AN ITERATIVE ALGORITHM FOR OPTIMAL DESIGN OF NON-FREQUENCY-SELECTIVE FIR DIGITAL FILTERS[1]

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Abstract This paper proposes a novel iterative algorithm for optimal design of non-frequency-selective Finite Impulse Response (FIR) digital filters based on the windowing method. Different from the traditional optimization concept of adjusting the window or the filter order in the windowing design of an FIR digital filter, the key idea of the algorithm is minimizing the approximation error by successively modifying the design result through an iterative procedure under the condition of a fixed window length. In the iterative procedure, the known deviation of the designed frequency response in each iteration from the ideal frequency response is used as a reference for the next iteration. Because the approximation error can be specified variably, the algorithm is applicable for the design of FIR digital filters with different technical requirements in the frequency domain. A design example is employed to illustrate the efficiency of the algorithm.

Key words Finite Impulse Response (FIR) digital filters; Optimal design; Windowing method; Approximation error; Iterative algorithm

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I. Introduction

Finite Impulse Response (FIR) filters play an important role in signal processing because of their extraordinary performance when compared with Infinite Impulse Response (IIR) filters. FIR filters can have the linear phase characteristic that is desired for pulse and data transmission, and can offer complete stability at all frequencies[1,2]. These filters are mainly used for two important reasons, separation of signals that are combined together and restoration of signals that are distorted due to some reasons. There are three basic methods for the design of FIR digital filters, namely windowing, frequency sampling, and minimax (optimum approximation) design[3]. Windowing is a simple method of obtaining the coefficients of an FIR filter and involves a minimal amount of calculation. But the method is suboptimal, *i.e.*, the filter order needed to satisfy a given set of prescribed specifications is not the lowest. In the frequency sampling method, the desired frequency response is sampled at equally-spaced points, and the result is inverse discrete Fourier transformed. The method only guarantees correct frequency response values at the points that were sampled. This sometimes leads to excessive ripples at intermediate points. Minimax design yields a filter which is optimum in the minimax sense, *i.e.*, the maximum error relative to the specification is minimized. Solutions to minimax design have been explored in many literatures[4,5]. Parks-McClellan algorithm is now the most widely used approach for minimax design which converts the filter design problem into a polynomial approximation problem. The algorithm is more sophisticated than the windowing and frequency sampling methods, but generally gives more satisfactory results.

In recent years, many approaches to optimal design of FIR digital filters based on the above three methods have been developed[6–10]. But these approaches are mainly applicable for frequency-selective FIR digital filters (lowpass filters, highpass filters, bandpass filters, *etc.*), and the design

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specifications of this kind of filters are commonly characterized by properties like sharp roll-off, passband ripple and stopband attenuation. They are not very suitable for the design of non-frequency-selective FIR digital filters such as FIR differentiators, FIR Hilbert transformers, and inverse-sinc compensation filters[11]. This paper proposes a novel iterative algorithm for optimal design of non-frequency-selective FIR digital filters based on the windowing method. The algorithm minimizes the approximation error by successively modifying the design result through an iterative procedure under the condition of a fixed window length. In the iterative procedure, the known deviation of the designed frequency response in each iteration from the ideal frequency response is used as a reference for the next iteration. Because the approximation error can be specified variably, the algorithm is applicable for the design of FIR digital filters with different technical requirements in the frequency domain.

II. The Fundamental and Disadvantages of the Windowing Method

In the design of an FIR filter, the ideal impulse response is often non-causal. Let $h_d(n)$ denote the ideal impulse response and $H_d(e^{jw})$ the ideal frequency response. When $H_d(e^{jw})$ is given, $h_d(n)$ could be determined by the inverse Fourier transform.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})e^{jw}d\omega \tag{1}$$

In the windowing method, the FIR filter is obtained by multiplying a window $w(n)$ with $h_d(n)$ to obtain a finite duration $h(n)$ of length N. This is required since $h_d(n)$ will in general be an infinite duration sequence, and the corresponding filter will therefore not be realizable[3]. If $h_d(n)$ is even or odd symmetric and $w(n)$ is even symmetric, the designed filter with the impulse response $h(n)$ is a linear phase filter. And $h(n)$ can be expressed as

$$h(n) = h_d(n)w(n) \tag{2}$$

Truncation by a window maps to convolution in the frequency domain which essentially introduces the non-ideal transition and the ripple effect in the passband and stopband for a frequency-selective digital filter[12]. The type of the window used affects these filter characteristics. And two important factors are the length and shape of the window $w(n)$. In practice, the window shape is chosen first based on passband and stopband tolerance requirements. The window size is then determined based on transition width requirements[11]. Let $W(e^{jw})$ denote the Fourier transform of $w(n)$ and $H(e^{jw})$ the Fourier transform of $h(n)$, then

$$\begin{aligned} H(e^{jw}) &= \sum_{n=0}^{N-1} h(n)e^{-jwn} \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta \end{aligned} \tag{3}$$

For any given window length, we would like $W(e^{jw})$ to be most like an impulse, with a narrow main lobe and low sidelobes (The main lobe width determines the transition band width and the sidelobe level affects the rejection characteristics and the ripple effect). The commonly used windows include rectangular, Bartlett, Hanning, Hamming, Blackman and Kaiser windows. Among them, the rectangular window has the narrowest main lobe; it gives the sharpest transition at the discontinuity of $H_d(e^{j\omega})$. The Bartlett window is triangular, with a slope discontinuity at its center, while the Hanning, Hamming and Blackman windows are smoother.

By tapering the window smoothly to zero, the sidelobes can be reduced in amplitudes, but the trade-off is the larger main lobes. The Kaiser windows are a family of near optimal windows that allow controlled trade-offs between the sidelobe amplitudes and main lobe widths. All of these windows are symmetric, hence their frequency responses have the linear phase characteristic.

It should be emphasized that the windowing design is usually performed iteratively, since it is difficult to predict the extent to which the transition band of the window will smear the frequency response[3]. To design a frequency-selective FIR filter to meet a specific frequency response, one computes the sequence of the desired impulse response by taking the inverse Fourier transform, applies the window, and then computes the actual response by taking the Fourier transform. The band edges or filter order are adjusted as necessary, and the process is repeated. Many approaches have been developed to perform this process by computer. But the windowing design does not generally yield the lowest possible order filter to meet the specifications.

III. An Iterative Algorithm for the Optimal Design

While the classical windowing method is very simple and reliable, the designs it produces are generally inferior to those produced by algorithms that employ some optimization criteria[13]. In this section, a novel iterative algorithm for optimal design of FIR digital filters based on the windowing method is presented, which is especially suitable for designing non-frequency-selective filters. The algorithm minimizes the approximation error through iterations, and the approximation error could be specified variably according to different technical requirements in the frequency domain. When performing the optimal design, the window length is firstly set and fixed, and then the approximation error is successively reduced by modifying the design result. And the known deviation of the designed frequency response in each iteration from the ideal frequency response is used as a reference for the next iteration. The parameters needed to be set and adjusted in the algorithm are few, and it could be easily implemented through programming by computer, and thus has high practicability.

Before performing the iterations, the approximation error needs to be specified according to the technical requirements and the characteristics of the desired filter. For a frequency-selective FIR digital filter, the approximation error R could be specified as

$$R = \max_{\omega \in A} \left| Q(\omega)[H_d(e^{j\omega}) - H(e^{j\omega})] \right| \tag{4}$$

where $Q(\omega)$ is an emphasis/de-emphasis weighting function, and A is a subset of frequencies from 0 to π. For a non-frequency-selective FIR digital filter, the approximation error could be specified variably. For example, the approximation error R could be specified as follows for an inverse-sinc compensation filter.

$$R = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H_d(e^{j\omega}) - H(e^{j\omega}) \right| d\omega \tag{5}$$

Then, the iterations could be performed basically as follows.

Step 1 Fix the window length N, the window $w(n)$, and the limit of the number of iterations M. Here N is equal to the length of the truncated impulse response which determines the filter order.

Step 2 Let $\bar{H}(e^{j\omega}) = H_d(e^{j\omega})$, and take the inverse Fourier transform

$$\begin{aligned} \bar{h}(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \bar{H}(e^{j\omega}) e^{jn\omega} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{jn\omega} d\omega \end{aligned} \tag{6}$$

Step 3 Let $h_0(n) = \bar{h}(n) w(n)$, $d_0(n) = 0$, $D_0(e^{jw}) = 0$, $i = 1$, and take the Fourier transform

$$H_0(e^{jw}) = \sum_{n=0}^{N-1} h_0(n) e^{-jwn} \tag{7}$$

Step 4 Let $h_i(n) = h_{i-1}(n) + d_{i-1}(n)$, and $H_i(e^{jw}) = H_{i-1}(e^{jw}) + D_{i-1}(e^{jw})$.

Step 5 Let $\bar{D}_i(e^{jw}) = H_d(e^{jw}) - H_i(e^{jw})$, and take the inverse Fourier transform

$$\bar{d}_i(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \bar{D}_i(e^{j\omega}) e^{jn\omega} d\omega \tag{8}$$

Step 6 Let $d_i(n) = \bar{d}_i(n) w(n)$, and take the Fourier transform

$$D_i(e^{j\omega}) = \sum_{n=0}^{N-1} d_i(n) e^{-jn\omega} \tag{9}$$

Step 7 Regard $H_i(e^{j\omega})$ as a designed frequency response, and compute the approximation error according to Eq.(4), Eq.(5), or other specifications. Let R_i denote the approximation error corresponding to $H_i(e^{j\omega})$. When Eq.(4) is employed, R_i could be computed as

$$R_i = \max_{\omega \in A} \left| Q(\omega)[H_d(e^{j\omega}) - H_i(e^{j\omega})] \right| \tag{10}$$

When Eq.(5) is employed, R_i could be computed as

$$R_i = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H_d(e^{j\omega}) - H_i(e^{j\omega}) \right| d\omega \tag{11}$$

Step 8 If $i = 1$, then $i := i + 1$ (the value of i is increased by 1), and go to Step 4. If $i = M$, stop the algorithm, and let $H_{i-1}(e^{j\omega})$ and $h_{i-1}(n)$ be the final design result, *i.e.*, let $H(e^{j\omega}) = H_{i-1}(e^{j\omega})$,and $h(n) = h_{i-1}(n)$. If $1 < i < M$, then go to Step 9.

Step 9 If $R_i < R_{i-1}$, then $i := i + 1$ (the value of

i is increased by 1), and go to Step 4. If $R_i \geq R_{i-1}$, stop the algorithm, and let $H_{i-1}(e^{j\omega})$ and $h_{i-1}(n)$ be the final design result, *i.e.*, let $H(e^{j\omega}) = H_{i-1}(e^{j\omega})$, and $h(n) = h_{i-1}(n)$.

In the application of the iterative algorithm described above, the following issues should be comprehended and considered.

(1) When changing the window $w(n)$ in Step 1, the final designed frequency response and the optimization effect will change. We should fix a suitable window in Step 1 according to $H_d(e^{j\omega})$, N and the specified approximation error, and thus make $H_0(e^{j\omega})$ approximate $H_d(e^{j\omega})$ to a great extent in the beginning of the iterations. If the suitable window is difficult to be found out, we can perform the algorithm using each of the commonly used windows respectively, and select the best design from the results.

(2) During performing the algorithm, the approximation error is minimized by successively reducing the magnitude (modulus) of $\overline{D}_i(e^{j\omega})$ (Here, $\overline{D}_i(e^{j\omega})$ is an error function which represents the deviation between the designed frequency response in each iteration and the ideal frequency response). And in each iteration, the modification of the designed frequency response is accomplished by designing another filter with the frequency response $\overline{D}_i(e^{j\omega})$ by windowing. But the window $w(n)$ in Step 6 is fixed and is not always optimal for $\overline{D}_i(e^{j\omega})$ and $\overline{d}_i(n)$. So we could use a variable window when executing Step 6 in practice, and ensure that the window is optimal for $\overline{D}_i(e^{j\omega})$ and $\overline{d}_i(n)$. This can improve the optimization effect.

(3) By analyzing design results in practical applications of the algorithm, we discovered that with the increment of i which is the number of iterations, the corresponding approximation error R_i does not always decrease, but fluctuates sometimes. This is mainly determined by $H_d(e^{j\omega})$, $w(n)$ and the specified approximation error. So when executing Step 9, If $R_i \geq R_{i-1}$, we could continue the iterations until $i = M$, and then select the best $H_i(e^{j\omega})$ and $h_i(n)$ as the final design result (Here, $0 \leq i \leq M$).

(4) It can be proved that if $w(n)$ fixed in Step 1 is a rectangular window, the value of $d_i(n)$ in Step 6 will equal 0 regardless of the value of i. Namely the temporary designed frequency response $H_i(e^{j\omega})$ in Step 7 and Step 4 will keep changeless with the increment of the number of iterations. So the algorithm is ineffective for rectangular windows.

(5) When designing linear phase filters, we can fix the magnitude (modulus) of $\overline{H}(e^{j\omega})$ arbitrarily while keeping the symmetry of $\overline{h}(n)$ in Step 2. Different magnitudes of $\overline{H}(e^{j\omega})$ will result in different design results. The optimization effect could be improved by adjusting the magnitude of $\overline{H}(e^{j\omega})$ in practice.

IV. Case Studies

In this section, the optimization effect of the proposed algorithm is examined by the design of an inverse-sinc compensation filter which is used to compensate the droop of the sinc envelope caused by a digital to analog converter in a telecommunication system[14]. The filter has a linear phase response, and the desired frequency response is

$$H_d(e^{j\omega}) = [\omega/(2\sin(\omega/2))]\exp(-j\omega a), \quad |\omega| \leq \pi \tag{12}$$

According to $H_d(e^{j\omega})$, the window length N should be odd. In the optimal design, we use $N = 11$, and perform the algorithm using a Hamming window and a Blackman window respectively. The Hamming window could be expressed as

$$w(n) = 0.54 - 0.46\cos[2\pi n/(N-1)], \quad 0 \leq n \leq N-1 \tag{13}$$

And the Blackman window could be expressed as

$$w(n) = 0.42 - 0.5\cos[2\pi n/(N-1)] + 0.08\cos[4\pi n/(N-1)], \quad 0 \leq n \leq N-1 \tag{14}$$

Fig.1 and Fig.2 illustrate the approximation error R_i computed in performing the algorithm using the Hamming window and the Blackman window respectively. In the two figures, the ring marks display the values of the approximation error specified by Eq.(10) where $Q(\omega) = 0.1$, and the star marks display the values of the approximation error specified by Eq.(11). As is shown in the figures, the approximation error specified by Eq.(10) decreases with the increment of the number of iterations i, while the error specified by Eq.(11) fluctuates with the increment of i. Fig.3 and Fig.4

display the designed magnitude frequency response $|H_i(e^{j\omega})|$ for $i=1$ and $i=30$ respectively, and the designed magnitude frequency response is computed in performing the algorithm using the Hamming window. In the two figures, the lines with “+” display the desired magnitude frequency response and the lines with “☆” display the designed magnitude frequency responses.

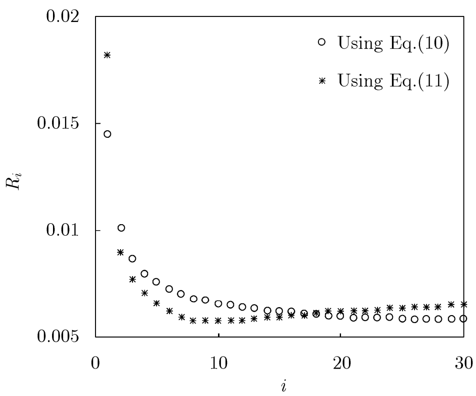

Fig.1 The approximation error for the Hamming window

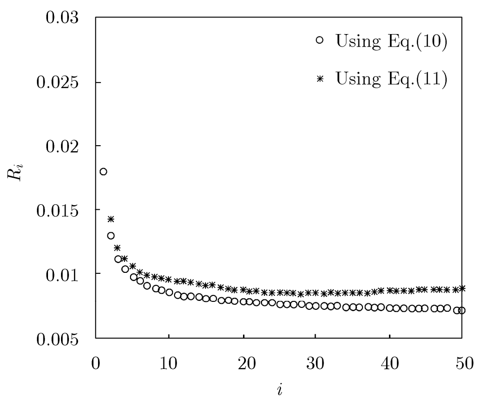

Fig.2 The approximation error for the Blackman window

As is shown in Fig.3 and Fig.4, the designed magnitude frequency response for $i=30$, namely $|H_{30}(e^{j\omega})|$, is much more satisfactory than that for $i=1$. The designed magnitude frequency response for $i=1$, namely $|H_1(e^{j\omega})|$, is the same as the designed magnitude frequency response produced directly by basic windowing method using the Hamming window. So the proposed algorithm can achieve evident optimization effect after 30 iterations. In order to test the effect of the window type and the number of iterations on the approximation error, we performed the algorithm using different windows and different numbers of iterations. The values of the approximation error specified by Eq.(11) in the test are presented in Tab.1.

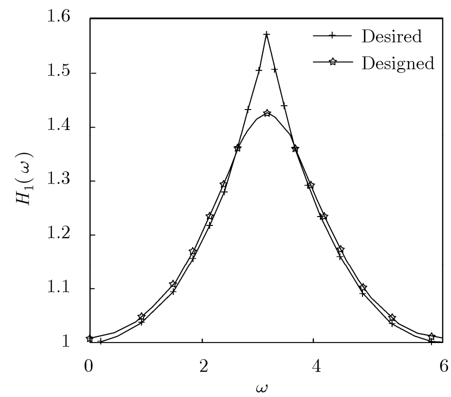

Fig.3 The designed magnitude frequency response for $i=1$

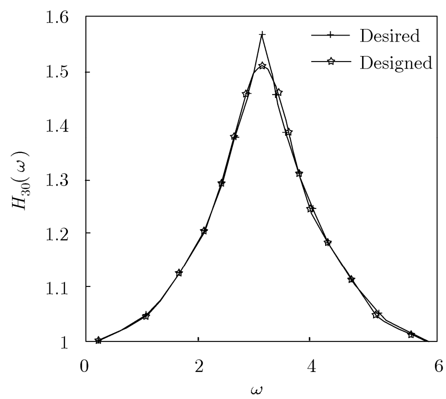

Fig.4 The designed magnitude frequency response for $i=30$

Tab.1 The approximation error in the test

Window	$i=1$	$i=15$	$i=30$	$i=60$
Bartlett	0.0195	0.0063	0.0065	0.0067
Hanning	0.0162	0.0058	0.0062	0.0065
Rectangular	0.0069	0.0069	0.0069	0.0069
Kaiser	0.0152	0.0058	0.0061	0.0064

The results in Tab.1 show that the optimization effect of the algorithm is evident for Bartlett, Hanning, and Kaiser windows, but except for the rectangular window. Furthermore, the approximation error in the test decreases generally when i increases from 1 to 15, and the error changes little when i increases on from 15. So, the limit of the number of iterations M in Step 1 could be set to be more than 15 in practical applications of the algorithm.

V. Conclusions

In recent years, many computationally sophisticated algorithms for optimal design of FIR digital filters have been developed, but there are few well-accepted algorithms in widespread use for non-frequency-selective filters. This paper proposed a novel iterative algorithm based on the windowing method for this problem. The key optimization concept of the algorithm is minimizing the approximation error by successively modifying the design result through an iterative procedure under the condition of a fixed window length. And the approximation error could be specified variably according to different technical requirements in the frequency domain. Our empirical results demonstrate that it is a promising algorithm for FIR digital filter design, and especially for optimal design of non-frequency-selective filters.

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