Computer Vision

Sheet 07

December 15, 2024

1 **Kalman Filter Implementation:** Consider a 2D signal with acceleration. The state vector includes position, velocity, and acceleration components: $x = [x, y, v_x, v_y, a_x, a_y]^T$.

System Model::

- Time Step: $\Delta t = 0.1$ seconds
- State transition matrix Ψ incorporating acceleration terms:
$$\Psi = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
- Process Noise Covariance (Σ_p):
Diagonal matrix with process noise parameter $sp = 0.001$
- Measurement Matrix (Φ) observing only positions:
$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
- Measurement Noise Covariance (Σ_m):
Diagonal matrix with measurement noise parameter $sm = 0.05$

1.1 What should be the time evolution equation? Explain each term.

Answer:

The time evolution equation describes how the state vector x evolves over time:

$$x_k = \Psi x_{k-1} + w_k$$

Explanation of Terms:

- x_k**:** The state vector at time k, which includes position, velocity, and acceleration components $[x, y, v_x, v_y, a_x, a_y]^T$.
- Ψ**:** The state transition matrix, which propagates the state from time $k-1$ to k. It incorporates motion equations with acceleration:
$$\Psi = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
Here, Δt is the time step (0.1 seconds).
- w_k**:** Process noise, assumed to be Gaussian with covariance Σ_p, representing uncertainties in the system dynamics.

1.2 What should be the measurement equation? Explain each term.

Answer:

The measurement equation relates the observed measurements z_k to the state vector x_k:

$$z_k = \Phi x_k + v_k$$

Explanation of Terms:

- z_k**:** The observation vector at time k, which includes the noisy 2D position measurements.
- Φ**:** The measurement matrix, which maps the state vector to the observed space:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

  This indicates that only the x and y positions are observed.
- v_k**:** Measurement noise, assumed to be Gaussian with covariance Σ_m, representing uncertainties in the measurements.

1.3 Implement a Kalman filter to track the signal:
Initialize the state vector with $x_0 = [-10, -150, 1, -2, 0, 0]^T$. Implement the prediction step (time update). and the correction step (measurement update).

Answer:

The Kalman filter algorithm includes two steps:

1. Prediction Step (Time Update):

$$\hat{x}_{k|k-1} = \Psi \hat{x}_{k-1|k-1}$$

$$P_{k|k-1} = \Psi P_{k-1|k-1} \Psi^T + \Sigma_p$$

2. Correction Step (Measurement Update):

$$K_k = P_{k|k-1} \Phi^T (\Phi P_{k|k-1} \Phi^T + \Sigma_m)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \Phi \hat{x}_{k|k-1})$$

$$P_{k|k} = (I - K_k \Phi) P_{k|k-1}$$

2 **Fixed lag smoothed implementation** As you can see in the chart, the output of smoothed implementation is closer to observations than Kalman filter's output.

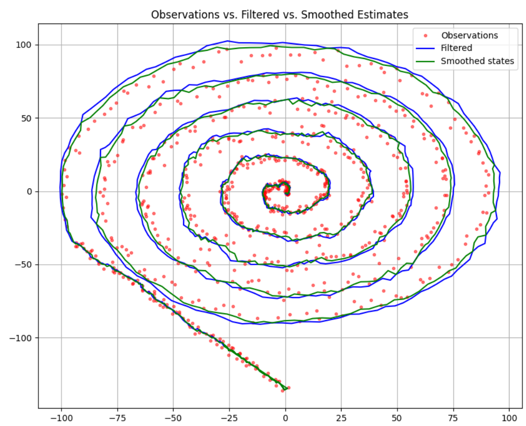

Figure 1: Observations vs. Filtered estimates vs. Smoothed estimates