Computer vision Sheet02 theory answer

Group members:

- Parsa Eissazadeh (s68peiss@uni-bonn.de, 50234732)
- Pranav Yadav (s52pyada@uni-bonn.de 50327698)

Q1

$$f(x) * g(x) = \int_{t=-\infty}^{+\infty} f(t)\, g(x-t)\, dt$$

$$F(\omega)\, G(\omega) = \int_{x=-\infty}^{+\infty} f(x) e^{\underbrace{-2\pi j \omega x}_{-i\omega x}} dx \cdot \int_{x=-\infty}^{+\infty} g(x) e^{\underbrace{-2\pi j \omega x}_{-i\omega x}} dx$$

$$F\{(f*g)(x)\} = \int_{-\infty}^{+\infty} (f*g)(x) e^{-i\omega x}\, dx$$

$$= \int_{-\infty}^{+\infty} e^{-i\omega x} \cdot \int_{-\infty}^{+\infty} f(t) g(x-t)\, dt\, dx$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(t) g(x-t) \cdot e^{-i\omega x}\, dx\, dt$$

$$= \int_{-\infty}^{+\infty} f(t) \left(\int_{-\infty}^{+\infty} g(x-t) e^{-i\omega x}\, dx \right) dt$$

$$\overset{u=x-t}{=\!=} \int_{-\infty}^{+\infty} f(t) \left(\int_{-\infty}^{+\infty} g(u) e^{-i\omega (u+t)}\, du \right) dt$$

$$= \int_{-\infty}^{+\infty} f(t) \cdot e^{-iwt} \left(\int_{-\infty}^{+\infty} g(u) \cdot e^{-iwu} \, du \right) dt$$

$$= \int_{-\infty}^{+\infty} f(t) \cdot e^{-iwt} \; G(w) \, dt$$

$$= G(w) \cdot \int_{-\infty}^{+\infty} f(t) \cdot e^{-iwt}$$

$$= \boxed{G(w) \cdot F(w)}$$

s

You can see the outputs of Q3 and Q5 codes in the "output" directory.