MODELADO MATEMÁTICO PARA LA DISTRIBUCIÓN DE TEMPERATURA EN UN CUERPO CILÍNDRICO A TRÁVES DEL TIEMPO (MÉTODO EXPLÍCITO E IMPLÍCITO)

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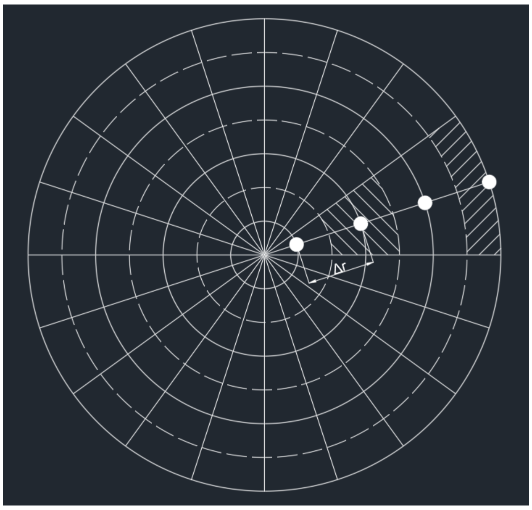
Datos conocidos:

$$T_\infty = temperatura\ del\ medio\ circundante$$
$$T_0 = temperatura\ inicial\ del\ cuerpo$$

Modelo:



Discretización:

$$M: número\ de\ nodos\ en\ una\ línea\ radial.$$
$$N: número\ de\ nodos\ en\ una\ línea\ transversal.$$
$$P: número\ de\ nodos\ temporales.$$

$$m = m\Delta r,\quad m: 0,1,2,\dots,M-1$$
$$n = n\Delta z,\quad n: 1,2,\dots,N$$
$$p = p\Delta t, p: 1,2,\dots,P$$

$$r = m\Delta r + \frac{\Delta r}{2}$$

Utilizando la ecuación del balance de energía:

$$\dot{E_{in}} - \dot{E_{out}} + \dot{E_g} = \dot{E_{st}}$$

MÉTODO EXPLÍCITO:

PUNTO INTERIOR:

$$\frac{kA_1\left(T_{m-1,n}^p - T_{m,n}^p\right)}{\Delta r} + \frac{kA_2\left(T_{m,n-1}^p - T_{m,n}^p\right)}{\Delta z} - \frac{kA_2\left(T_{m,n}^p - T_{m,n+1}^p\right)}{\Delta z} - \frac{kA_3\left(T_{m,n}^p - T_{m+1,n}^p\right)}{\Delta r} = \rho cV\frac{\left(T_{m,n}^{p+1} - T_{m,n}^p\right)}{\Delta t}$$

$$A_1 = 2\pi\left(r - \frac{\Delta r}{2}\right)\Delta z$$
$$A_2 = \pi\left(\left(r + \frac{\Delta r}{2}\right)^2 - \left(r - \frac{\Delta r}{2}\right)^2\right)$$

$$A_3 = 2\pi\left(r + \frac{\Delta r}{2}\right)\Delta z$$

$$V = A_2 \Delta z$$

Para simplificar la ecuación se tomará $\Delta z = \Delta r$, por lo tanto:

$$A_1 = \pi(\Delta r)^2(2m)$$
$$A_2 = \pi(\Delta r)^2(2m+1)$$
$$A_3 = \pi(\Delta r)^2(2m+2)$$

$$V = A_2 \Delta r$$

Reemplazando en la ecuación principal y tomando $F_o = \frac{\alpha \Delta t}{(\Delta r)^2}$, con $\alpha = \frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T_{m-1,n}^p - T_{m,n}^p\right) + F_o\left(T_{m,n-1}^p + T_{m,n+1}^p - 2T_{m,n}^p\right) - F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^p - T_{m+1,n}^p\right) = \left(T_{m,n}^{p+1} - T_{m,n}^p\right)$$

$$T_{m,n}^{p+1} = T_{m,n}^p(1-4F_o) + F_o\left(T_{m,n-1}^p + T_{m,n+1}^p + \left(\frac{2m}{2m+1}\right)T_{m-1,n}^p + \left(\frac{2m+2}{2m+1}\right)T_{m+1,n}^p\right)$$

PUNTO EXTERIOR (CARA LATERAL):

$$\frac{kA_1\left(T_{m-1,n}^p - T_{m,n}^p\right)}{\Delta r} + \frac{kA_2\left(T_{m,n-1}^p - T_{m,n}^p\right)}{\Delta z} - \frac{kA_2\left(T_{m,n}^p - T_{m,n+1}^p\right)}{\Delta z} - \bar{h}A_3\left(T_{m,n}^p - T_\infty\right) = \rho c V \frac{\left(T_{m,n}^{p+1} - T_{m,n}^p\right)}{\Delta t}$$

$$A_1 = 2\pi\left(r - \frac{\Delta r}{2}\right)\Delta z$$
$$A_2 = \pi\left((r)^2 - \left(r - \frac{\Delta r}{2}\right)^2\right)$$
$$A_3 = 2\pi(r)\Delta z$$

$$V = A_2 \Delta z$$

Para simplificar la ecuación se tomará $\Delta z = \Delta r$, por lo tanto:

$$A_1 = \pi(\Delta r)^2(2m)$$
$$A_2 = \pi(\Delta r)^2\left(m + \frac{1}{4}\right)$$
$$A_3 = \pi(\Delta r)^2(2m+1)$$

$$V = A_2 \Delta r$$

Reemplazando en la ecuación principal y tomando $Bi = \left(\frac{\bar{h}\Delta r}{k}\right)$ $F_o = \frac{\alpha \Delta t}{(\Delta r)^2}, con\ \alpha = \frac{k}{\rho c}$:

$$F_o\left(\frac{8m}{4m+1}\right)\left(T_{m-1,n}^p - T_{m,n}^p\right) + F_o\left(T_{m,n-1}^p + T_{m,n+1}^p - 2T_{m,n}^p\right) - F_o Bi\left(\frac{8m+4}{4m+1}\right)\left(T_{m,n}^p - T_\infty\right) = \left(T_{m,n}^{p+1} - T_{m,n}^p\right)$$

$$T_{m,n}^{p+1} = T_{m,n}^p\left(1 - 2F_o - F_o\left(\frac{8m}{4m+1}\right) - F_o Bi\left(\frac{8m+4}{4m+1}\right)\right)$$
$$+ F_o\left(\left(\frac{8m}{4m+1}\right)T_{m-1,n}^p + T_{m,n-1}^p + T_{m,n+1}^p + Bi\left(\frac{8m+4}{4m+1}\right)T_\infty\right)$$

PUNTO EXTERIOR (BASE SUPERIOR):

$$\frac{kA_1\left(T^p_{m-1,n}-T^p_{m,n}\right)}{\Delta r}+\frac{kA_2\left(T^p_{m,n-1}-T^p_{m,n}\right)}{\Delta z}-\bar{h}A_2\left(T^p_{m,n}-T_\infty\right)-\frac{kA_3\left(T^p_{m,n}-T^p_{m+1,n}\right)}{\Delta z}=\rho cV\,\frac{\left(T^{p+1}_{m,n}-T^p_{m,n}\right)}{\Delta t}$$

$$A_1=\pi\left(r-\frac{\Delta r}{2}\right)\Delta z$$
$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
$$A_3=\pi\left(r+\frac{\Delta r}{2}\right)\Delta z$$

$$V=A_2\left(\frac{\Delta z}{2}\right)$$

Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

$$A_1=\pi(\Delta r)^2(m)$$
$$A_2=\pi(\Delta r)^2(2m+1)$$
$$A_3=\pi(\Delta r)^2(m+1)$$

$$V=A_2\left(\frac{\Delta r}{2}\right)$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T^p_{m-1,n}-T^p_{m,n}\right)+2F_o\left(T^p_{m,n-1}-T^p_{m,n}\right)-2F_oBi\left(T^p_{m,n}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T^p_{m,n}-T^p_{m+1,n}\right)$$
$$=\left(T^{p+1}_{m,n}-T^p_{m,n}\right)$$

$$T^{p+1}_{m,n}=T^p_{m,n}(1-4F_o-2F_oBi)+2F_o\left(T^p_{m,n-1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T^p_{m+1,n}+\left(\frac{m}{2m+1}\right)T^p_{m-1,n}\right)$$

PUNTO EXTERIOR (BASE INFERIOR):

$$\frac{kA_1\left(T^p_{m-1,n}-T^p_{m,n}\right)}{\Delta r}+\frac{kA_2\left(T^p_{m,n+1}-T^p_{m,n}\right)}{\Delta z}-\bar{h}A_2\left(T^p_{m,n}-T_\infty\right)-\frac{kA_3\left(T^p_{m,n}-T^p_{m+1,n}\right)}{\Delta z}=\rho cV\,\frac{\left(T^{p+1}_{m,n}-T^p_{m,n}\right)}{\Delta t}$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T^p_{m-1,n}-T^p_{m,n}\right)+2F_o\left(T^p_{m,n+1}-T^p_{m,n}\right)-2F_oBi\left(T^p_{m,n}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T^p_{m,n}-T^p_{m+1,n}\right)$$
$$=\left(T^{p+1}_{m,n}-T^p_{m,n}\right)$$

$$T^{p+1}_{m,n}=T^p_{m,n}(1-4F_o-2F_oBi)+2F_o\left(T^p_{m,n+1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T^p_{m+1,n}+\left(\frac{m}{2m+1}\right)T^p_{m-1,n}\right)$$

PUNTO EXTERIOR (ARISTA SUPERIOR):

$$\frac{kA_1\left(T_{m-1,n}^p - T_{m,n}^p\right)}{\Delta r} + \frac{kA_2\left(T_{m,n-1}^p - T_{m,n}^p\right)}{\Delta z} - \bar{h}A_2\left(T_{m,n}^p - T_\infty\right) - \bar{h}A_3\left(T_{m,n}^p - T_\infty\right) = \rho c V\,\frac{\left(T_{m,n}^{p+1} - T_{m,n}^p\right)}{\Delta t}$$

$$A_1 = \pi\left(r - \frac{\Delta r}{2}\right)\Delta z$$

$$A_2 = \pi\left((r)^2 - \left(r - \frac{\Delta r}{2}\right)^2\right)$$

$$A_3 = \pi(r)\Delta z$$

$$V = A_2\left(\frac{\Delta z}{2}\right)$$

Para simplificar la ecuación se tomará $\Delta z = \Delta r$, por lo tanto:

$$A_1 = \pi(\Delta r)^2(m)$$

$$A_2 = \pi(\Delta r)^2\left(m + \frac{1}{4}\right)$$

$$A_3 = \pi(\Delta r)^2\left(m + \frac{1}{2}\right)$$

$$V = A_2\left(\frac{\Delta r}{2}\right)$$

Reemplazando en la ecuación principal y tomando $Bi = \left(\frac{\bar{h}\Delta r}{k}\right)\ F_o = \frac{\alpha \Delta t}{(\Delta r)^2}, con\ \alpha = \frac{k}{\rho c}$:

$$2\left(\frac{A_1}{A_2}\right)F_o\left(T_{m-1,n}^p - T_{m,n}^p\right) + 2F_o\left(T_{m,n-1}^p - T_{m,n}^p\right) - 2\left(\frac{A_2 + A_3}{A_2}\right)F_oBi\left(T_{m,n}^p - T_\infty\right) = \left(T_{m,n}^{p+1} - T_{m,n}^p\right)$$

$$T_{m,n}^{p+1} = T_{m,n}^p\left(1 - 2\left(\frac{8m+1}{4m+1}\right)F_o - 2\left(\frac{8m+3}{4m+1}\right)F_oBi\right) + 2F_o\left(T_{m,n-1}^p + \left(\frac{4m}{4m+1}\right)T_{m-1,n}^p + Bi\left(\frac{8m+3}{4m+1}\right)T_\infty\right)$$

PUNTO EXTERIOR (ARISTA INFERIOR):

$$\frac{kA_1\left(T_{m-1,n}^p - T_{m,n}^p\right)}{\Delta r} + \frac{kA_2\left(T_{m,n+1}^p - T_{m,n}^p\right)}{\Delta z} - \bar{h}A_2\left(T_{m,n}^p - T_\infty\right) - \bar{h}A_3\left(T_{m,n}^p - T_\infty\right) = \rho c V\,\frac{\left(T_{m,n}^{p+1} - T_{m,n}^p\right)}{\Delta t}$$

Reemplazando en la ecuación principal y tomando $Bi = \left(\frac{\bar{h}\Delta r}{k}\right)\ F_o = \frac{\alpha \Delta t}{(\Delta r)^2}, con\ \alpha = \frac{k}{\rho c}$:

$$2\left(\frac{A_1}{A_2}\right)F_o\left(T_{m-1,n}^p - T_{m,n}^p\right) + 2F_o\left(T_{m,n+1}^p - T_{m,n}^p\right) - 2\left(\frac{A_2 + A_3}{A_2}\right)F_oBi\left(T_{m,n}^p - T_\infty\right) = \left(T_{m,n}^{p+1} - T_{m,n}^p\right)$$

$$T_{m,n}^{p+1} = T_{m,n}^p\left(1 - 2\left(\frac{8m+1}{4m+1}\right)F_o - 2\left(\frac{8m+3}{4m+1}\right)F_oBi\right) + 2F_o\left(T_{m,n+1}^p + \left(\frac{4m}{4m+1}\right)T_{m-1,n}^p + Bi\left(\frac{8m+3}{4m+1}\right)T_\infty\right)$$

ECUACIONES PARA EL CENTRO DEL CILINDRO:

PUNTO INTERIOR:

$$\frac{kA_2\left(T_{m,n-1}^{p}-T_{m,n}^{p}\right)}{\Delta z}-\frac{kA_2\left(T_{m,n}^{p}-T_{m,n+1}^{p}\right)}{\Delta z}-\frac{kA_3\left(T_{m,n}^{p}-T_{m+1,n}^{p}\right)}{\Delta r}=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
$$A_3=2\pi\left(r+\frac{\Delta r}{2}\right)\Delta z$$

$$V=A_2\Delta z$$

Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

$$A_2=\pi(\Delta r)^2(2m+1)$$
$$A_3=\pi(\Delta r)^2(2m+2)$$

$$V=A_2\Delta r$$

Reemplazando en la ecuación principal y tomando $F_o=\frac{\alpha\Delta t}{(\Delta r)^2}$, con $\alpha=\frac{k}{\rho c}$:

$$F_o\left(T_{m,n-1}^{p}+T_{m,n+1}^{p}-2T_{m,n}^{p}\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p}-T_{m+1,n}^{p}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p+1}=T_{m,n}^{p}\left(1-2F_o-F_o\left(\frac{2m+2}{2m+1}\right)\right)+F_o\left(T_{m,n-1}^{p}+T_{m,n+1}^{p}+\left(\frac{2m+2}{2m+1}\right)T_{m+1,n}^{p}\right)$$

PUNTO EXTERIOR (BASE SUPERIOR):

$$\frac{kA_2\left(T_{m,n-1}^{p}-T_{m,n}^{p}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_{\infty}\right)-\frac{kA_3\left(T_{m,n}^{p}-T_{m+1,n}^{p}\right)}{\Delta z}=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
$$A_3=\pi\left(r+\frac{\Delta r}{2}\right)\Delta z$$

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$$2F_o\left(T_{m,n-1}^{p}-T_{m,n}^{p}\right)-2F_oBi\left(T_{m,n}^{p}-T_{\infty}\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p}-T_{m+1,n}^{p}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p+1}=T_{m,n}^{p}\left(1-2F_o-2F_oBi-F_o\left(\frac{2m+2}{2m+1}\right)\right)+2F_o\left(T_{m,n-1}^{p}+BiT_{\infty}+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p}\right)$$

PUNTO EXTERIOR (BASE INFERIOR):

$$\frac{kA_2\left(T_{m,n+1}^{p}-T_{m,n}^{p}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_{\infty}\right)-\frac{kA_3\left(T_{m,n}^{p}-T_{m+1,n}^{p}\right)}{\Delta z}=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$T_{m,n}^{p+1}=T_{m,n}^{p}\left(1-2F_o-2F_oBi-F_o\left(\frac{2m+2}{2m+1}\right)\right)+2F_o\left(T_{m,n+1}^{p}+BiT_{\infty}+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p}\right)$$

MÉTODO IMPLÍCITO:

PUNTO INTERIOR:

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\frac{kA_2\left(T_{m,n}^{p+1}-T_{m,n+1}^{p+1}\right)}{\Delta z}-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta r}=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_1=2\pi\left(r-\frac{\Delta r}{2}\right)\Delta z$$
$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
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Reemplazando en la ecuación principal y tomando $F_o=\frac{\alpha\Delta t}{(\Delta r)^2}$, con $\alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+F_o\left(T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}-2T_{m,n}^{p+1}\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}(1+4F_o)-F_o\left(T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}+\left(\frac{2m}{2m+1}\right)T_{m-1,n}^{p+1}+\left(\frac{2m+2}{2m+1}\right)T_{m+1,n}^{p+1}\right)$$

PUNTO EXTERIOR (CARA LATERAL):

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\frac{kA_2\left(T_{m,n}^{p+1}-T_{m,n+1}^{p+1}\right)}{\Delta z}-\bar{h}A_3\left(T_{m,n}^{p+1}-T_\infty\right)=\rho cV\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_1=2\pi\left(r-\frac{\Delta r}{2}\right)\Delta z$$

$$A_2=\pi\left((r)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$

$$A_3=2\pi(r)\Delta z$$

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Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

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$$F_o\left(\frac{8m}{4m+1}\right)\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+F_o\left(T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}-2T_{m,n}^{p+1}\right)-F_oBi\left(\frac{8m+4}{4m+1}\right)\left(T_{m,n}^{p+1}-T_\infty\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}\left(1+2F_o+F_o\left(\frac{8m}{4m+1}\right)+F_oBi\left(\frac{8m+4}{4m+1}\right)\right)$$
$$-F_o\left(\left(\frac{8m}{4m+1}\right)T_{m-1,n}^{p+1}+T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}+Bi\left(\frac{8m+4}{4m+1}\right)T_\infty\right)$$

PUNTO EXTERIOR (BASE SUPERIOR):

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta z}=\rho cV\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_1=\pi\left(r-\frac{\Delta r}{2}\right)\Delta z$$
$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
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Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+2F_o\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)-2F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}(1+4F_o+2F_oBi)-2F_o\left(T_{m,n-1}^{p+1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p+1}+\left(\frac{m}{2m+1}\right)T_{m-1,n}^{p+1}\right)$$

PUNTO EXTERIOR (BASE INFERIOR):

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta z}=\rho cV\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+2F_o\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)-2F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}(1+4F_o+2F_oBi)-2F_o\left(T_{m,n+1}^{p+1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p+1}+\left(\frac{m}{2m+1}\right)T_{m-1,n}^{p+1}\right)$$

PUNTO EXTERIOR (ARISTA SUPERIOR):

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\bar{h}A_3\left(T_{m,n}^{p+1}-T_\infty\right)=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_1=\pi\left(r-\frac{\Delta r}{2}\right)\Delta z$$
$$A_2=\pi\left((r)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
$$A_3=\pi(r)\Delta z$$

$$V=A_2\left(\frac{\Delta z}{2}\right)$$

Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

$$A_1=\pi(\Delta r)^2(m)$$
$$A_2=\pi(\Delta r)^2\left(m+\frac{1}{4}\right)$$
$$A_3=\pi(\Delta r)^2\left(m+\frac{1}{2}\right)$$

$$V=A_2\left(\frac{\Delta r}{2}\right)$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right)$ $F_o=\frac{\alpha\Delta t}{(\Delta r)^2}, con\ \alpha=\frac{k}{\rho c}$:

$$2\left(\frac{A_1}{A_2}\right)F_o\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+2F_o\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)-2\left(\frac{A_2+A_3}{A_2}\right)F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=\ T_{m,n}^{p+1}\left(1+2\left(\frac{8m+1}{4m+1}\right)F_o+2\left(\frac{8m+3}{4m+1}\right)F_oBi\right)-2F_o\left(T_{m,n-1}^{p+1}+\left(\frac{4m}{4m+1}\right)T_{m-1,n}^{p+1}+Bi\left(\frac{8m+3}{4m+1}\right)T_\infty\right)$$

PUNTO EXTERIOR (ARISTA INFERIOR):

$$\frac{kA_1\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta r}+\frac{kA_2\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\bar{h}A_3\left(T_{m,n}^{p+1}-T_\infty\right)=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right)$ $F_o=\frac{\alpha\Delta t}{(\Delta r)^2}, con\ \alpha=\frac{k}{\rho c}$:

$$2\left(\frac{A_1}{A_2}\right)F_o\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+2F_o\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)-2\left(\frac{A_2+A_3}{A_2}\right)F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=\ T_{m,n}^{p+1}\left(1+2\left(\frac{8m+1}{4m+1}\right)F_o+2\left(\frac{8m+3}{4m+1}\right)F_oBi\right)-2F_o\left(T_{m,n+1}^{p+1}+\left(\frac{4m}{4m+1}\right)T_{m-1,n}^{p+1}+Bi\left(\frac{8m+3}{4m+1}\right)T_\infty\right)$$

ECUACIONES PARA EL CENTRO DEL CILINDRO:

PUNTO INTERIOR:

$$\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\frac{kA_2\left(T_{m,n}^{p+1}-T_{m,n+1}^{p+1}\right)}{\Delta z}-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta r}=\rho cV\,\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$

$$A_3=2\pi\left(r+\frac{\Delta r}{2}\right)\Delta z$$

$$V=A_2\Delta z$$

Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

$$A_2=\pi(\Delta r)^2(2m+1)$$

$$A_3=\pi(\Delta r)^2(2m+2)$$

$$V=A_2\Delta r$$

Reemplazando en la ecuación principal y tomando $F_o=\frac{\alpha\Delta t}{(\Delta r)^2}$, con $\alpha=\frac{k}{\rho c}$:

$$F_o\left(T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}-2T_{m,n}^{p+1}\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}\left(1+2F_o+F_o\left(\frac{2m+2}{2m+1}\right)\right)-F_o\left(T_{m,n-1}^{p+1}+T_{m,n+1}^{p+1}+\left(\frac{2m+2}{2m+1}\right)T_{m+1,n}^{p+1}\right)$$

PUNTO EXTERIOR (BASE SUPERIOR):

$$\frac{kA_2\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta z}=\rho cV\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

$$A_2=\pi\left(\left(r+\frac{\Delta r}{2}\right)^2-\left(r-\frac{\Delta r}{2}\right)^2\right)$$
$$A_3=\pi\left(r+\frac{\Delta r}{2}\right)\Delta z$$

$$V=A_2\left(\frac{\Delta z}{2}\right)$$

Para simplificar la ecuación se tomará $\Delta z=\Delta r$, por lo tanto:

$$A_2=\pi(\Delta r)^2(2m+1)$$
$$A_3=\pi(\Delta r)^2(m+1)$$

$$V=A_2\left(\frac{\Delta r}{2}\right)$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$2F_o\left(T_{m,n-1}^{p+1}-T_{m,n}^{p+1}\right)-2F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}\left(1+2F_o+2F_oBi+F_o\left(\frac{2m+2}{2m+1}\right)\right)-2F_o\left(T_{m,n-1}^{p+1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p+1}\right)$$

PUNTO EXTERIOR (BASE INFERIOR):

$$\frac{kA_2\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)}{\Delta z}-\bar{h}A_2\left(T_{m,n}^{p+1}-T_\infty\right)-\frac{kA_3\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)}{\Delta z}=\rho cV\frac{\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)}{\Delta t}$$

Reemplazando en la ecuación principal y tomando $Bi=\left(\frac{\bar{h}\Delta r}{k}\right), F_o=\frac{\alpha\Delta t}{(\Delta r)^2}\ con\ \alpha=\frac{k}{\rho c}$:

$$F_o\left(\frac{2m}{2m+1}\right)\left(T_{m-1,n}^{p+1}-T_{m,n}^{p+1}\right)+2F_o\left(T_{m,n+1}^{p+1}-T_{m,n}^{p+1}\right)-2F_oBi\left(T_{m,n}^{p+1}-T_\infty\right)-F_o\left(\frac{2m+2}{2m+1}\right)\left(T_{m,n}^{p+1}-T_{m+1,n}^{p+1}\right)$$
$$=\left(T_{m,n}^{p+1}-T_{m,n}^{p}\right)$$

$$T_{m,n}^{p}=T_{m,n}^{p+1}\left(1+2F_o+2F_oBi+F_o\left(\frac{2m+2}{2m+1}\right)\right)-2F_o\left(T_{m,n+1}^{p+1}+BiT_\infty+\left(\frac{m+1}{2m+1}\right)T_{m+1,n}^{p+1}\right)$$

MATLAB:

Debido a que en Matlab la indexación comienza en 1, se tuvo que hacer un cambio de variable m_a = m-1; para reemplazar en las fórmulas deducidas en el modelo matemático, ya que m en el modelado matemático comienza en 0.

Código para método implícito:

```
%r = (m-1/2)*dr
%z = (n-1)*dz
%p = (p-1)*dt
%M = numero de nodos radiales;
%N = numero de nodos en el eje z;
%P = numero de nodos temporales;
%tc = tiempo de congelamiento;
% dt = tc/(P-1);
% dr = 2*R/(2*M-1);
% dz = L/(N-1);
%F_o = constante de Fourier
%Bi = numero de Biot
%T_o = Temperatura inicial del cuerpo
%T_amb = Temperatura del ambiente
%R = radio del cilindro
%L = longitud del cilindro

function [T]=metodo_implicito(F_o, Bi, T_o, T_amb, R, L, M, P)

N = 1+(L/R)*(M-0.5);
%T: Matriz de temperatura para el analisis transitorio.
T = zeros(M,N,P);
%T_v: Matriz simbolica usada para guardar los valores de las ecuaciones
%resueltas
T_v = sym('T_%d_%d', [M N]);
%Estableciendo la temperatura inicial del cilindro
T(:,:,1)=ones(M,N)*T_o;
%Inicializando una matriz para guardar las ecuaciones
EQN = sym('eqn',[M N]);
%vector auxiliar para ordenamiento de variables antes de
%resolver sistema de ecuaciones
VAR = sym('var', [1 M*N]);

for p=1:P-1
    for m=1:M
        m_a = m-1;
        for n=1:N
            %punto interior
            if (m<M && m>1 && n<N && n>1)
                EQN(m,n) = T_v(m,n)*(1+4*F_o)-F_o*(T_v(m,n-1)...
                +T_v(m,n+1)+((2*m_a)/(2*m_a+1))*T_v(m-1,n)+...
                ((2*m_a+2)/(2*m_a+1))*T_v(m+1,n)) == T(m,n,p);
            end

            %punto exterior cara lateral
            if (m==M && n<N && n>1)
                EQN(m,n) = T_v(m,n)*(1+2*F_o+F_o*((8*m_a)/(4*m_a+1))+...
                F_o*Bi*((8*m_a+4)/(4*m_a+1)))-F_o*(((8*m_a)/(4*m_a+1))*T_v(m-1,n)...
                +T_v(m,n-1)+T_v(m,n+1)+Bi*((8*m_a+4)/(4*m_a+1))*T_amb)...
                == T(m,n,p);
            end
```

```
            %punto exterior base superior
            if (m>1 && m<M && n==N)
                EQN(m,n) = T_v(m,n)*(1+4*F_o+2*F_o*Bi)-2*F_o*...
                (T_v(m,n-1)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T_v(m+1,n)...
                +(m_a/(2*m_a+1))*T_v(m-1,n))==T(m,n,p);
            end

            %punto exterior base inferior
            if (m>1 && m<M && n==1)
                EQN(m,n) = T_v(m,n)*(1+4*F_o+2*F_o*Bi)-2*F_o*...
                (T_v(m,n+1)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T_v(m+1,n)...
                +(m_a/(2*m_a+1))*T_v(m-1,n))==T(m,n,p);
            end

            %punto exterior arista superior
            if (m==M && n==N)
                EQN(m,n) = T_v(m,n)*(1+2*((8*m_a+1)/(4*m_a+1))*F_o+...
                2*((8*m_a+3)/(4*m_a+1))*F_o*Bi)-2*F_o*(T_v(m,n-1)+(4*m_a/(4*m_a+1))*...
                T_v(m-1,n)+Bi*((8*m_a+3)/(4*m_a+1))*T_amb)==T(m,n,p);
            end

            %punto exterior arista inferior
            if (m==M && n==1)
                EQN(m,n) = T_v(m,n)*(1+2*((8*m_a+1)/(4*m_a+1))*F_o+...
                2*((8*m_a+3)/(4*m_a+1))*F_o*Bi)-2*F_o*(T_v(m,n+1)+(4*m_a/(4*m_a+1))*...
                T_v(m-1,n)+Bi*((8*m_a+3)/(4*m_a+1))*T_amb)==T(m,n,p);
            end

            %ecuaciones centro de cilindro
            %punto interior
            if (m==1 && n<N && n>1)
                EQN(m,n) = T_v(m,n)*(1+2*F_o+F_o*((2*m_a+2)/(2*m_a+1)))-F_o*(T_v(m,n-1)...
                +T_v(m,n+1)+ ((2*m_a+2)/(2*m_a+1))*T_v(m+1,n)) == T(m,n,p);
            end
            %punto exterior base superior
            if (m==1 && n==N)
                EQN(m,n) = T_v(m,n)*(1+2*F_o+2*F_o*Bi+F_o*((2*m_a+2)/(2*m_a+1)))-2*F_o*...
                (T_v(m,n-1)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T_v(m+1,n))==T(m,n,p);
            end

            %punto exterior base inferior
            if (m==1 && n==1)
                EQN(m,n) = T_v(m,n)*(1+2*F_o+2*F_o*Bi+F_o*((2*m_a+2)/(2*m_a+1)))-2*F_o*...
                (T_v(m,n+1)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T_v(m+1,n))==T(m,n,p);
            end

        end
    end

    for m=1:M
        VAR(((m-1)*N+1):(N*m))= sort(symvar(T_v(m,:)));
    end
    S = solve(EQN,VAR);
    S1 = struct2cell(S);

    for m=1:M
        for n=1:N
            T(m,n,p+1) = S1{N*(m-1)+n};
        end
    end
end
```

Código para método explícito:

```
function [T]=metodo_explicito(F_o, Bi, T_o, T_amb, R, L, M, P)

N = 1+(L/R)*(M-0.5);
%T: Matriz de temperatura para el analisis transitorio.
T = zeros(M,N,P);
%Estableciendo la temperatura inicial del cilindro
T(:,:,1)=ones(M,N)*T_o;


for p=1:P-1
    for m=1:M
        m_a = m-1;
        for n=1:N
            %punto interior
            if (m<M && m>1 && n<N && n>1)
                T(m,n,p+1)= T(m,n,p)*(1-4*F_o)+F_o*(T(m,n-1,p)...
                +T(m,n+1,p)+((2*m_a)/(2*m_a+1))*T(m-1,n,p)+...
                ((2*m_a+2)/(2*m_a+1))*T(m+1,n,p));
            end

            %punto exterior cara lateral
            if (m==M && n<N && n>1)
                T(m,n,p+1) = T(m,n,p)*(1-2*F_o-F_o*((8*m_a)/(4*m_a+1))-...
                F_o*Bi*((8*m_a+4)/(4*m_a+1)))+F_o*(((8*m_a)/(4*m_a+1))*T(m-1,n,p)...
                +T(m,n-1,p)+T(m,n+1,p)+Bi*((8*m_a+4)/(4*m_a+1))*T_amb);
            end

            %punto exterior base superior
            if (m>1 && m<M && n==N)
                T(m,n,p+1) = T(m,n,p)*(1-4*F_o-2*F_o*Bi)+2*F_o*...
                (T(m,n-1,p)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T(m+1,n,p)...
                +(m_a/(2*m_a+1))*T(m-1,n,p));
            end

            %punto exterior base inferior
            if (m>1 && m<M && n==1)
                T(m,n,p+1) = T(m,n,p)*(1-4*F_o-2*F_o*Bi)+2*F_o*...
                (T(m,n+1,p)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T(m+1,n,p)...
                +(m_a/(2*m_a+1))*T(m-1,n,p));
            end

            %punto exterior arista superior
            if (m==M && n==N)
                T(m,n,p+1) = T(m,n,p)*(1-2*((8*m_a+1)/(4*m_a+1))*F_o-...
                2*((8*m_a+3)/(4*m_a+1))*F_o*Bi)+2*F_o*(T(m,n-1,p)+(4*m_a/(4*m_a+1))*...
                T(m-1,n,p)+Bi*((8*m_a+3)/(4*m_a+1))*T_amb);
            end

            %punto exterior arista inferior
            if (m==M && n==1)
                T(m,n,p+1) = T(m,n,p)*(1-2*((8*m_a+1)/(4*m_a+1))*F_o-...
                2*((8*m_a+3)/(4*m_a+1))*F_o*Bi)+2*F_o*(T(m,n+1,p)+(4*m_a/(4*m_a+1))*...
                T(m-1,n,p)+Bi*((8*m_a+3)/(4*m_a+1))*T_amb);
            end

            %ecuaciones centro de cilindro
            %punto interior
            if (m==1 && n<N && n>1)
```

```
                T(m,n,p+1) = T(m,n,p)*(1-2*F_o-F_o*((2*m_a+2)/(2*m_a+1)))+F_o*(T(m,n-
1,p)...
                +T(m,n+1,p)+ ((2*m_a+2)/(2*m_a+1))*T(m+1,n,p));
            end
            %punto exterior base superior
            if (m==1 && n==N)
                T(m,n,p+1) = T(m,n,p)*(1-2*F_o-2*F_o*Bi-
F_o*((2*m_a+2)/(2*m_a+1)))+2*F_o*...
                (T(m,n-1,p)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T(m+1,n,p));
            end

            %punto exterior base inferior
            if (m==1 && n==1)
                T(m,n,p+1) = T(m,n,p)*(1-2*F_o-2*F_o*Bi-
F_o*((2*m_a+2)/(2*m_a+1)))+2*F_o*...
                (T(m,n+1,p)+Bi*T_amb+((m_a+1)/(2*m_a+1))*T(m+1,n,p));
            end

        end
    end
end
```

Código para gráfica de distribución de temperaturas:

```
function graf_dist_temp(R, L, M, T, p_e, T_amb, T_o)
%p_e = valor del tiempo discreto
%N = numero de nodos en el eje z
%M = numero de nodos radiales
%dr = delta de r
%dz = delta de z
%r = parametro radial del cilindro
%z = parametro z del cilindro
%T = matriz de distribucion de temperatura
%T_amb = Temperatura del ambiente
%T_o = Temperatura inicial del cuerpo

N = 1+(L/R)*(M-0.5);
dr = 2*R/(2*M-1);
r = zeros(1,M);
dz = L/(N-1);
z = zeros(1,N);
for m = 1:M
    r(m) = (m-1/2)*dr;
end
for n = 1:N
    z(n) = (n-1)*dz;
end
T_a = T(:,:,p_e);
imshow(T_a, [T_amb T_o], 'XData', z, 'Ydata', r, 'InitialMagnification', 1600);
colormap(gca, jet(256));
colorbar(gca);
axis('on','image');
title('T(r,z)', 'Fontsize', 15);
xlabel('z', 'Fontsize', 15);
ylabel('r', 'Fontsize', 15);
```