# Computer Vision
Sheet 07

December 15, 2024

1 **Kalman Filter Implementation:** Consider a 2D signal with acceleration. The state vector includes position, velocity, and acceleration components: $x = [x, y, v_x, v_y, a_x, a_y]^T$ .

**System Model:**:

- Time Step: $\Delta t = 0.1$ seconds
- State transition matrix $\Psi$ incorporating acceleration terms:
$\Psi = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
- Process Noise Covariance ($\Sigma_p$):
Diagonal matrix with process noise parameter $sp = 0.001$
- Measurement Matrix ($\Phi$) observing only positions:
$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$
- Measurement Noise Covariance ($\Sigma_m$):
Diagonal matrix with measurement noise parameter $sm = 0.05$

1.1 What should be the time evolution equation? Explain each term.
**Answer:**
The time evolution equation describes how the state vector $x$ evolves over time:

$$x_k = \Psi x_{k-1} + w_k$$

**Explanation of Terms:**

- $x_k$**:** The state vector at time $k$, which includes position, velocity, and acceleration components $[x, y, v_x, v_y, a_x, a_y]^T$.
- $\Psi$**:** The state transition matrix, which propagates the state from time $k-1$ to $k$. It incorporates motion equations with acceleration:

$$\Psi = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

  Here, $\Delta t$ is the time step (0.1 seconds).
- $w_k$**:** Process noise, assumed to be Gaussian with covariance $\Sigma_p$, representing uncertainties in the system dynamics.

1.2 What should be the measurement equation? Explain each term.
**Answer:**
The measurement equation relates the observed measurements $z_k$ to the state vector $x_k$:

$$z_k = \Phi x_k + v_k$$

**Explanation of Terms:**

- $z_k$**:** The observation vector at time $k$, which includes the noisy 2D position measurements.
- $\Phi$**:** The measurement matrix, which maps the state vector to the observed space:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

  This indicates that only the $x$ and $y$ positions are observed.
- $v_k$**:** Measurement noise, assumed to be Gaussian with covariance $\Sigma_m$, representing uncertainties in the measurements.

1.3 Implement a Kalman filter to track the signal:
Initialize the state vector with $x_0 = [-10, -150, 1, -2, 0, 0]^T$ . Implement the prediction step (time update). and the correction step (measurement update).

**Answer:**

The Kalman filter algorithm includes two steps:

**1. Prediction Step (Time Update):**

$$\hat{x}_{k|k-1} = \Psi \hat{x}_{k-1|k-1}$$

$$P_{k|k-1} = \Psi P_{k-1|k-1} \Psi^T + \Sigma_p$$

**2. Correction Step (Measurement Update):**

$$K_k = P_{k|k-1} \Phi^T (\Phi P_{k|k-1} \Phi^T + \Sigma_m)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \Phi \hat{x}_{k|k-1})$$

$$P_{k|k} = (I - K_k \Phi) P_{k|k-1}$$

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