Computer Vision

Exercise 5

November 27, 2024

1 A function is submodular when it satisfies the equation ...

Answer 1.1:

$$P(\beta,\gamma)+P(\alpha,\delta)-P(\beta,\delta)-P(\alpha,\gamma)=k((\beta-\gamma)^2+(\alpha-\delta)^2-(\beta-\delta)^2-(\alpha-\gamma)^2)=$$

$$k(\beta^2+\gamma^2-2\beta\gamma+\alpha^2+\delta^2-2\alpha\delta-(\beta^2+\delta^2-2\beta\delta+\alpha^2+\gamma^2-2\alpha\gamma))$$

$$-2k(\gamma(\beta-\alpha)-\delta(\beta-\alpha))=-2k(\beta-\alpha)(\gamma-\delta)$$

k is positive, $\beta>\alpha$ and $\gamma<\delta$ so we have negative*positive*negative which is **positive**. Therefore, this function **is sobmodular**.

Answer 1.2:

The δ function is:

$$\delta_{ij}=\begin{cases}1 & \text{if } x=0,\\ 0 & \text{O.W}\end{cases}$$

Then $P(\omega_m,\omega_n)$ is:

$$P(\omega_m,\omega_n)=\begin{cases}k(1-1)=0 & \text{if } \omega_m=\omega_n,\\ k(1-0)=k & \text{O.W}\end{cases}$$

So we should survey the equality of α and β to δ and γ.

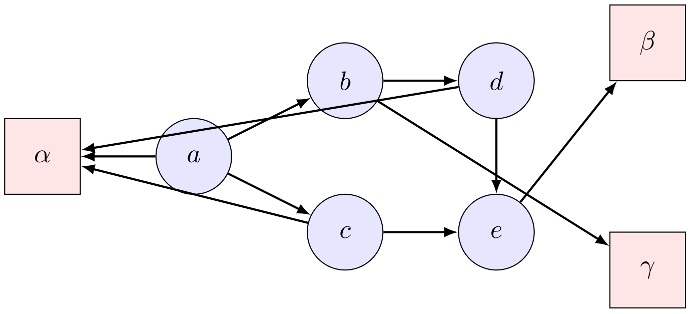
Since $\beta>\alpha$ and $\delta>\gamma$:

(a) if $\beta=\gamma$, then α cannot be equal to δ and γ.
(b) if $\alpha=\delta$, then β cannot be equal to δ and γ.
(c) $\alpha=\delta$ and $\beta=\gamma$ are not possible together.

In the case (a), we have $0+k-k-k=-k$, As k is positive, the result is negative and **we cannot say** this function is modular.

2 Provide a graph structure using the Alpha Expansion model for the 5 nodes (a,b,c,d,e) with states before: $\beta|\gamma|\alpha|\alpha|\beta$ where the label α is expanded.

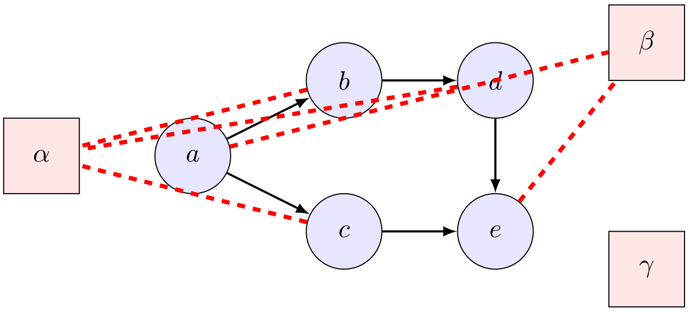
Answer:



3 Show the cut on the graph in Question 2 with states after: $\beta|\gamma|\alpha|\alpha|\beta$. Write down the total cost which includes unary and pairwise costs.

Answer:

Graph with Cut for States $\beta|\alpha|\alpha|\alpha|\beta$



States After the Cut

- $a \to \beta$ (retained)
- $b \to \alpha$ (changed)
- $c \to \alpha$ (changed)
- $d \to \alpha$ (changed)
- $e \to \beta$ (retained)

Total Cost Calculation

The total cost C includes:

- Unary Costs:

$$U(a,\beta) = U_a^\beta, \quad U(b,\alpha) = U_b^\alpha, \quad U(c,\alpha) = U_c^\alpha, \quad U(d,\alpha) = U_d^\alpha, \quad U(e,\beta) = U_e^\beta$$

- Pairwise Costs:

$$\begin{aligned} &P(a,b,\beta,\alpha) = P_{ab}^{\beta\alpha}, \quad P(a,c,\beta,\alpha) = P_{ac}^{\beta\alpha}, \\ &P(b,d,\alpha,\alpha) = P_{bd}^{\alpha\alpha}, \quad P(c,e,\alpha,\beta) = P_{ce}^{\alpha\beta}, \\ &P(d,e,\alpha,\beta) = P_{de}^{\alpha\beta} \end{aligned}$$

The total cost C is:

$$C = \sum_{\text{nodes } x} U(x, l_x) + \sum_{\text{edges } (x,y)} P(x, y, l_x, l_y)$$

Substituting the given states:

$$C = U_a^\beta + U_b^\alpha + U_c^\alpha + U_d^\alpha + U_e^\beta + P_{ab}^{\beta\alpha} + P_{ac}^{\beta\alpha} + P_{bd}^{\alpha\alpha} + P_{ce}^{\alpha\beta} + P_{de}^{\alpha\beta}$$