Technische Universität Bergakademie Freiberg
Institut für Numerische Mathematik und Optimierung
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Wintersemester 2024/25
06.11.2024

Introduction to High Performance Computing and Optimization

1. part PVL

To get points for the parts of the PVL, the C++ code, the job scripts and, depending on the task, the times achieved must be submitted. The submission takes place via the upload in OPAL in the corresponding course element. Copying or typing code from fellow students is not permitted. You must write and submit your own code. The specified submission deadline must be adhered to. Your code will be tested. Faulty code and code that delivers significantly different times than specified will result in zero points for the PVL part. So make sure that the code has been compiled on the cluster and executed on the compute nodes.

Exercise 6

The Jacobi method is an iterative methods to solve a linear system

$$Ax = b.$$

The matrix-based formulation of the Jacobi method is defined as

$$x^{(k+1)} = -D^{-1}(L+U)x^{(k)} + D^{-1}b,$$

where x_k are iterates, $k \geq 0$, $A = L + D + U$ and $D = \mathrm{diag}(A)$ is the diagonal of A, L a lower triangular matrix and U an upper triangular matrix.
The element-based formulation of the Jacobi method is defined as

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j \neq i} a_{ij} x_j^{(k)} \right), \qquad i = 1, 2, \ldots, n$$

Use $x_0 = \vec{0}$ as a start vector. Remark: If $\|D^{-1}(L+U)\|_\infty < 1$ then this iteration converges.

(a) Define the $n \times n$-Matrix

$$A = \begin{pmatrix} +2^0 & -2^{-2} & -2^{-4} & -2^{-8} & \ldots & -2^{-2^{n-1}} \\ -2^{-2} & +2^0 & -2^{-2} & -2^{-4} & \ldots & -2^{-2^{n-2}} \\ -2^{-4} & -2^{-2} & +2^0 & -2^{-2} & \ldots & -2^{-2^{n-3}} \\ -2^{-8} & -2^{-4} & -2^{-2} & +2^0 & \ldots & -2^{-2^{n-4}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ -2^{-2^{n-1}} & -2^{-2^{n-2}} & -2^{-2^{n-3}} & -2^{-2^{n-4}} & & +2^0 \end{pmatrix}$$

and the right hand side $b = (1, \ldots, 1)^T$.

(b) Implement a parallel Jacobi method using OpenMP. The iterations stop when the norm of the residual $r = b - Ax$ has been reduced by 10^{-6}, i. e., $\|r^{(k)}\|_2 < 10^{-6} \cdot \|r^{(0)}\|_2$.

(c) Parallelize the code using OpenMP.

(d) Perform a strong scaling test using $1, 2, 4, 8$ threads and a problem size $n = 10000$. The obtained times should be close to

```
#Threads 1: 57s
#Threads 2: 30s
#Threads 4: 17s
#Threads 8: 8s
```

These values are for orientation purposes only.

Submission deadline is 23:59 20.11.2024.