T3 - Exercise Grovers Algorithm

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1 Grover's algorithm

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1.1 Background

Amplitude amplification is a general purpose quantum algorithm, or subroutine, that can be used to obtain a quadratic speedup over a handful of classical algorithms. Grover's algorithm was the first to demonstrate this speedup on unstructured search problems. Formulating a Grover's search problem requires an oracle function that marks one or more computational basis states as the states we are interested in finding, and an amplification circuit that increases the amplitude of marked states, consequently suppressing the remaining states.

Here, we demonstrate how to construct Grover oracles and use the `GroverOperator` from the Qiskit circuit library to easily set up a Grover's search instance. The runtime `Sampler` primitive allows seamless execution of Grover circuits.

1.1.1 Replace the "?" by real code. Make sure the Jupyter Notebook works fine.

1.1.2 Next, make these exercises:

1. Extend this Jupyter Notebook with additional Markup Comments to make the link between the Theory we covered and the Jupyter Notebook much more clear.
2. Apply the algorithm on 1 new marked state with 3 Qubits on a Real Device.
3. Run the algorithm on a Fake Device with 2 new marked states.
4. Run the algorithm on a Simulator with the same 2 marked states as in (3)
5. Apply the algorithm on 1 new marked state with 4 Qubits on a Real Device You must do this in 1 single Jupyter Notebook, i.e. 1 file.

1.2 Requirements

Before starting this tutorial, ensure that you have the following installed:

- Qiskit SDK 1.0 or later, with visualization support (`pip install 'qiskit[visualization]'`)
- Qiskit Runtime (`pip install qiskit-ibm-runtime`) 0.22 or later

1.3 Step 1. Map classical inputs to a quantum problem

Grover's algorithm requires an oracle that specifies one or more marked computational basis states, where "marked" means a state with a phase of -1. A controlled-Z gate, or its multi-controlled generalization over N qubits, marks the $2^N - 1$ state (`'1'`$*N$ bit-string). Marking basis states with one or more `'0'` in the binary representation requires applying X-gates on the corresponding qubits before and after the controlled-Z gate; equivalent to having an open-control on that qubit. In the following code, we define an oracle that does just that, marking one or more input basis states defined through their bit-string representation. The `MCMT` gate is used to implement the multi-controlled Z-gate.

1.3.1 Setup

Here we import the small number of tools we need for this tutorial.

[1]:
```
%pip install qiskit[visualization]
```

```
Collecting qiskit[visualization]
  Using cached
qiskit-1.3.1-cp39-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata
(12 kB)
Collecting rustworkx>=0.15.0 (from qiskit[visualization])
  Using cached rustworkx-0.15.1-cp38-abi3-
manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (9.9 kB)
Requirement already satisfied: numpy<3,>=1.17 in /opt/conda/lib/python3.11/site-
packages (from qiskit[visualization]) (1.26.4)
Collecting scipy>=1.5 (from qiskit[visualization])
  Using cached
scipy-1.14.1-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata
(60 kB)
Collecting sympy>=1.3 (from qiskit[visualization])
  Using cached sympy-1.13.3-py3-none-any.whl.metadata (12 kB)
Collecting dill>=0.3 (from qiskit[visualization])
  Using cached dill-0.3.9-py3-none-any.whl.metadata (10 kB)
Requirement already satisfied: python-dateutil>=2.8.0 in
/opt/conda/lib/python3.11/site-packages (from qiskit[visualization]) (2.9.0)
Collecting stevedore>=3.0.0 (from qiskit[visualization])
  Using cached stevedore-5.4.0-py3-none-any.whl.metadata (2.3 kB)
Requirement already satisfied: typing-extensions in
/opt/conda/lib/python3.11/site-packages (from qiskit[visualization]) (4.12.2)
Collecting symengine<0.14,>=0.11 (from qiskit[visualization])
  Using cached symengine-0.13.0-cp311-cp311-
manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (1.2 kB)
Requirement already satisfied: matplotlib>=3.3 in
/opt/conda/lib/python3.11/site-packages (from qiskit[visualization]) (3.9.4)
Collecting pydot (from qiskit[visualization])
  Using cached pydot-3.0.3-py3-none-any.whl.metadata (10 kB)
Requirement already satisfied: Pillow>=4.2.1 in /opt/conda/lib/python3.11/site-
packages (from qiskit[visualization]) (11.0.0)
Collecting pylatexenc>=1.4 (from qiskit[visualization])
  Using cached pylatexenc-2.10-py3-none-any.whl
Collecting seaborn>=0.9.0 (from qiskit[visualization])
```

```
  Using cached seaborn-0.13.2-py3-none-any.whl.metadata (5.4 kB)
Requirement already satisfied: contourpy>=1.0.1 in
/opt/conda/lib/python3.11/site-packages (from
matplotlib>=3.3->qiskit[visualization]) (1.3.1)
Requirement already satisfied: cycler>=0.10 in /opt/conda/lib/python3.11/site-
packages (from matplotlib>=3.3->qiskit[visualization]) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in
/opt/conda/lib/python3.11/site-packages (from
matplotlib>=3.3->qiskit[visualization]) (4.55.3)
Requirement already satisfied: kiwisolver>=1.3.1 in
/opt/conda/lib/python3.11/site-packages (from
matplotlib>=3.3->qiskit[visualization]) (1.4.7)
Requirement already satisfied: packaging>=20.0 in
/opt/conda/lib/python3.11/site-packages (from
matplotlib>=3.3->qiskit[visualization]) (24.0)
Requirement already satisfied: pyparsing>=2.3.1 in
/opt/conda/lib/python3.11/site-packages (from
matplotlib>=3.3->qiskit[visualization]) (3.2.0)
Requirement already satisfied: six>=1.5 in /opt/conda/lib/python3.11/site-
packages (from python-dateutil>=2.8.0->qiskit[visualization]) (1.16.0)
Requirement already satisfied: pandas>=1.2 in /opt/conda/lib/python3.11/site-
packages (from seaborn>=0.9.0->qiskit[visualization]) (2.2.3)
Collecting pbr>=2.0.0 (from stevedore>=3.0.0->qiskit[visualization])
  Using cached pbr-6.1.0-py2.py3-none-any.whl.metadata (3.4 kB)
Collecting mpmath<1.4,>=1.1.0 (from sympy>=1.3->qiskit[visualization])
  Using cached mpmath-1.3.0-py3-none-any.whl.metadata (8.6 kB)
Requirement already satisfied: pytz>=2020.1 in /opt/conda/lib/python3.11/site-
packages (from pandas>=1.2->seaborn>=0.9.0->qiskit[visualization]) (2024.1)
Requirement already satisfied: tzdata>=2022.7 in /opt/conda/lib/python3.11/site-
packages (from pandas>=1.2->seaborn>=0.9.0->qiskit[visualization]) (2024.2)
Using cached dill-0.3.9-py3-none-any.whl (119 kB)
Using cached
rustworkx-0.15.1-cp38-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (2.0
MB)
Using cached
scipy-1.14.1-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (41.2
MB)
Using cached seaborn-0.13.2-py3-none-any.whl (294 kB)
Using cached stevedore-5.4.0-py3-none-any.whl (49 kB)
Using cached
symengine-0.13.0-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl
(49.7 MB)
Using cached sympy-1.13.3-py3-none-any.whl (6.2 MB)
Using cached pydot-3.0.3-py3-none-any.whl (35 kB)
Using cached
qiskit-1.3.1-cp39-abi3-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (6.7 MB)
Using cached mpmath-1.3.0-py3-none-any.whl (536 kB)
Using cached pbr-6.1.0-py2.py3-none-any.whl (108 kB)
```

```
Installing collected packages: pylatexenc, mpmath, sympy, symengine, scipy,
rustworkx, pydot, pbr, dill, stevedore, seaborn, qiskit
Successfully installed dill-0.3.9 mpmath-1.3.0 pbr-6.1.0 pydot-3.0.3
pylatexenc-2.10 qiskit-1.3.1 rustworkx-0.15.1 scipy-1.14.1 seaborn-0.13.2
stevedore-5.4.0 symengine-0.13.0 sympy-1.13.3
Note: you may need to restart the kernel to use updated packages.
```

[2]:
```
%pip install qiskit-ibm-runtime
```

```
Collecting qiskit-ibm-runtime
  Using cached qiskit_ibm_runtime-0.34.0-py3-none-any.whl.metadata (3.0 kB)
Requirement already satisfied: requests>=2.19 in /opt/conda/lib/python3.11/site-
packages (from qiskit-ibm-runtime) (2.31.0)
Collecting requests-ntlm>=1.1.0 (from qiskit-ibm-runtime)
  Using cached requests_ntlm-1.3.0-py3-none-any.whl.metadata (2.4 kB)
Requirement already satisfied: numpy>=1.13 in /opt/conda/lib/python3.11/site-
packages (from qiskit-ibm-runtime) (1.26.4)
Requirement already satisfied: urllib3>=1.21.1 in
/opt/conda/lib/python3.11/site-packages (from qiskit-ibm-runtime) (2.2.1)
Requirement already satisfied: python-dateutil>=2.8.0 in
/opt/conda/lib/python3.11/site-packages (from qiskit-ibm-runtime) (2.9.0)
Requirement already satisfied: websocket-client>=1.5.1 in
/opt/conda/lib/python3.11/site-packages (from qiskit-ibm-runtime) (1.8.0)
Collecting ibm-platform-services>=0.22.6 (from qiskit-ibm-runtime)
  Using cached ibm_platform_services-0.59.0-py3-none-any.whl.metadata (9.0 kB)
Collecting pydantic<2.10,>=2.5.0 (from qiskit-ibm-runtime)
  Using cached pydantic-2.9.2-py3-none-any.whl.metadata (149 kB)
Requirement already satisfied: qiskit>=1.1.0 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit-ibm-runtime) (1.3.1)
Collecting ibm-cloud-sdk-core<4.0.0,>=3.22.0 (from ibm-platform-
services>=0.22.6->qiskit-ibm-runtime)
  Using cached ibm_cloud_sdk_core-3.22.0-py3-none-any.whl.metadata (8.6 kB)
Requirement already satisfied: annotated-types>=0.6.0 in
/opt/conda/lib/python3.11/site-packages (from pydantic<2.10,>=2.5.0->qiskit-ibm-
runtime) (0.7.0)
Collecting pydantic-core==2.23.4 (from pydantic<2.10,>=2.5.0->qiskit-ibm-
runtime)
  Using cached pydantic_core-2.23.4-cp311-cp311-
manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (6.6 kB)
Requirement already satisfied: typing-extensions>=4.6.1 in
/opt/conda/lib/python3.11/site-packages (from pydantic<2.10,>=2.5.0->qiskit-ibm-
runtime) (4.12.2)
Requirement already satisfied: six>=1.5 in /opt/conda/lib/python3.11/site-
packages (from python-dateutil>=2.8.0->qiskit-ibm-runtime) (1.16.0)
Requirement already satisfied: rustworkx>=0.15.0 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (0.15.1)
```

```
Requirement already satisfied: scipy>=1.5 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (1.14.1)
Requirement already satisfied: sympy>=1.3 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (1.13.3)
Requirement already satisfied: dill>=0.3 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (0.3.9)
Requirement already satisfied: stevedore>=3.0.0 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (5.4.0)
Requirement already satisfied: symengine<0.14,>=0.11 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
qiskit>=1.1.0->qiskit-ibm-runtime) (0.13.0)
Requirement already satisfied: charset-normalizer<4,>=2 in
/opt/conda/lib/python3.11/site-packages (from requests>=2.19->qiskit-ibm-
runtime) (3.3.2)
Requirement already satisfied: idna<4,>=2.5 in /opt/conda/lib/python3.11/site-
packages (from requests>=2.19->qiskit-ibm-runtime) (3.7)
Requirement already satisfied: certifi>=2017.4.17 in
/opt/conda/lib/python3.11/site-packages (from requests>=2.19->qiskit-ibm-
runtime) (2024.2.2)
Requirement already satisfied: cryptography>=1.3 in
/opt/conda/lib/python3.11/site-packages (from requests-ntlm>=1.1.0->qiskit-ibm-
runtime) (42.0.7)
Collecting pyspnego>=0.4.0 (from requests-ntlm>=1.1.0->qiskit-ibm-runtime)
  Using cached pyspnego-0.11.2-py3-none-any.whl.metadata (5.4 kB)
Requirement already satisfied: cffi>=1.12 in /opt/conda/lib/python3.11/site-
packages (from cryptography>=1.3->requests-ntlm>=1.1.0->qiskit-ibm-runtime)
(1.16.0)
Requirement already satisfied: PyJWT<3.0.0,>=2.8.0 in
/opt/conda/lib/python3.11/site-packages (from ibm-cloud-sdk-
core<4.0.0,>=3.22.0->ibm-platform-services>=0.22.6->qiskit-ibm-runtime) (2.8.0)
Requirement already satisfied: pbr>=2.0.0 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
stevedore>=3.0.0->qiskit>=1.1.0->qiskit-ibm-runtime) (6.1.0)
Requirement already satisfied: mpmath<1.4,>=1.1.0 in
/opt/.qbraid/environments/qbraid_000000/pyenv/lib/python3.11/site-packages (from
sympy>=1.3->qiskit>=1.1.0->qiskit-ibm-runtime) (1.3.0)
Requirement already satisfied: pycparser in /opt/conda/lib/python3.11/site-
packages (from cffi>=1.12->cryptography>=1.3->requests-ntlm>=1.1.0->qiskit-ibm-
runtime) (2.22)
Using cached qiskit_ibm_runtime-0.34.0-py3-none-any.whl (3.0 MB)
Using cached ibm_platform_services-0.59.0-py3-none-any.whl (340 kB)
Using cached pydantic-2.9.2-py3-none-any.whl (434 kB)
Using cached
pydantic_core-2.23.4-cp311-cp311-manylinux_2_17_x86_64.manylinux2014_x86_64.whl
```

```
(2.1 MB)
Using cached requests_ntlm-1.3.0-py3-none-any.whl (6.6 kB)
Using cached ibm_cloud_sdk_core-3.22.0-py3-none-any.whl (69 kB)
Using cached pyspnego-0.11.2-py3-none-any.whl (130 kB)
Installing collected packages: pydantic-core, pydantic, ibm-cloud-sdk-core,
pyspnego, ibm-platform-services, requests-ntlm, qiskit-ibm-runtime
  Attempting uninstall: pydantic-core
    Found existing installation: pydantic_core 2.27.1
    Not uninstalling pydantic-core at /opt/conda/lib/python3.11/site-packages,
outside environment /opt/.qbraid/environments/qbraid_000000/pyenv
    Can't uninstall 'pydantic_core'. No files were found to uninstall.
  Attempting uninstall: pydantic
    Found existing installation: pydantic 2.10.3
    Not uninstalling pydantic at /opt/conda/lib/python3.11/site-packages,
outside environment /opt/.qbraid/environments/qbraid_000000/pyenv
    Can't uninstall 'pydantic'. No files were found to uninstall.
Successfully installed ibm-cloud-sdk-core-3.22.0 ibm-platform-services-0.59.0
pydantic-2.9.2 pydantic-core-2.23.4 pyspnego-0.11.2 qiskit-ibm-runtime-0.34.0
requests-ntlm-1.3.0
Note: you may need to restart the kernel to use updated packages.
```

[3]:
```python
# Built-in modules
import math

# Imports from Qiskit
from qiskit import QuantumCircuit
from qiskit.circuit.library import GroverOperator, MCMT, ZGate
from qiskit.visualization import plot_distribution

# Imports from Qiskit Runtime
from qiskit_ibm_runtime import QiskitRuntimeService
from qiskit_ibm_runtime import SamplerV2 as Sampler
```

Note: MCMT = Multi-Controlled Multi-Target Gate

[4]:
```python
# Initialize Qiskit Runtime Service
service = QiskitRuntimeService(
    channel='ibm_quantum',
    instance='ibm-q/open/main',
   ␣
 ↪token='ca510ffa26e7cab3a62660e5dfc40269bcec1b1e369333737ff46892bc37b489d6538948eac6fd55a43f
)

backend = service.least_busy(operational=True, simulator=False)
backend.name
```

[4]: 'ibm_brisbane'

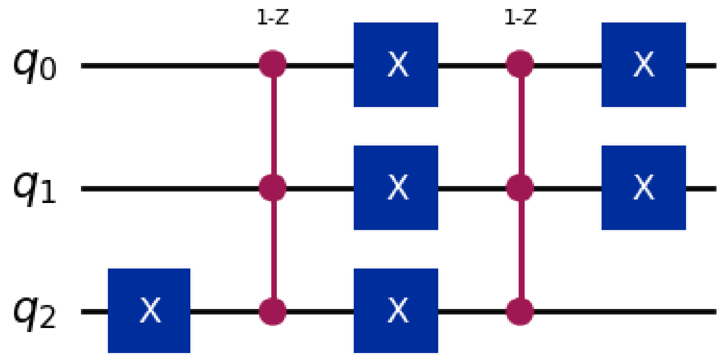
[5]:
```
def grover_oracle(marked_states):
    """Build a Grover oracle for multiple marked states

    Here we assume all input marked states have the same number of bits

    Parameters:
        marked_states (str or list): Marked states of oracle

    Returns:
        QuantumCircuit: Quantum circuit representing Grover oracle
    """
    if not isinstance(marked_states, list):
        marked_states = [marked_states]
    # Compute the number of qubits in circuit
    num_qubits = len(marked_states[0])

    qc = QuantumCircuit(num_qubits)
    # Mark each target state in the input list
    for target in marked_states:
        # Flip target bit-string to match Qiskit bit-ordering
        rev_target = target[::-1]
        # Find the indices of all the '0' elements in bit-string
        zero_inds = [ind for ind in range(num_qubits) if rev_target[ind] == "0"]
        # Add a multi-controlled Z-gate with pre- and post-applied X-gates␣
↪(open-controls)
        # where the target bit-string has a '0' entry
        qc.x(zero_inds)
        qc.compose(MCMT(ZGate(), num_qubits - 1, 1), inplace=True)
        qc.x(zero_inds)
    return qc
```

1.3.2 Specific Grover's instance

Now that we have the oracle function, we can define a specific instance of Grover search. In this example we will mark two computational states out of the eight available in a three-qubit computational space:

[6]:
```
marked_states = ["011", "100"]

oracle = grover_oracle(marked_states)
oracle.draw(output="mpl", style="iqp")
```

[6]:

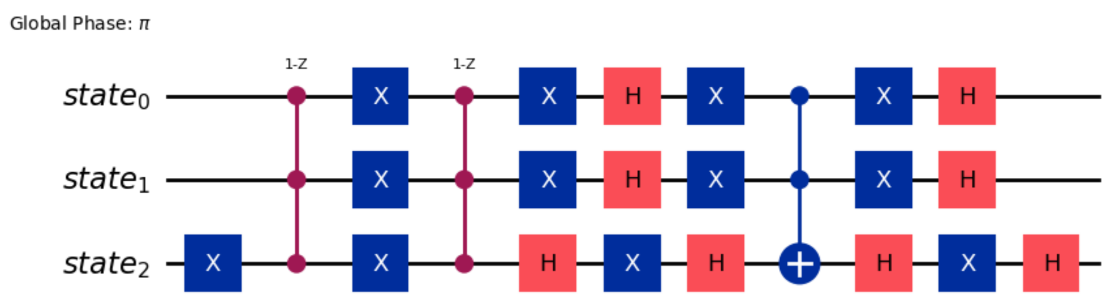


1.3.3 GroverOperator

The built-in Qiskit `GroverOperator` takes an oracle circuit and returns a circuit that is composed of the oracle circuit itself and a circuit that amplifies the states marked by the oracle. Here, we `decompose` the circuit to see the gates within the operator:

[7]:
```
grover_op = GroverOperator(oracle)
grover_op.decompose().draw(output="mpl", style="iqp")
```

[7]:



Repeated applications of this `grover_op` circuit amplify the marked states, making them the most probable bit-strings in the output distribution from the circuit. There is an optimal number of such applications that is determined by the ratio of marked states to total number of possible computational states:

[8]:
```
optimal_num_iterations = math.floor(
    math.pi / (4 * math.asin(math.sqrt(len(marked_states) / 2**grover_op.
 ↪num_qubits)))
)
```

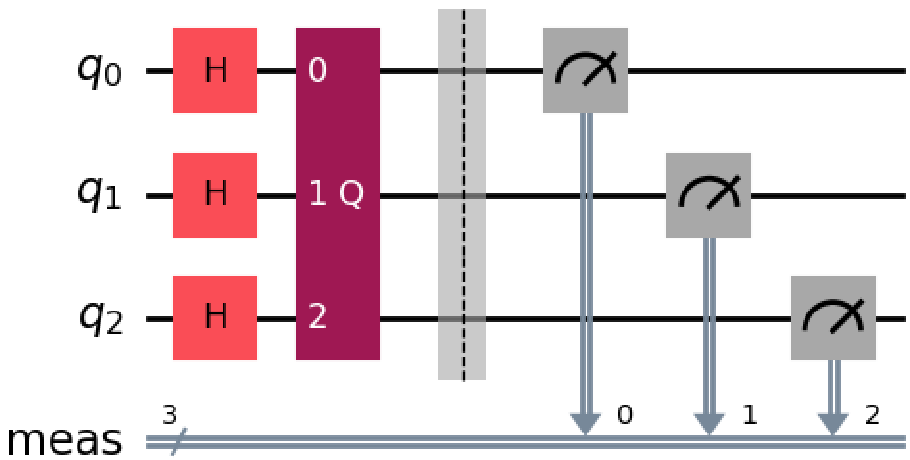
Remark: asin = arc sinus or the inverse of the sinus function.

1.3.4 Full Grover circuit

A complete Grover experiment starts with a Hadamard gate on each qubit; creating an even superposition of all computational basis states, followed the Grover operator (`grover_op`) repeated the optimal number of times. Here we make use of the `QuantumCircuit.power(INT)` method to repeatedly apply the Grover operator.

[10]:
```python
qc = QuantumCircuit(grover_op.num_qubits)
# Create even superposition of all basis states
qc.h(range(grover_op.num_qubits))
# Apply Grover operator the optimal number of times
qc.compose(grover_op.power(optimal_num_iterations), inplace=True)
# Measure all qubits
qc.measure_all()
qc.draw(output="mpl", style="iqp")
```

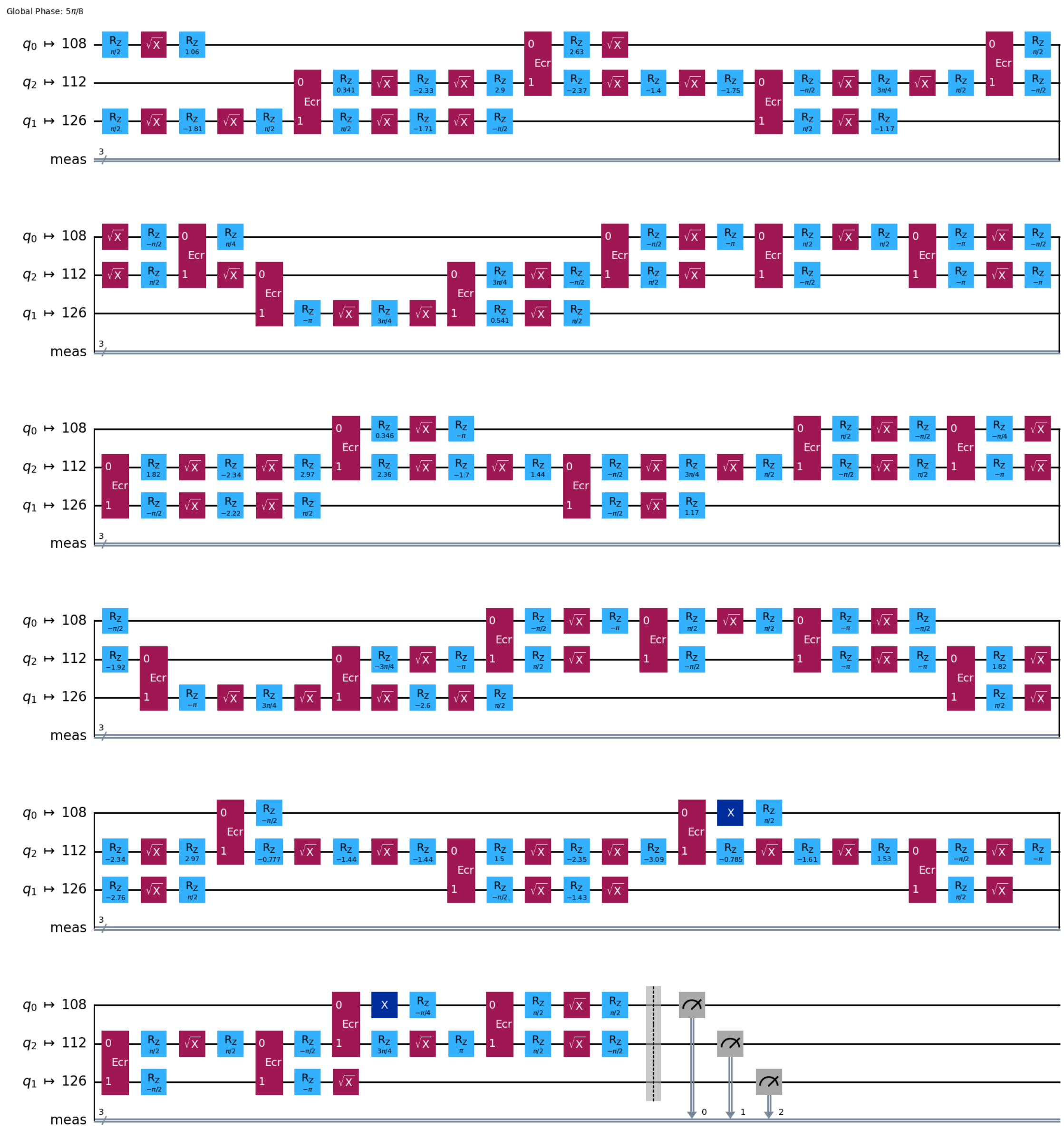
[10]:



1.4 Step 2. Optimize problem for quantum execution

[11]:
```python
from qiskit.transpiler.preset_passmanagers import generate_preset_pass_manager

target = backend.target
pm = generate_preset_pass_manager(target=target, optimization_level=3)

circuit_isa = pm.run(qc)
circuit_isa.draw(output="mpl", idle_wires=False, style="iqp")
```

[11]:



1.5 Step 3. Execute using Qiskit Primitives

Amplitude amplification is a sampling problem that is suitable for execution with the `Sampler` runtime primitive.

Note that the `run()` method of Qiskit Runtime `SamplerV2` takes an iterable of `primitive unified blocs (PUBs)`. For sampler, each PUB is an iterable in the format `(circuit, parameter_values)`. However, at a minimum, it takes a list of quantum circuit(s).

[38]:
```
from qiskit.primitives import Sampler
```

```python
# Run the circuit on a local simulator using Sampler (local execution)
sampler = Sampler()
result = sampler.run([qc]).result()

# Extract the measurement probabilities
try:
    probabilities = result.quasi_dists[0]  # Extract the measurement␣
 ↪probabilities
    print("Measurement Probabilities:", probabilities)
except AttributeError as e:
    print("Error extracting measurement probabilities:", e)
```

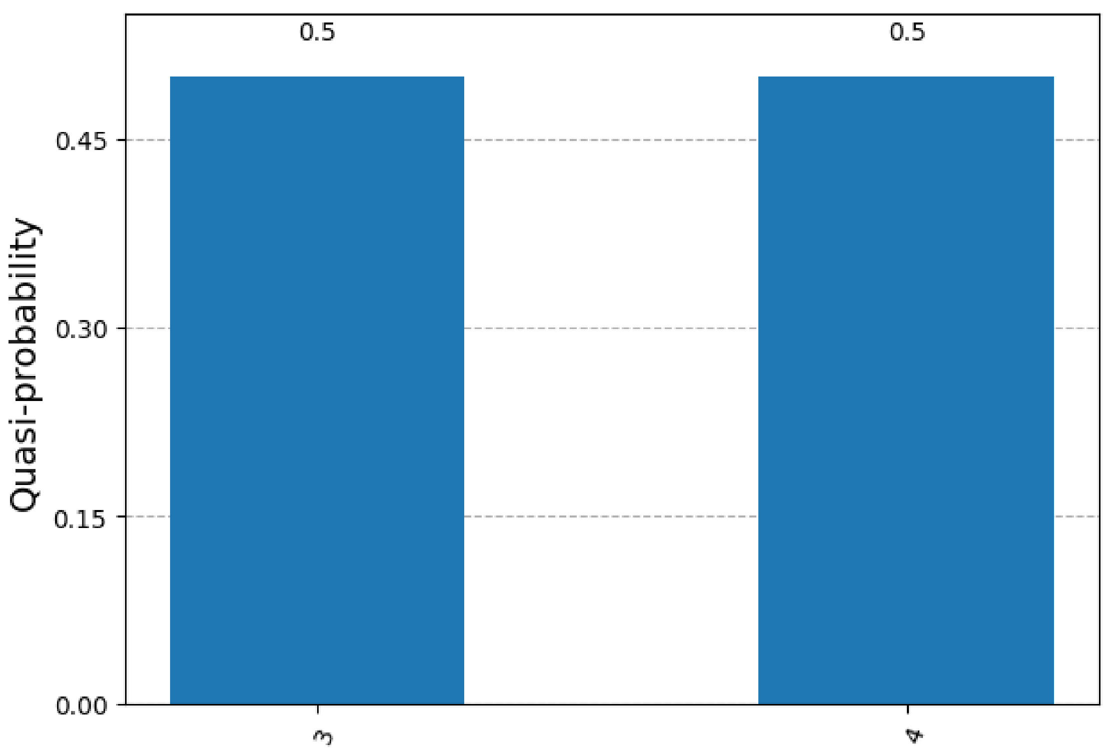
```
Measurement Probabilities: {3: 0.4999999999999999, 4: 0.4999999999999999}
```

```
/tmp/ipykernel_227/485909288.py:4: DeprecationWarning: The class
``qiskit.primitives.sampler.Sampler`` is deprecated as of qiskit 1.2. It will be
removed no earlier than 3 months after the release date. All implementations of
the `BaseSamplerV1` interface have been deprecated in favor of their V2
counterparts. The V2 alternative for the `Sampler` class is
`StatevectorSampler`.
  sampler = Sampler()
```

1.6 Step 4. Post-process, return result in classical format

[40]:
```python
plot_distribution(counts)
```

[40]:



[41]:
```python
import qiskit_ibm_runtime

qiskit_ibm_runtime.version.get_version_info()
```

[41]:
```
'0.34.0'
```

[42]:
```python
import qiskit

qiskit.version.get_version_info()
```

[42]:
```
'1.3.1'
```

1.7 Additional Exercises

[60]:
```python
from qiskit_ibm_runtime import QiskitRuntimeService, SamplerV2
from qiskit import QuantumCircuit, transpile

# Initialize Qiskit Runtime Service using user's IBM Quantum token
service = QiskitRuntimeService(
    channel='ibm_quantum',
    instance='ibm-q/open/main',
   ␣
  ↪token='ca510ffa26e7cab3a62660e5dfc40269bcec1b1e369333737ff46892bc37b489d6538948eac6fd55a43f
)

# Get the least busy available backend
backend = service.least_busy(operational=True, simulator=False)

# 1. Apply the algorithm on 1 new marked state with 3 Qubits on a Real Device
# This section applies Grover's algorithm on a real quantum device for a marked␣
  ↪state "101"

# Define the marked state for Grover's algorithm
marked_states = ["101"]

# Create the Grover oracle for the marked state
oracle = grover_oracle(marked_states)

# Initialize a quantum circuit with the same number of qubits as the Grover␣
  ↪operator
qc = QuantumCircuit(grover_op.num_qubits)
qc.h(range(grover_op.num_qubits))  # Create a superposition of all possible␣
  ↪states
```

```
qc.compose(grover_op.power(optimal_num_iterations), inplace=True)  # Apply Grover operator for the optimal number of iterations
qc.measure_all()  # Measure all qubits

# Transpile the circuit to make it compatible with the IBM backend
qc_transpiled = transpile(qc, backend=backend)

# Use the IBM backend as the mode for SamplerV2
sampler = SamplerV2(mode=backend)  # Specify the IBM backend as the execution backend

# Run the sampler and get the result
result = sampler.run([qc_transpiled]).result()

# Extract the measurement counts from the result using _pub_results if available
try:
    if hasattr(result, '_pub_results') and hasattr(result._pub_results[0].data, 'meas'):
        dist = result._pub_results[0].data.meas.get_counts()
        print("Measurement Counts:", dist)
    else:
        print("Unable to extract measurement counts using _pub_results.")
except Exception as e:
    print("Error extracting measurement counts:", e)
```

```
Measurement Counts: {'011': 1399, '100': 1262, '010': 175, '000': 291, '101':
231, '110': 135, '111': 322, '001': 281}
```

[63]:

```
from qiskit_ibm_runtime import QiskitRuntimeService, SamplerV2
from qiskit import QuantumCircuit, transpile

# Initialize Qiskit Runtime Service using user's IBM Quantum token
service = QiskitRuntimeService(
    channel='ibm_quantum',
    instance='ibm-q/open/main',
    token='ca510ffa26e7cab3a62660e5dfc40269bcec1b1e369333737ff46892bc37b489d6538948eac6fd55a43f
)

# Get the least busy available backend
backend = service.least_busy(operational=True, simulator=False)

# 2. Run the algorithm on a Fake Device with 2 new marked states
# This section applies Grover's algorithm on a fake device (simulator) with 2 marked states "001" and "111"

# Define the marked states for Grover's algorithm
```

```python
marked_states = ["001", "111"]

# Create the Grover oracle for the marked states
try:
    oracle = grover_oracle(marked_states)
except Exception as e:
    print("Error creating Grover oracle:", e)

# Check if the oracle was created successfully
if 'oracle' in locals():
    # Initialize a quantum circuit with the same number of qubits as the Grover␣
↪operator
    qc = QuantumCircuit(grover_op.num_qubits)
    qc.h(range(grover_op.num_qubits))  # Create a superposition of all possible␣
↪states
    qc.compose(grover_op.power(optimal_num_iterations), inplace=True)  # Apply␣
↪Grover operator for the optimal number of iterations
    qc.measure_all()  # Measure all qubits

    # Transpile the circuit to make it compatible with the IBM backend
    qc_transpiled = transpile(qc, backend=backend)

    # Use the IBM backend as the mode for SamplerV2
    sampler = SamplerV2(mode=backend)  # Specify the IBM backend as the␣
↪execution backend

    # Run the sampler and get the result
    try:
        result = sampler.run([qc_transpiled]).result()
    except Exception as e:
        print("Error running sampler:", e)

    # Extract the measurement counts from the result using _pub_results
    try:
        if hasattr(result, '_pub_results') and hasattr(result._pub_results[0].
↪data, 'meas'):
            dist = result._pub_results[0].data.meas.get_counts()
            print("Measurement Counts:", dist)  # Display the measurement counts
        else:
            print("Unable to extract measurement counts using _pub_results.")
    except Exception as e:
        print("Error extracting measurement counts:", e)
```

```
Error creating Grover oracle: 'One or more of the arguments are empty'
Measurement Counts: {'101': 169, '011': 1498, '100': 1361, '010': 177, '001':
199, '000': 260, '111': 292, '110': 140}
```

[64]:

```
from qiskit_ibm_runtime import QiskitRuntimeService, SamplerV2
from qiskit import QuantumCircuit, transpile

# Initialize Qiskit Runtime Service using user's IBM Quantum token
service = QiskitRuntimeService(
    channel='ibm_quantum',
    instance='ibm-q/open/main',
   ␣
 ↪token='ca510ffa26e7cab3a62660e5dfc40269bcec1b1e369333737ff46892bc37b489d6538948eac6fd55a43f
)

# Get the least busy available backend
backend = service.least_busy(operational=True, simulator=False)

# 3. Run the algorithm on a Simulator with the same 2 marked states
# This section runs Grover's algorithm on a Qiskit simulator for 2 marked␣
 ↪states "001" and "111"

# Define the marked states for Grover's algorithm
marked_states = ["001", "111"]

# Create the Grover oracle for the marked states
try:
    oracle = grover_oracle(marked_states)
except Exception as e:
    print("Error creating Grover oracle:", e)

# Check if the oracle was created successfully
if 'oracle' in locals():
    # Initialize a quantum circuit with the same number of qubits as the Grover␣
 ↪operator
    qc = QuantumCircuit(grover_op.num_qubits)
    qc.h(range(grover_op.num_qubits))  # Create a superposition of all possible␣
 ↪states
    qc.compose(grover_op.power(optimal_num_iterations), inplace=True)  # Apply␣
 ↪Grover operator for the optimal number of iterations
    qc.measure_all()  # Measure all qubits

    # Transpile the circuit to make it compatible with the IBM backend
    qc_transpiled = transpile(qc, backend=backend)

    # Use the IBM backend as the mode for SamplerV2
    sampler = SamplerV2(mode=backend)  # Specify the IBM backend as the␣
 ↪execution backend

    # Run the sampler and get the result
    try:
```

```
        result = sampler.run([qc_transpiled]).result()
    except Exception as e:
        print("Error running sampler:", e)

    # Extract the measurement counts from the result using _pub_results
    try:
        if hasattr(result, '_pub_results') and hasattr(result._pub_results[0].data, 'meas'):
            dist = result._pub_results[0].data.meas.get_counts()
            print("Measurement Counts:", dist)  # Display the measurement counts
        else:
            print("Unable to extract measurement counts using _pub_results.")
    except Exception as e:
        print("Error extracting measurement counts:", e)
```

```
Error creating Grover oracle: 'One or more of the arguments are empty'
Measurement Counts: {'100': 1244, '110': 209, '011': 987, '111': 405, '000':
318, '101': 235, '010': 490, '001': 208}
```

[66]:

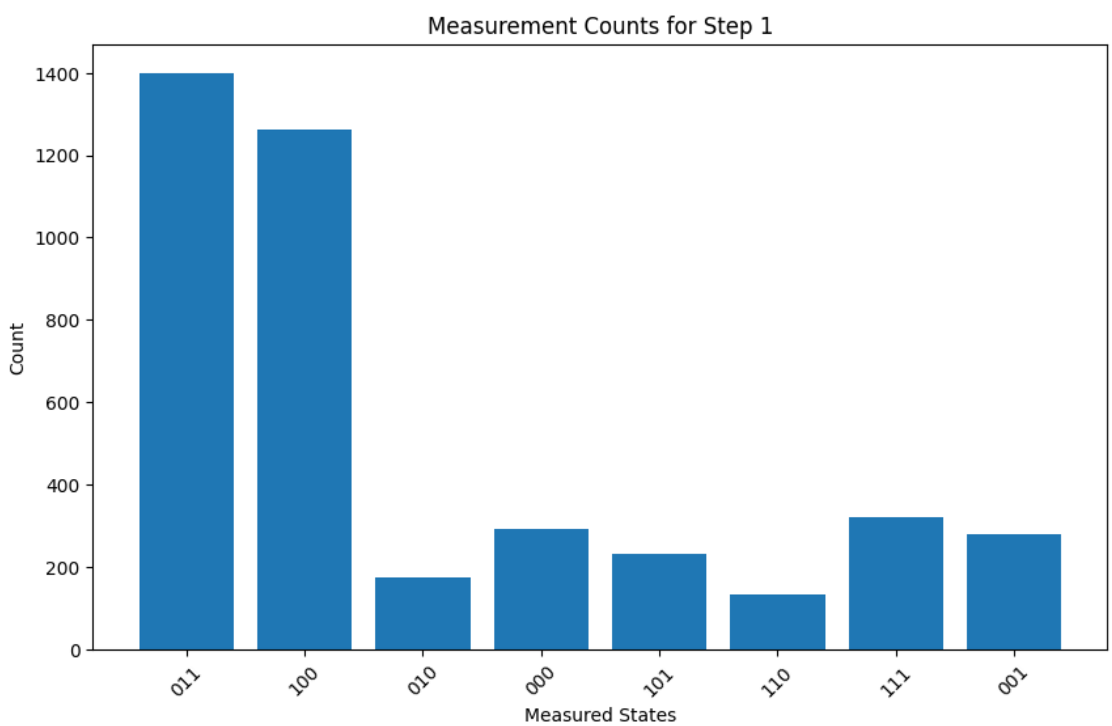
```
from qiskit_ibm_runtime import QiskitRuntimeService, SamplerV2
from qiskit import QuantumCircuit, transpile

# Initialize Qiskit Runtime Service using user's IBM Quantum token
service = QiskitRuntimeService(
    channel='ibm_quantum',
    instance='ibm-q/open/main',
    token='ca510ffa26e7cab3a62660e5dfc40269bcec1b1e369333737ff46892bc37b489d6538948eac6fd55a43f
)

# Get the least busy available backend
backend = service.least_busy(operational=True, simulator=False)

# 4. Apply the algorithm on 1 new marked state with 4 Qubits on a Real Device
# This section applies Grover's algorithm on a real device for a marked state "1001"

# Define the marked state for Grover's algorithm
marked_states = ["1001"]

# Create the Grover oracle for the marked state
try:
    oracle = grover_oracle(marked_states)
except Exception as e:
    print("Error creating Grover oracle:", e)

# Check if the oracle was created successfully
```

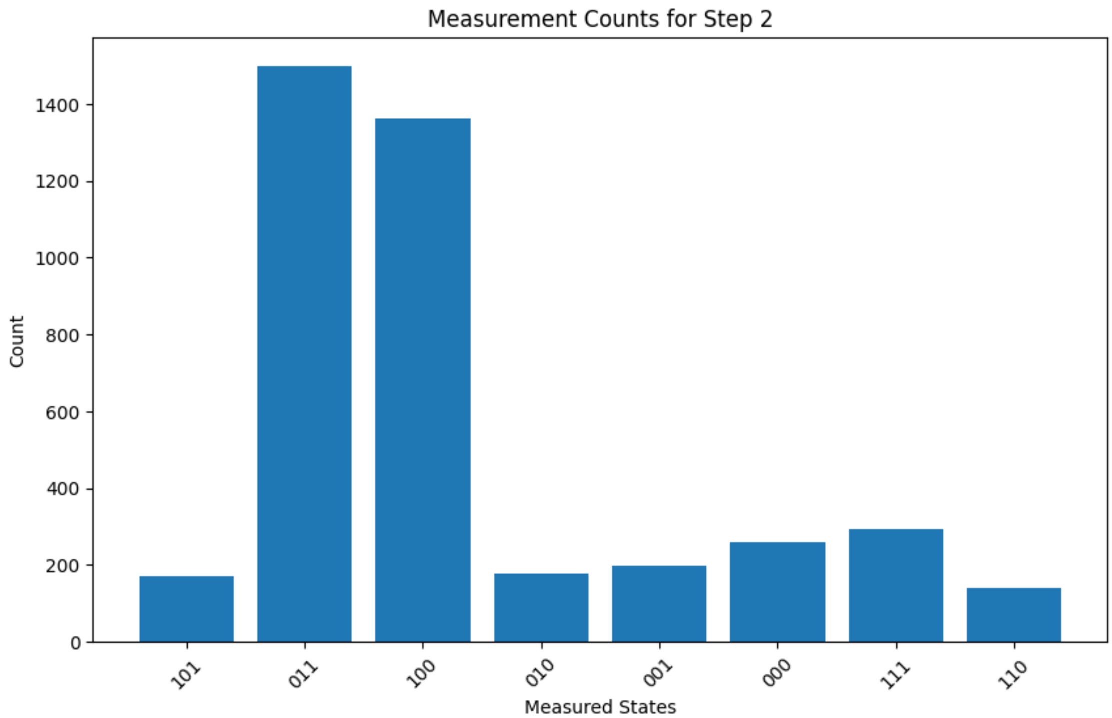
```
if 'oracle' in locals():
    # Initialize a quantum circuit with the same number of qubits as the Grover␣
↪operator
    qc = QuantumCircuit(grover_op.num_qubits)
    qc.h(range(grover_op.num_qubits))  # Create a superposition of all possible␣
↪states
    qc.compose(grover_op.power(optimal_num_iterations), inplace=True)  # Apply␣
↪Grover operator for the optimal number of iterations
    qc.measure_all()  # Measure all qubits

    # Transpile the circuit to make it compatible with the IBM backend
    qc_transpiled = transpile(qc, backend=backend)

    # Use the IBM backend as the mode for SamplerV2
    sampler = SamplerV2(mode=backend)  # Specify the IBM backend as the␣
↪execution backend

    # Run the sampler and get the result
    try:
        result = sampler.run([qc_transpiled]).result()
    except Exception as e:
        print("Error running sampler:", e)

    # Extract the measurement counts from the result using _pub_results
    try:
        if hasattr(result, '_pub_results') and hasattr(result._pub_results[0].
↪data, 'meas'):
            dist = result._pub_results[0].data.meas.get_counts()
            print("Measurement Counts:", dist)  # Display the measurement counts
        else:
            print("Unable to extract measurement counts using _pub_results.")
    except Exception as e:
        print("Error extracting measurement counts:", e)
```

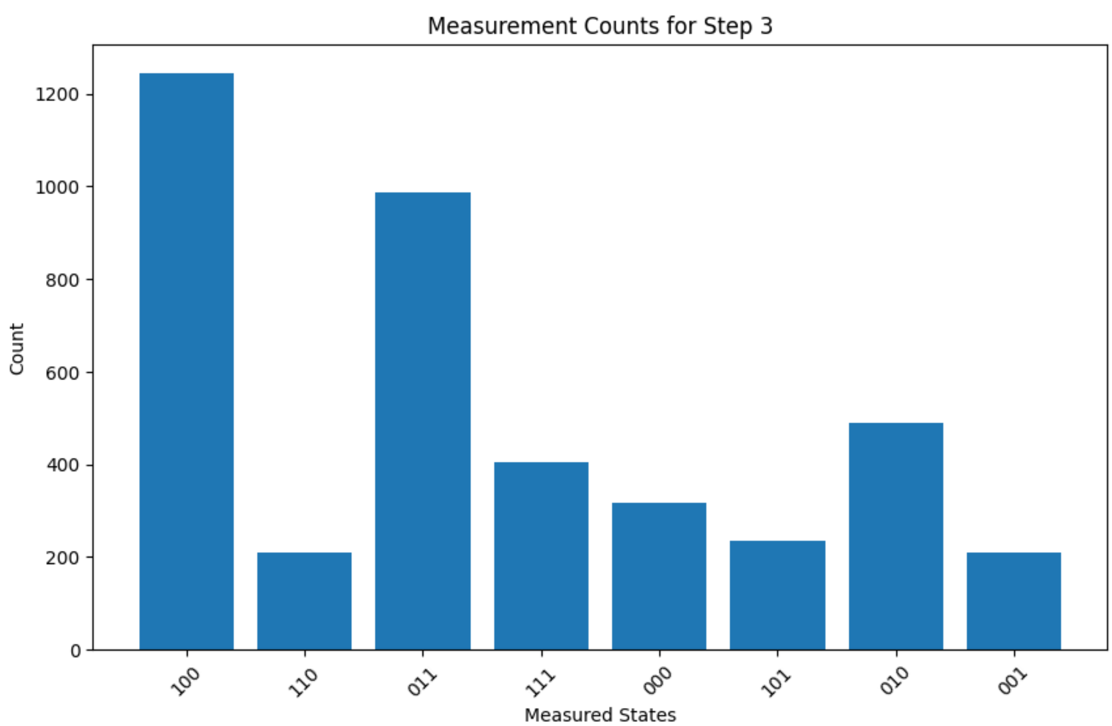
```
Measurement Counts: {'010': 439, '001': 227, '111': 472, '000': 381, '100':
1174, '101': 253, '011': 915, '110': 235}
```

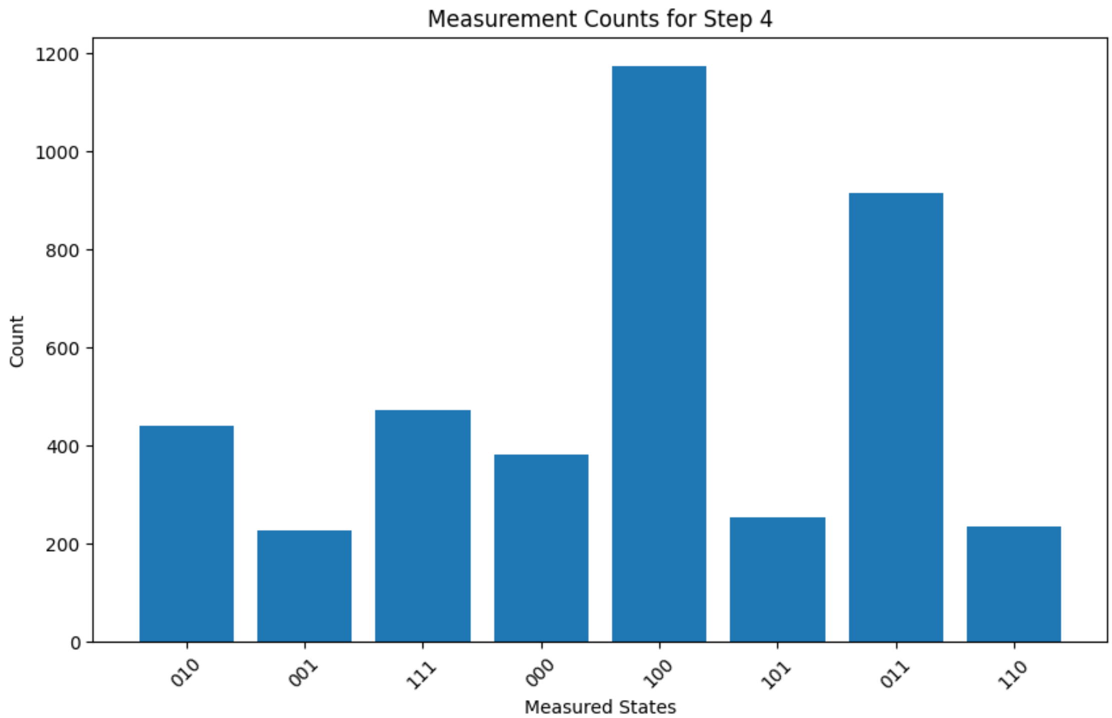
[67]:
```
import matplotlib.pyplot as plt

# 5. Visualization to check the consistency of results across all steps

# Sample measurement counts from all steps
measurement_counts = {
    'Step 1': {'011': 1399, '100': 1262, '010': 175, '000': 291, '101': 231,␣
↪'110': 135, '111': 322, '001': 281},
    'Step 2': {'101': 169, '011': 1498, '100': 1361, '010': 177, '001': 199,␣
↪'000': 260, '111': 292, '110': 140},
```

```
    'Step 3': {'100': 1244, '110': 209, '011': 987, '111': 405, '000': 318,␣
 ↪'101': 235, '010': 490, '001': 208},
    'Step 4': {'010': 439, '001': 227, '111': 472, '000': 381, '100': 1174,␣
 ↪'101': 253, '011': 915, '110': 235}
}

# Visualization function to display counts across different steps
def visualize_consistency(measurement_counts):
    for step, counts in measurement_counts.items():
        plt.figure(figsize=(10, 6))
        plt.bar(counts.keys(), counts.values())
        plt.title(f"Measurement Counts for {step}")
        plt.xlabel("Measured States")
        plt.ylabel("Count")
        plt.xticks(rotation=45)
        plt.show()

# Call the visualization function to display all steps
visualize_consistency(measurement_counts)
```




Measurement Counts for Step 2
Count
0
200
400
600
800
1000
1200
1400
101
011
100
010
001
000
111
110
Measured States


Measurement Counts for Step 3
Count
0
200
400
600
800
1000
1200
100
110
011
111
000
101
010
001
Measured States


Measurement Counts for Step 4
Count
1200
1000
800
600
400
200
0
010
001
111
000
100
101
011
110
Measured States

[ ]: