method, namely *novel*-IMAT is as described in Table 2.

Table 2. The novel-IMAT algorithm

S_j : The signal in the time domain in the j^th iteration
F_j : The signal in frequency domain in the j^th iteration
$A_j \triangleq \sum_{i=0}^{k} abs(F_j^i)$ = Summation of frequency samples
$D_j \triangleq A_{j+1} - 2 \times A_j + A_{j-1}$ = Approximation of the second derivative

Initialization:

$\beta = \max_i \left(abs(F_0^i)\right)$

$\alpha = \log\left(\max_i \left(abs\left(\frac{abs(F_0^m)}{abs(F_0^{m+1})}\right)\right)\right.$

M = Maximum Number of Steps

$Flag = 0$

While $j < M$ do
1. $F_j = DFT\,(S_j)$
2. Compute A_j
3. If $j > 2$ or $Flag = 1$ then compute D_j
4. If $D_{j-1} \times D_j < 0$, then $Thr_j = Thr_{j-1}$ and $Flag = 1$ (stop decreasing the threshold)
5. If $Flag = 0$ then $Thr_j = \beta e^{-\alpha j}$
6. Use hard Thresholding to distinguish Non-zero samples and name the output F_j^*
7. $S_{j+1} = IDT(F_j^*)$
8. Replace the known samples in S_{j+1}
9. j= j+1

4. IMPULSIVE NOISE CANCELLATION

4.1. Proposed Soft-IMAT

As mentioned in Section 1, in the thresholding stage of IMAT, all the samples of the input signal which are less than the threshold will be set to zero. In noiseless cases and when we have enough information this approach ends up extremely fast, however, in downsampled cases and in the presence of white noise, the method would not perform properly as good.

In fact, the hard decision in eliminating below threshold samples reduces the performance of IMAT in highly noised or downsampled cases. In order to be robust against these setbacks, we have substituted the hard decision with a soft decision basis. In our soft decision procedure, a thresholding mask, with components in the [0,1] interval, is generated. After specifying the adaptive threshold in each iteration the mask function for the k^th sample is defined as follow:

$$mask(k) = \begin{cases} 1 & |X_{in}(k)| \geq T \\ e^{-i|X_{in}(k)-T|} & |X_{in}(k)| \leq T \end{cases} \qquad (1)$$

where i is the index of the iteration, T is the threshold in the current iteration which is specified according to the original IMAT algorithm, and X_{in} is the input to the thresholding (see Fig. 1).

According to (1), the mask function attributes a number to each sample, which is in fact the probability of that sample to be reconstructed correctly. In soft-IMAT, the samples less than the threshold are attenuated with a factor based on the difference between their value and the threshold, instead of being eliminated. As the mask is greater; i.e., is closer to 1, the sample is considered to be more accurately reconstructed. Consequently, since the samples greater than the soft threshold are due to impulses, the mask is 1, and for the others it is exponentially related to the difference between the sample and threshold.

Now, the output of the thresholding block, X_{out}, is defined as follows,

$$X_{out}(k) = X_{in}(k) \times mask(k) \qquad (2)$$

This approach makes the method capable of compensating its faults in detecting the impulses. Since the soft decision method is very slowly convergent in comparison with hard thresholding, it is logical to convert to the hard decision, as the method proceeds. Thus, we have included the ascending parameter i, in the power of the exponential threshold mask. So, in the early steps of the method, where there an inaccurate estimation of the signal is available, soft decision is used and then we convert to hard thresholding.

Table 3. The minimum number of required samples for IMAT ($\alpha = 0.1$, $\beta = \max$ {impulsive noised signal})

Sparsity Number (m)	Signal Length (n)	Minimum number of required samples
5	400	130
10	400	192
15	400	210
20	400	240
30	400	250
40	400	260
50	400	270
60	400	280
70	400	290
80	400	300
90	400	310
100	400	315
110	400	315
120	400	320
130	400	324
140	400	330
150	400	340

In the next subsection a discussion about the implementation of soft-IMAT in impulsive noise cancellation as well as a useful parameter table are presented.

4.2. Impulsive Noise Cancellation Using Soft-IMAT

As discussed previously, impulsive noise is sparse in the time domain and we have assumed that the signal information is in the frequency domain; specifically in the DFT domain. Then according to Fig. 1, the DFT is our discrete transform. In the meanwhile, we assume that the original signal, i.e. the signal without impulsive noise, is low-pass. So, in the DFT domain the high-frequency components are only pertained to impulsive noise.

In IMAT, we use these high-frequency samples to reconstruct the impulsive noise. Consequently, we can remove additive noise and clean up the signal. As a result, we do not use the original signal itself, which is another advantage over other possible methods, such as RDE.

In this technique however, according to the sparsity number of the sparse signal we need a minimum number of samples to be able to reconstruct the signal adequately. In Table 3, we have presented the minimum number of required samples for different cases of m and n. This table is based on the condition that the reconstructed to original signal MSE is -300 dB. The important virtue of the proposed table is that we can multiply its three columns by a constant and obtain other tables which represent other cases of m and n.

5. COMPARISON AND RESULTS

5.1. Spectral Estimation

From the literature, we know that some of the greedy and basis pursuit sparse signal reconstruction methods are robust against white noise, whereas, others are incapable to perform in the presence of white noise. According to Fig. 3, our proposed novel IMAT outperforms the L1-minimization method as well as OMP. At the same time, IMAT introduces far less complexity than L1.

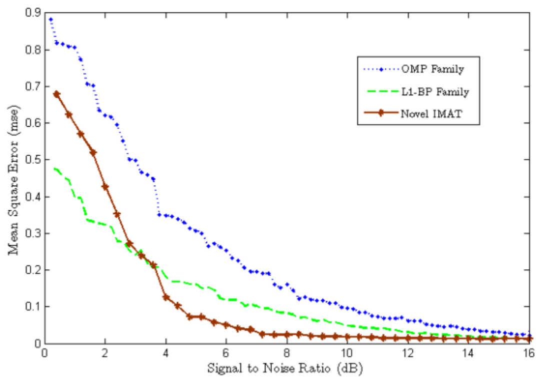

Fig. 2. Performance comparison between the proposed novel IMAT, Orthogonal Matching Pursuit and L1 minimization in spectral estimation (block size: 1000, sampling ratio: 0.1)

5.2. Impulsive Noise Cancellation

We have simulated the proposed methods, IMAT and soft-IMAT, for different situations using table 3 and compared the results with the other methods. The results indicated that in the noiseless situations IMAT is able to reach -300 dB for its output MSE. As the white noise is incremented, the performance of IMAT decreases and soft-IMAT comes in. Table 4, represents the performance and complexity of different methods in different cases. It worth mentioning that in IHT we have to know the number of sparses as extra information. According to Table 4, it can be concluded that in low-noise situations soft-IMAT surpasses the other methods considering its performance, complexity and the needed information.

As is shown in the results, soft-IMAT is performing as effective as the modified RDE, but in soft-IMAT the complexity has been decreased in a large scale. It worth mentioning that RDE has a practical impairment due to setting its parameters. However, In this paper, we have implemented a modified version of RDE, which is automatically parameterized, in order to be fairly compared.

TABALE 4 PERFORMANCE AND COMPLEXITY OF DIFFERENT METHODS IN IMPULSIVE NOISE CANCELLATION

Methods	Sparsity =25 in a block length of 1000, SNR =20 dB		Sparsity =375 for the same block length, SNR =10 dB	
	MSE (dB)	Complexity	MSE (dB)	Complexity
soft -IMAT	-21	2	-14	3
IMAT	-16	1.6	-11	2
RDE	-22	147	-16	150
OMP	-14	20	-9	600
K	-19	6	-9	7
L1	-21	166	-13	170

6. CONCLUSION

As clearly observed in section 5, IMAT is shown to be a powerful algorithm in many sparse signal processing applications. Its advantages are not only in precision, but also in its low computational complexity. Moreover, the adapted versions of IMAT exhibit even better characteristics in the applications mentioned in this paper.

The extension of IMAT into other sparse signal processing fields alongside devising a fully automatic procedure based on IMAT, are possible future works. Also, the analytic justification of the algorithm's convergence in noisy conditions may be of interest.

7. REFERENCES

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