OR - HW #3   ٨١.١٧٧٤٢٨   باسمه تعالی

$$\text{Min } J = 3x_1 - x_2 - 3x_3 \qquad (٣$$

$$\text{s.t} \begin{cases} -x_1 + x_2 + 2x_3 \leq 6 \\ x_1 - \frac{1}{3}x_2 - x_3 \geq -3 \\ 2x_1 + x_2 = 3 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

$$\Rightarrow \text{Min } J = 3x_1 - x_2 - 3x_3$$

$$\text{s.t} \begin{cases} -x_1 + x_2 + 2x_3 + x_4 = 6 \\ x_1 - \frac{1}{3}x_2 - x_3 - x_5 = -3 \\ 2x_1 + x_2 = 3 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

$$-x_1 + x_2 + 2x_3 + x_4 = 6 \rightarrow x_2, x_3, x_4 \rightarrow \underline{x_4}$$

$$x_1 - \frac{1}{3}x_2 - x_3 - x_5 = -3 \rightarrow x_2, x_3, x_5 \rightarrow \underline{x_5}$$

$$2x_0 + x_2 = 3 \rightarrow x_1, x_2 \rightarrow \underline{x_1}$$

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	x_4	x_5	x_1	x_2	x_3	RHS
$-Z$	0	0	3	-1	-3	0
x_4	1	0	-1	1	2	6
x_5	0	-1	1	$-1/3$	-1	$3-3 \times \frac{-1}{3}$
x_1	0	0	2	$1/2$	0	$3 \frac{3}{2} \times 1/2$

	x_4	x_5	u_1	u_2	u_3	RHS
$=> -Z$	0	0	3	-1 $-2,5$	-3 -3	$\frac{-9}{2}$
x_4	1	0	-1	1 $3/2$	2 2	$\frac{15}{2}$ 6
x_5	0	1	0	$5/6$	1	$9/2$
u_1	0	0	1	$1/2$	0	$3/2$

$\times -3$

	x_4	x_5	u_1	u_2	u_3	RHS
$=> -Z$	0	0	0	$-2,5$	(-3)	$-4,5$
x_4	1	0	0	$1,5$	$\frac{2}{5}$	$7,5$
x_5	0	1	0	$5/6 \approx 0,83$	1	$4,5$
u_1	0	0	1	$0,5$	0	$1,5$

	x_3	x_5	u_1	u_2	u_4	RHS
$=> -Z$	-3	0	0	$-2,5 + \frac{4,5}{2}$	$3/2$	$-4,5 + \frac{3 \times 7,5}{2}$
$1/2 \times u_3$	2 1	0	0	$1,5$ $\frac{1,5}{2}$	$1 \frac{1}{2}$	$7,5$ $\frac{7,5}{2}$
x_5	1	1	1	$5/6$ $\frac{1,5}{2}$	$-1/2$	$4,5$ $4,5 - \frac{7,5}{2}$
u_1	0	0	1	$0,5$	0	$1,5$

	x_3	x_5	x_1	x_2	x_4	RHS
$-Z$	0	0	0	(-0.25)	1.5	6.75
x_3	1	0	-	0.75	0.5	3.75
x_5	0	1	0	$1/12$	-0.5	0.75
x_1	0	0	1	$\frac{0.5}{5}$	0	1.5

$\Longrightarrow$

	x_3	x_5	x_2	x_1	x_4	RHS
$-Z$	0	0	-0.25 0	0.5	1.5	6.75 7.5
x_3	1	0	0.75 0	-1.5	0.5	3.75 1.5
x_5	0	1	$1/12$ 0	$-\frac{1}{6}$	-0.5	0.75 0.5
x_2	0	0	0.5 1	2	0	1.5 3

$\Longrightarrow$

	x_3	x_5	x_2	x_1	x_4	RHS
$-Z$	0	0	0	0.5	1.5	7.5
x_3	1	0	0	-1.5	0.5	1.5
x_5	0	1	0	$-\frac{1}{6}$	-0.5	0.5
x_2	0	0	1	2	0	3

$\Longrightarrow$ in $\begin{cases} x_3^* = 1.5 \\ x_5^* = 0.5 \\ x_2^* = 3 \end{cases}$ out $\begin{cases} x_1^\alpha = 0 \\ x_4^\alpha = - \end{cases}$ feď dines $\Longrightarrow$ $\begin{cases} J^* = -7.5 \\ x_3^* = 1.5 \\ x_2^\alpha = 3 \\ x_1^{\alpha} = 0 \end{cases}$

$\int \min J = J^\alpha = -7.5$

$\alpha_1, \beta_1, y_3 = ?$ → x_3 حذف   12

	x_1	x_2	x_3	S_1	S_2	S_3	RHS
Z	0	b	0	-2	+3	0	a
x_1	1	-4	0	9	1	0	y_1
x_3	0	α_1	1	-6	-1	0	1
S_3	0	β_1	0	0	2	1	y_3

$\{y_1, y_3, a, b, \alpha_1, \beta_1\} \geq 0$

$\Rightarrow$

	x_1	x_3	S_3	S_1	S_2	x_2	RHS
-Z	0	0	0	+2	$\boxed{-3}$	+b	-a
x_1	1	0	0	9	1	-4	y_1
x_3	0	1	0	-6	-1	α_1	1
S_3	0	0	1	0	$\frac{2}{3}$	β_1	y_3

$\longrightarrow \frac{y_3}{2} < y_1$

	x_1	x_3	S_2	S_1	S_3	x_2	RHS
-Z	0	0	-3/0	2	$\emptyset 1.5$	$b+\frac{\beta_1}{2}\times 3$	$-a+\frac{3}{2}y_3$
x_1	1	0	-1/0	9	$\emptyset -1.5$	$-4-\frac{\beta_1}{2}$	$y_1-\frac{y_3}{2}$
x_3	0	1	-1/0	-6	$\emptyset\frac{1}{2}$	$\alpha_1\,\alpha_1+\frac{\beta_1}{2}$	$1+\frac{y_3}{2}$
S_2	0	0	2/1	0	$\emptyset\frac{1}{2}$	$\beta_1\,\frac{\beta_1}{2}$	$y_3\,\frac{y_3}{2}$

$\longrightarrow \dot{X}$

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$$\left\{ y_1 < \frac{y_3}{2} \right.$$

	S_2	x_3	S_3	S_1	x_1	x_2	RHS
-2	$\not> 0$	0	0	229	$\not> 3$	$\cancel{b-12}$	$\not> a - a + 3y_1$
S_2	1	0	0	9	1	-4	y_1
x_3	$\not> 0$	1	0	$\not> 3$	0 1	$\cancel{\alpha_1 - 4} \not X \, 1 + y_1$	
S_3	$\not> 0$	0	1	$0-18$	$0-2$	β_1	$y_3 \; y_3 - 2y_1$
						$\beta_1 + 8$	

if $b - 12 < 0 \rightarrow \left\{ b < 12 \right.$

$\&$ $\left\{ \dfrac{1 + y_1}{\alpha_1 - 4} < \dfrac{y_3 - 2y_1}{\beta_1 + 8} \right\} \rightarrow \dfrac{x_3 \; out}{x_2 \; in}$

$\&$ $\alpha_1 - 4 > 0 \Rightarrow \left\{ \alpha_1 > 4 \right.$

$$\Rightarrow \begin{cases} y_1 < \dfrac{y_3}{2} & \text{(I)} \\[2mm] b < 12 & \text{(II)} \\[2mm] \alpha_1 > 4 & \text{(III)} \\[2mm] \dfrac{1 + y_1}{\alpha_1 - 4} < \dfrac{y_3 - 2y_1}{\beta_1 + 8} & \text{(VI)} \end{cases}$$

$$\text{Max } J = 12x_1 + 8x_2 \tag{1}$$

$$\text{s.t} \begin{cases} 5x_1 + x_2 \leqslant 15. \\ x_1 + 3x_2 \leqslant 1.. \\ 4x_1 + x_2 \leqslant 8. \\ \forall i \in \{1,2\} \to x_i \geqslant. \end{cases}$$

$\to \text{Min } J' = -J = -12x_1 - 8x_2$

$$\text{s.t} \begin{cases} 5x_1 + x_2 + x_3 = 15. \to x_1, x_2, x_3 \\ x_1 + 3x_2 + x_4 = 1.. \to x_1, x_2, x_4 \\ 4x_1 + x_2 + x_5 = 8. \to x_1, x_2, x_5 \\ \forall i \in \{1,2,3,4,5\} \to x_i \geqslant. \end{cases}$$

	x_3	x_4	x_5	x_1	x_2	RHS
$-Z$	0	0	0	(-12)	-8	0
x_3	1	0	0	5	1	15.
x_4	0	1	0	1	3	1..
x_5	0	0	1	4	1	8.

ROHAM

mplex Dual

$-2''' = -3\lambda_1 - 3\lambda_2$

$1 - \lambda_2 + \lambda_3 = 2$

$-2\lambda_2 + \lambda_4 = 1$

$\lambda_3, \lambda_4 \geq 0$

(1%bl)

	x_3	x_4	x_1	x_5	x_2	RHS
$-z$	0	0	-12	3	-8 $\boxed{-5}$	24
x_3	1	0	5	$-5/4$ $\cdot 1.25$		15 5
x_4	0	1	1	$-1/4$ 32.75	1	8
x_1	0	0	1	$1/4$	$1/4$	2

~~~~~~~~~~~~~~

	x_3	x_2	x_1	x_5	x_4	RHS
$-z$	0	-5	0	$3 - \frac{5}{9}$	$\frac{2}{9}$	$24 + \frac{2 \cdot x \cdot 8}{9}$
x_3	1	$-1/4$	0	$-\frac{5}{4} - \frac{1}{36}$	$1/9$	$5 + \frac{8}{9}$
x_2	0	$\frac{4}{1}$	0	$-\frac{1}{4} - \frac{1}{9}$	$4/9$	$8 - \frac{4 \times 8}{9}$
x_1	0	$\frac{1}{4}$	1	$\frac{1}{4} + \frac{1}{36}$	$1/9$	$2 - \frac{8}{9}$

$\Rightarrow$

$\begin{cases} x_3^* = 58,89 \\ x_2^* = 35,56 \\ x_1^* = 11,11 \end{cases}$
$\begin{cases} x_5^* = 0 \\ x_4^* = 0 \end{cases}$

$\{ J'^* = -417,78 \longrightarrow \{ J^* = -J'^* = 417,78$

$\Rightarrow \begin{cases} x_1^* = 11,11 \\ x_2^* = 35,56 \\ \\ J^* = 417,78 \end{cases}$

Max $Z = -2x_1 - x_2$ (4

s.t $\begin{cases} -3x_1 - x_2 \le -3 \\ -4x_1 - 3x_2 \le -6 \\ -x_1 - 2x_2 \le -3 \\ x_1, x_2 \ge 0 \end{cases}$ → simplex Dual

$\Rightarrow$ Min $z' = -z = 2x_1 + x_2$

s.t $\begin{cases} +3x_1 + x_2 \ge 3 \rightsquigarrow \lambda_1 \\ \\ +x_1 + 2x_2 \ge 3 \rightsquigarrow \lambda_2 \\ x_1, x_2 \ge 0 \end{cases}$

→ Min $z''' = -3\lambda_1 - 3\lambda_2$

Dual

→ Max $z'' = 3\lambda_1 + 3\lambda_2$

s.t $\begin{cases} 3\lambda_1 + \lambda_2 \le 2 \rightarrow 3\lambda_1 + \lambda_2 + \lambda_3 = 2 \\ \lambda_1 + 2\lambda_2 \le 1 \rightarrow \lambda_1 + 2\lambda_2 + \lambda_4 = 1 \\ \\ \lambda_1, \lambda_2 \ge 0 \rightarrow \lambda_1, \lambda_2, \lambda_3, \lambda_4 \ge 0 \end{cases}$

	x_3
$-z$	0
x_3	1
x_4	0
x_1	0

	x_3
$-z$	0
x_3	1
x_2	0
x_1	0

$\Rightarrow \begin{cases} x_3^* = \\ x_2^* = \\ x_1^* = \end{cases}$

$\begin{cases} J'^* = \end{cases}$

$\Rightarrow$

Min $z' = -3\lambda_1$ ∓ $3\lambda_2$

s.t
$\begin{cases} 3\lambda_1 + \lambda_2 + \lambda_3 = +2 \longrightarrow \dfrac{\lambda_3}{?} \\[2mm] \lambda_1 + 2\lambda_2 + \lambda_4 = +1 \longrightarrow \underline{\lambda_4} \\[2mm] \forall i \in \{1,2,3,4\} \longrightarrow \lambda_i \geq 0 \end{cases}$

	λ_3	λ_4	λ_1	λ_2	RHS
$-Z'$	0	0	(−3)	−3	0
λ_3	1	0	3	1	2
λ_4	0	1	1	2	1

	λ_1	λ_4	λ_3	λ_2	RHS
$-Z$	−3 0	0	1	−3 (−2)	0 2
λ_1	3 1	0	1/3	1/3	2/3
λ_4	1 0	1	−1/3	2−1/3 5/3	1−2/3 1/3

	λ_1	λ_2	λ_3	λ_4	RHS
$-Z$	0	−2 0	1+2/5	6/5	2+2/5
λ_1	1	1/3	$1/3-\frac{1}{15}$	−1/5	$2/3-1/15$
λ_2	0	5/3 1	$-1/5-1/5$	+3/5	1/3 1/5

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$$\begin{cases} \lambda_1^* = \dfrac{3}{5} \\ \lambda_2^* = 1/5 \end{cases} \qquad \begin{cases} \lambda_3^*, \lambda_4^* = 0 \end{cases}$$

$$\begin{cases} J''^*_{\min} = -2{,}4 \end{cases} \xrightarrow{\;\text{max}\;} +2{,}4 \xrightarrow{\;\text{min}\;} -2{,}4$$

$$\Rightarrow \begin{cases} 3\lambda_1^* + \lambda_2^* + \lambda_3^* = +2 \longrightarrow \underline{\text{active}} \\[2mm] \lambda_1^* + 2\lambda_2^* + \lambda_4^* = +1 \longrightarrow \underline{\text{active}} \end{cases}$$

$$\Rightarrow \begin{cases} -3x_1^* - x_2^* = -3 \\[2mm] -x_1^* - 2x_2^* = -3 \end{cases} \Rightarrow \begin{cases} x_1^* = ,6 \\ x_2^* = 1{,}2 \end{cases}$$

$$\Rightarrow \begin{cases} z^* = \dfrac{-2{,}4}{5} \end{cases}$$

Right page:

$$\text{Min } z' = -3\lambda_1 \qquad -3\lambda_2$$

$$\text{s.t} \begin{cases} 3\lambda_1 \qquad +\lambda_2 - \lambda_3 \\ \lambda_1 \qquad +2\lambda_2 - \lambda_4 \\ \forall i \in \{1,2,3,4\} \end{cases} \longrightarrow$$

	λ_3	λ_4	λ_1
$-z'$	0	0	-3
λ_3	1	0	3
λ_4	0	1	1

	λ_1	λ_4	λ_2
$-z$	-3_0	0	
λ_1	3_1	0	
λ_4	$\times_0$	1	

	λ_1	λ_2
$-z$	0	-2_0
λ_1	1	$1/3$
λ_2	0	$5/3_1$

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