Computer vision

Sheet 01

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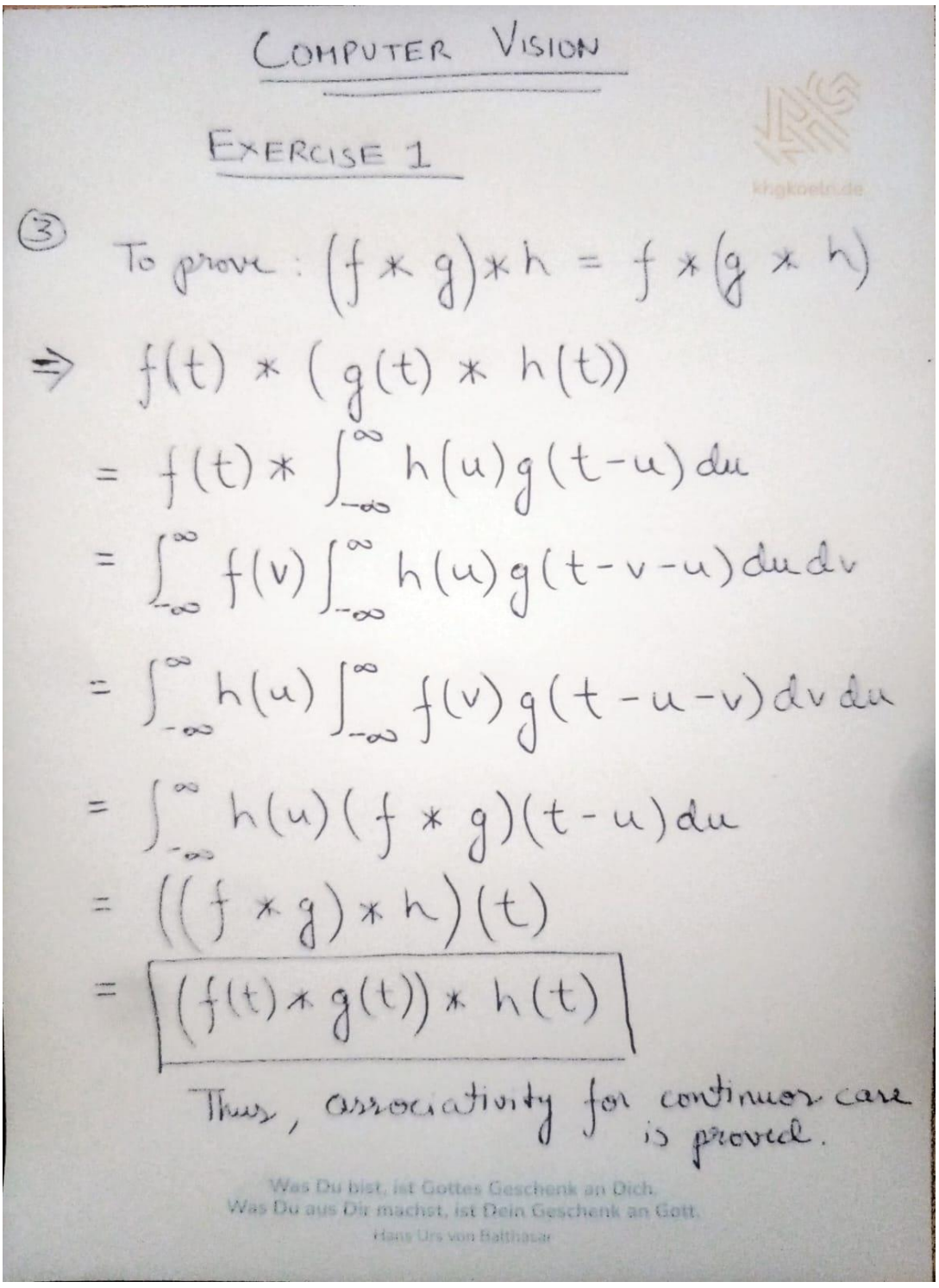
Q3



COMPUTER VISION

EXERCISE 1

(3) To prove: $(f * g) * h = f * (g * h)$

$\Rightarrow f(t) * (g(t) * h(t))$

$= f(t) * \int_{-\infty}^{\infty} h(u)\, g(t-u)\, du$

$= \int_{-\infty}^{\infty} f(v) \int_{-\infty}^{\infty} h(u)\, g(t-v-u)\, du\, dv$

$= \int_{-\infty}^{\infty} h(u) \int_{-\infty}^{\infty} f(v)\, g(t-u-v)\, dv\, du$

$= \int_{-\infty}^{\infty} h(u)\, (f * g)(t-u)\, du$

$= ((f * g) * h)(t)$

$= \boxed{(f(t) * g(t)) * h(t)}$

Thus, associativity for continuous case is proved.

Was Du bist, ist Gottes Geschenk an Dich.
Was Du aus Dir machst, ist Dein Geschenk an Gott.
Hans Urs von Balthasar

Q6

$$f(x) * G(x) * G(x) = f(x) * \left(G(x) * G(x) \right)$$

$$\rightarrow G(x) * G(x) = \int_{-\infty}^{+\infty} G(t) \times G(-t+x)\, dt =$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{(x-t)^2}{2\sigma^2}} dt =$$

$$= \frac{1}{2\pi\cdot\sigma^2} \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}\left[(t^2 + (x-t)^2) \right]} dt \quad (1)$$

(1) $\rightarrow$ $t^2 + (x-t)^2 = t^2 + x^2 - 2xt + t^2 =$

$$= 2t^2 - 2xt + x^2$$

first equasion $=$ $\frac{1}{2\pi\cdot\sigma^2} \int_{-\infty}^{+\infty} e^{-\frac{2t^2-2xt}{2\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}} dt$

$$= \underbrace{\frac{e^{-\frac{x^2}{2\sigma^2}}}{2\pi\sigma^2}}_{A} \int_{-\infty}^{+\infty} e^{\frac{xt-t^2}{\sigma^2}} dt = A \int_{-\infty}^{+\infty} e^{-\frac{2(t-\frac{x}{2})^2 - \frac{x^2}{4}}{\sigma^2}}$$

$$= A e^{\frac{x^2}{4\sigma^2}} \int_{-\infty}^{+\infty} e^{\frac{-1}{\sigma^2}\left(t-\frac{x}{2}\right)^2} dt = A e^{\left(\frac{x}{4\sigma}\right)^2} \cdot \sqrt{\frac{\pi}{\frac{1}{\sigma^2}}}$$

$$= A e^{\left(\frac{x}{4\sigma}\right)^2} \cdot \sigma \cdot \sqrt{\pi} = \frac{e^{-\frac{x^2}{2\sigma^2}} \cdot e^{\frac{x^2}{4\sigma^2}} \cdot \sigma\sqrt{\pi}}{2\pi\sigma^2}$$

$$= \frac{e^{-\frac{x^2}{2\sigma^2}}}{2\sqrt{\pi}\cdot\sigma} \longrightarrow \sigma_2 = \sqrt{2}\cdot\sigma \longrightarrow \frac{e^{-\frac{x^2}{2\sigma^2}}}{2\sigma\sqrt{\pi}} = \frac{e^{-\frac{x^2}{\sigma_2^2}}}{\sigma_2\sqrt{2\pi}}$$

$$\rightsquigarrow G_{\sigma}(x) * G_{\sigma}(x) = G_{\sigma\sqrt{2}}(x)$$

$$\rightsquigarrow f(x) * G_{\sigma}(x) * G_{\sigma}(x) = f(x) * G_{\sigma\sqrt{2}}(x)$$