ME 206 – Statics and Dynamics

Experiment 1 - Measurement of Position, Velocity, and Acceleration of a Particle in 2D

Experiment Topic - Tracking the trajectory of a volleyball served underhand to determine its position, velocity, and acceleration in various coordinate frames.

Group 12

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Introduction:

Motion analysis is the basis of engineering, physics, biomechanics, robotics, aerospace, and many other disciplines. Accurately measuring and describing the motion of an object or a particle can enable engineers and scientists to construct safer structures that are mechanically superior and able to predict behaviour and understand the fundamental physical laws governing motion. Objects never move along straight lines or along and over simple curvatures for practical applications. Their movement is generally arbitrary, force-, initial velocity-, environment-, and constraint-dependent.

Motion analysis needs a strong framework capable of expressing position, velocity, and acceleration in multiple ways. Depending upon the scenario, there are situations where different coordinate systems are preferable. E.g.,

Cartesian coordinates permit movement in horizontal and vertical directions to be tracked directly.

Normal and tangential coordinates are convenient for curved motion, breaking acceleration down into elements along and normal to the path.

Radial and angular coordinates facilitate the analysis of motion around a point or an axis more efficiently.

The experiment uses such coordinate systems to find the position, the velocity, and the acceleration of an arbitrarily selected particle moving along any path within a two-dimensional plane. The objectives are to:

Find the motion through video analysis.

Apply mathematical models for extracting physical quantities.

Compare predictions of theories with observations.

Determine error sources and suggest improvements.

Practical Exercise – Volleyball Path

An underhand-served volleyball motion was considered a study example to demonstrate these rules under controlled but practical situations. The path of the volleyball is not a line or a small curve but a dynamically changing path governed by initial velocity, gravity, aerodynamic drag, and human intervention. The motion was recorded through a camera held stationary on a tripod, and the video recording was played back and analysed frame by frame for the ball position over time to be determined.

Mathematical equations calculated acceleration and velocity for the three coordinate frames from the recorded motion data. Comparisons with models were conducted to identify reasons for differences introduced by practicalities such as environmental factors, resolution of cameras, and noise in the measurements.

Although the resulting volleyball movement was the experimental configuration, the analysis methods and experimental strategies employed here are widely applicable. They can be used in:

Tracking movements in sport science and performance investigations of players,

Autonomous vehicle and robotic arm controlling

Creating aerospace trajectories and satellites,

Investigating any system for an irregular trajectory-travelling object.

Hence, the experiment illustrates the measurement and analysis of motion and the necessity for selecting proper coordinate systems and mathematical models based on the problem. Combining theories and practices enables us to tackle problems of complex motions systematically and accurately.

Experimental Design:

Aim of the Experiment

This experiment calculates and analyses the position, velocity, and acceleration of a particle moving along an indefinite trajectory on a two-dimensional plane using three coordinate systems—Cartesian, normal-tangential, and radial-angular. The focus is on tracking the motion of an underhand-served volleyball, recording its trajectory, and applying mathematical models to determine motion parameters.

Experimental Setup

To study the volleyball's motion, we created a controlled environment where the ball was served and tracked as it followed a curved path influenced by gravity and air resistance.

Equipment Used

- A volleyball
- A high-speed camera mounted on a tripod
- Calibration scale (marked tape or ruler)
- Open-source video analysis software (Tracker)

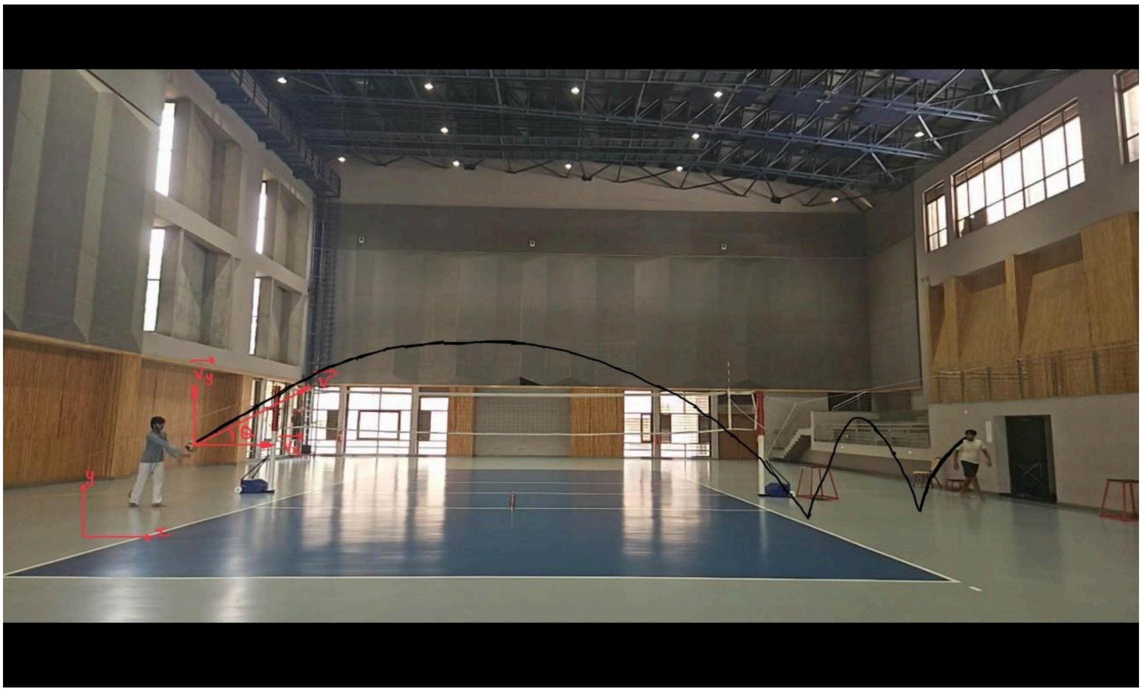
Setup Description:

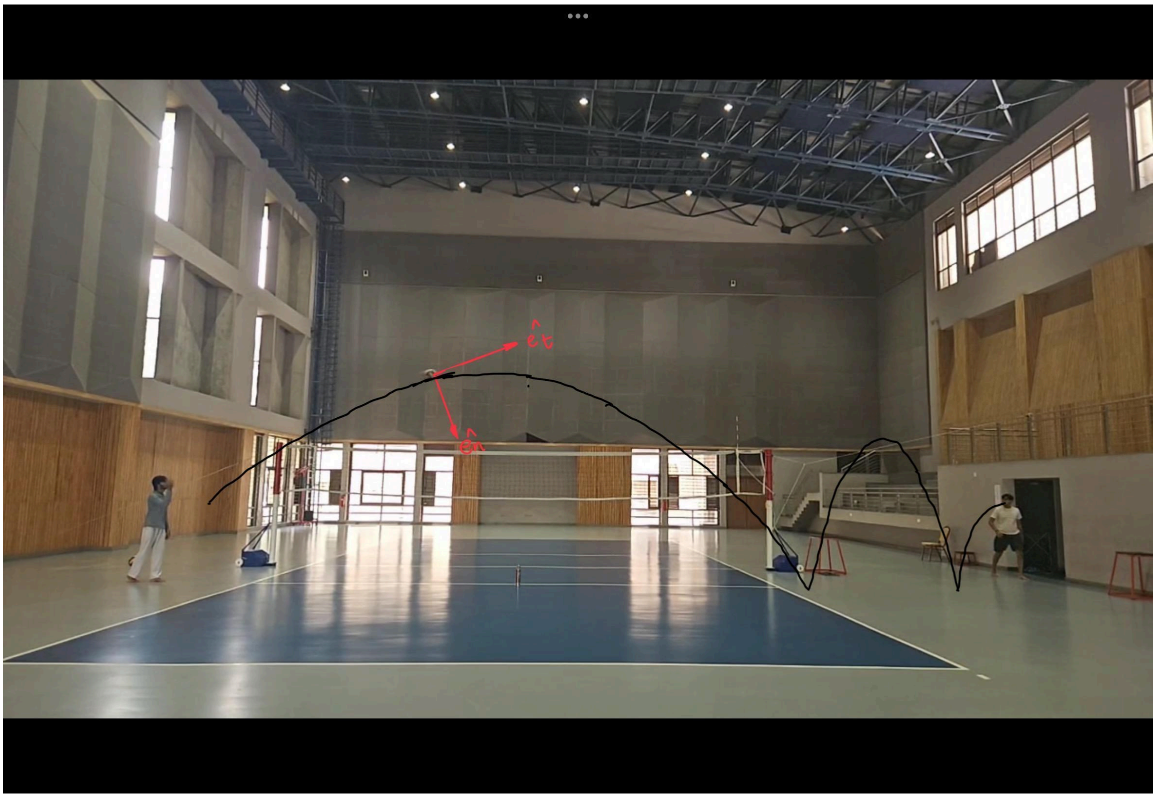
- The volleyball was served underhand from a fixed position.
- The camera was mounted on a tripod, aligned perpendicular to the plane of motion to minimise parallax errors.
- A calibration scale was placed in the frame to convert pixel distances into real-world measurements.
- Motion was recorded at 60 frames per second.
- The video was analysed frame by frame to determine the ball's position at different timestamps.

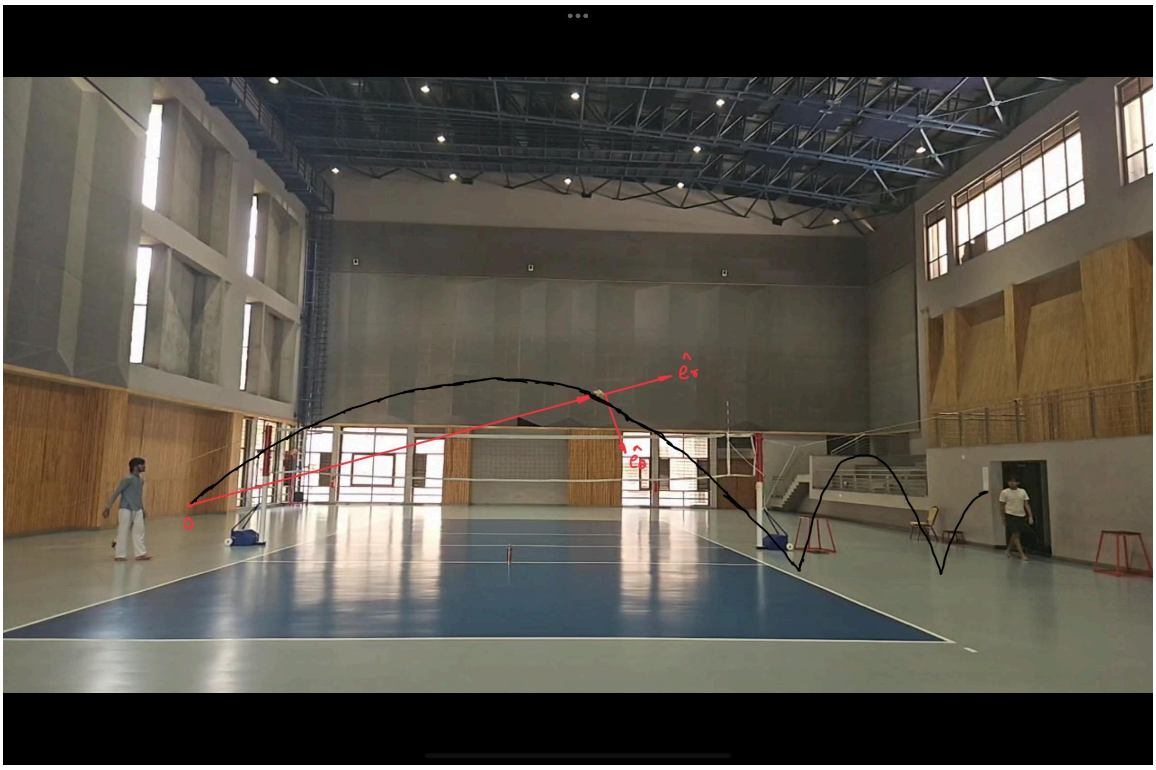
Procedure:

1. The volleyball was served underhand from a designated starting point.
2. The camera recorded the ball's motion until it landed or left the frame.
3. Using Tracker software, the ball's position was marked frame by frame.
4. The calibration scale was used to convert pixel data into meters.
5. Velocity and acceleration were calculated by differentiating position data over time.
6. The motion was modelled using:
    - Cartesian coordinates (x, y)
    - Normal and tangential coordinates
    - Radial and angular coordinates
2. Theoretical predictions were calculated with standard motion equations.
3. Experimental data were compared with theoretical results to identify discrepancies.

Design Illustration







Key Considerations

- The camera was aligned carefully to reduce perspective distortion.
- Lighting and resolution were controlled to improve accuracy.
- Multiple trials were conducted to ensure consistency.

- The software provided tools to smooth data and correct small tracking errors.

Measurement Techniques:

The primary measurement technique employed in this experiment was Digital Particle Tracking Velocimetry (DPTV)using the open-source video analysis software, ***Video Tracker***.

Methodology

1. **High-Speed Video Capture:** The trajectory of the underhand-served volleyball was recorded using a digital camera operating at 240 frames per second. The camera was kept as stable as possible and kept parallel to the plane of the trajectory of the ball.

2. **Spatial Calibration:** A bottle with known measurements was placed in front of the camera and in the same plane as the volleyball's trajectory. This allowed for the conversion of pixel coordinates from the video into real-world metric units (meters).

3. **Kinematic Data Extraction:** The recorded video was imported into the Video Tracker software. The position of the volleyball's centre was marked, generating a time series of Cartesian (x, y) coordinates.

4. **Numerical Differentiation:** The software calculated the instantaneous velocity and acceleration components (v_x , v_y, a_x, a_y) by performing numerical differentiation on the position-time data. The velocity was computed as the change in position over the time interval between frames ($\frac{dx}{dt}$, $\frac{dy}{dt}$), and acceleration was similarly computed as the change in velocity over the time interval ($\frac{dVx}{dt}$,$\frac{dVy}{dt}$).

Mathematical and Theoretical Modelling:

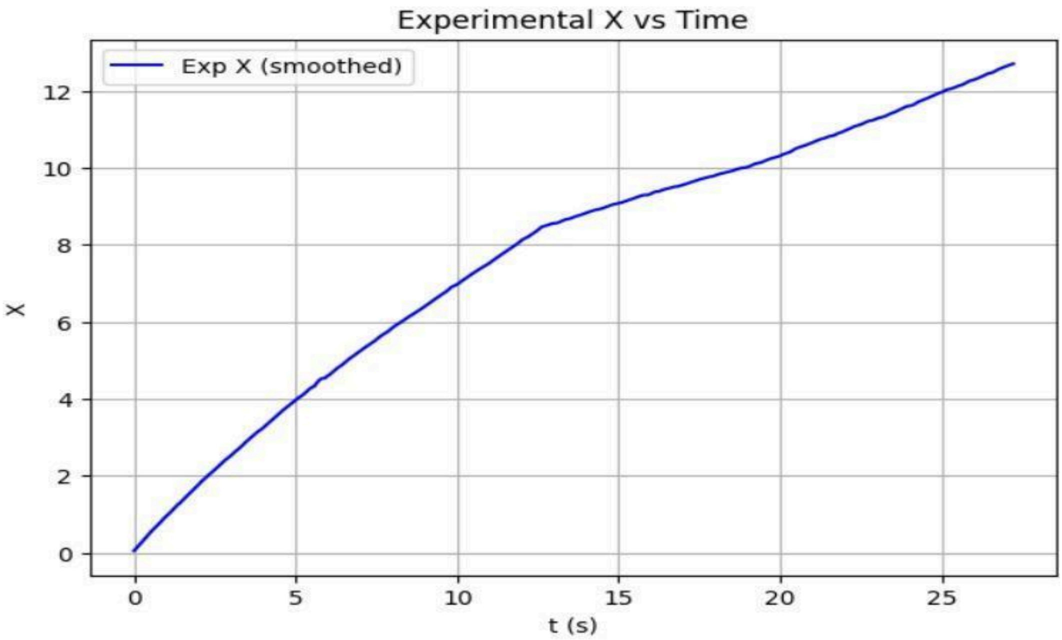
1) Cartesian Coordinate System:

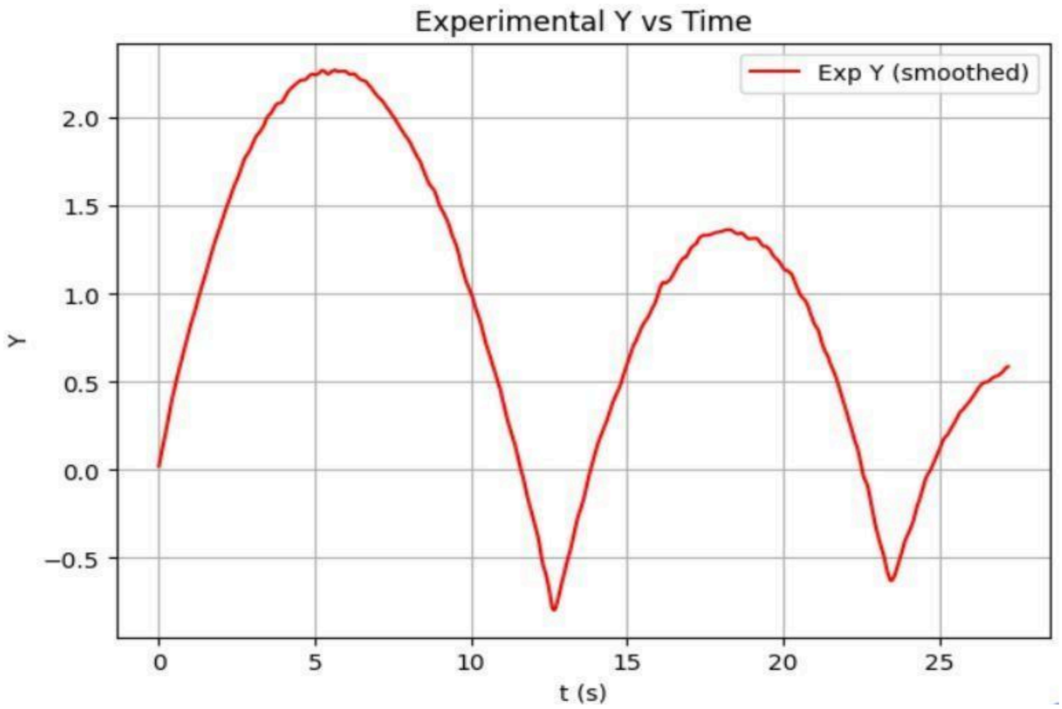
The theoretical motion of the volleyball in Cartesian coordinates is modelled as an ideal projectile. This model operates under the assumptions that the only force acting on the ball post-launch is gravity and that air resistance is negligible. The governing equations are:

Position Analysis:

$$x(t)=x_0+v_{0x}t\ ;$$

$$y(t)=y_0+v_{0y}t-\frac{1}{2}gt^2$$

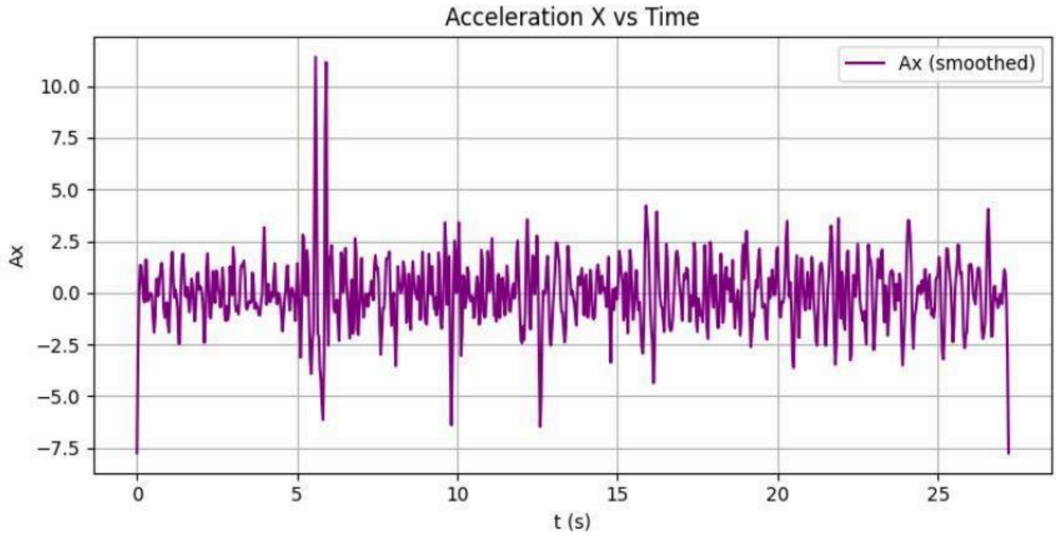




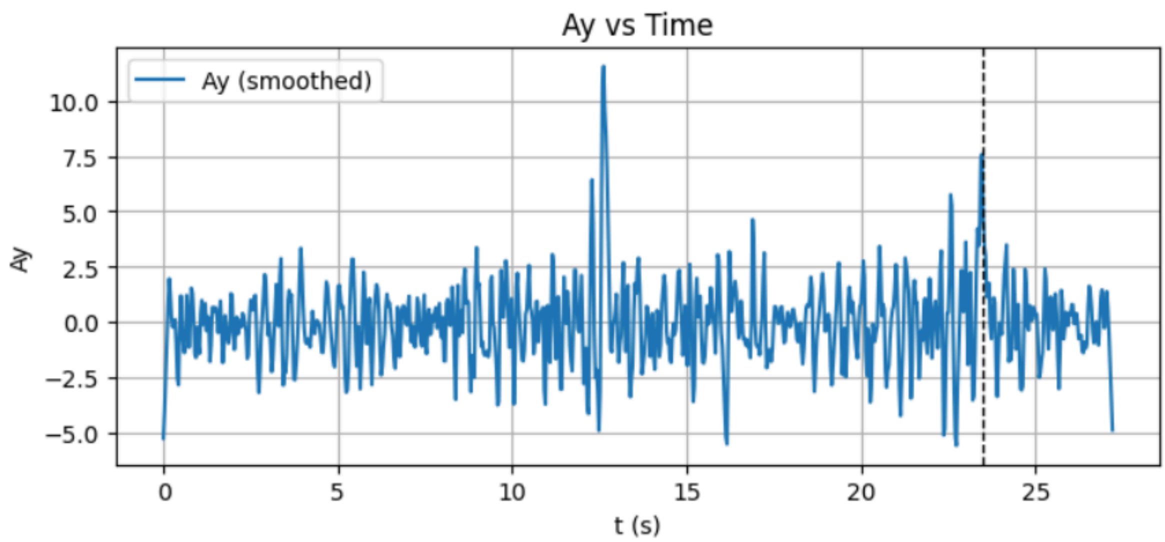
This model predicts a constant horizontal velocity, a linearly decreasing vertical velocity, zero horizontal acceleration, a constant downward vertical acceleration equal to g, and a parabolic trajectory.

Acceleration Analysis:

- **Horizontal Acceleration (a_x):** The theoretical value is 0 m/s². The experimental data a_x show significant fluctuations around zero, with a mean value close to zero but a high standard deviation.



- **Vertical Acceleration (a_y):** The theoretical value is a constant -9.81 m/s². The experimental data a_y fluctuates wildly and does not maintain a constant value near -9.81 m/s².

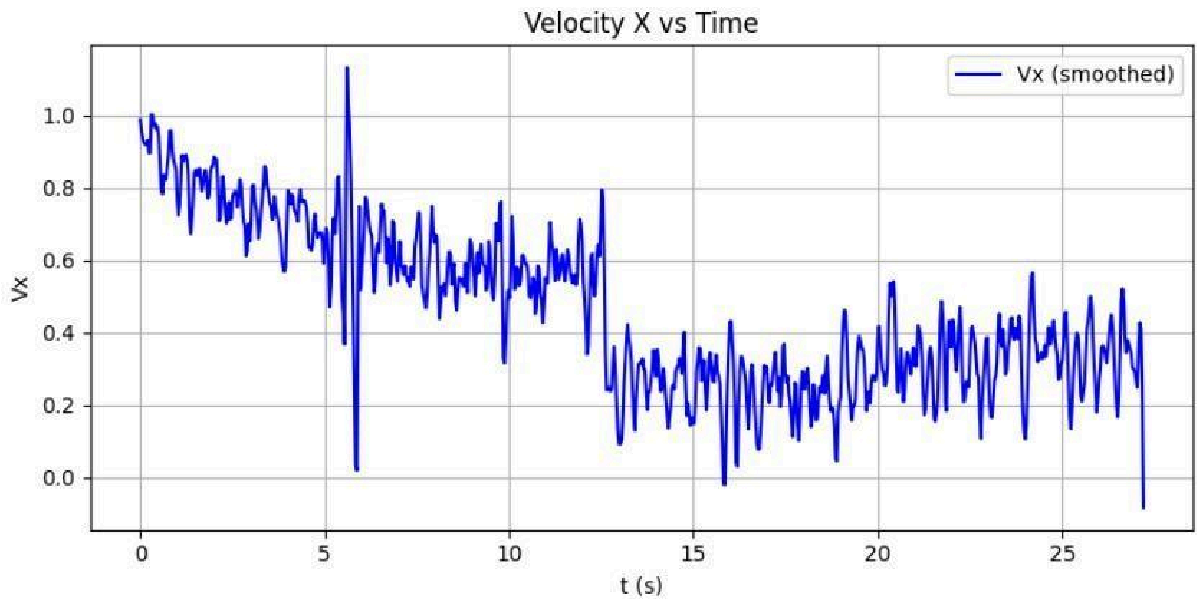


Velocity Analysis:

- **Horizontal Velocity (v_x):** The theoretical model predicts a constant v_x.

$$v_x(t)=v_{0x}\,\mathrm{i}$$

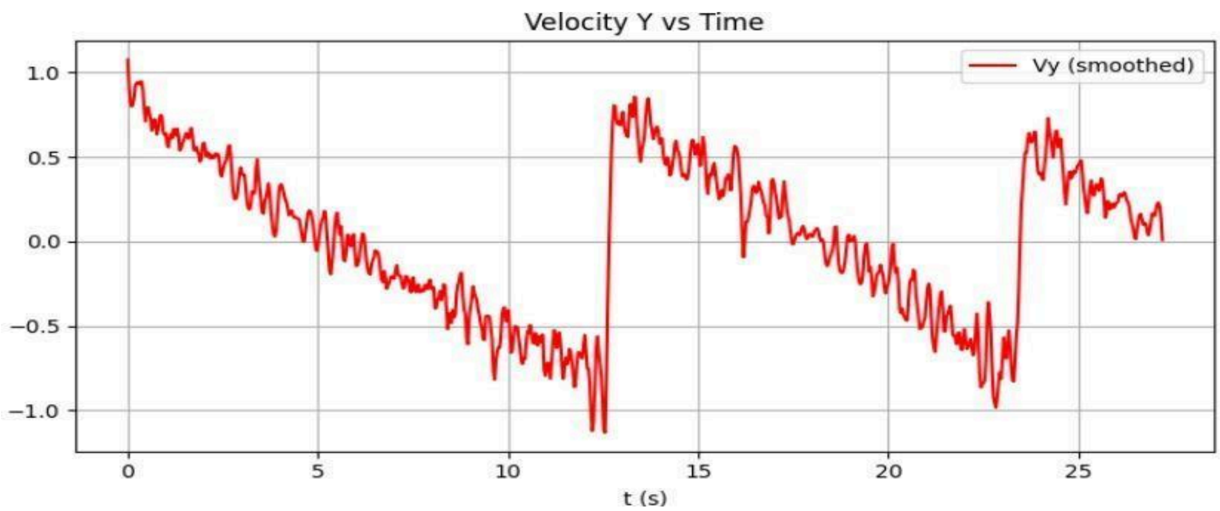
- The experimental data show a general trend of decreasing horizontal velocity over time, which contradicts the model and points to the presence of a resistive force (air drag).



- **Vertical Velocity (v_y):**

$$v_y(t)=(v_{0y}-gt)\,\mathrm{j}$$

- The experimental plot of v_y versus time shows a roughly linear decrease, which is consistent with the theoretical model. However, the slope of this line, representing the vertical acceleration, is not perfectly constant.



2) **Tangential Normal Coordinate System:**

Position Analysis:

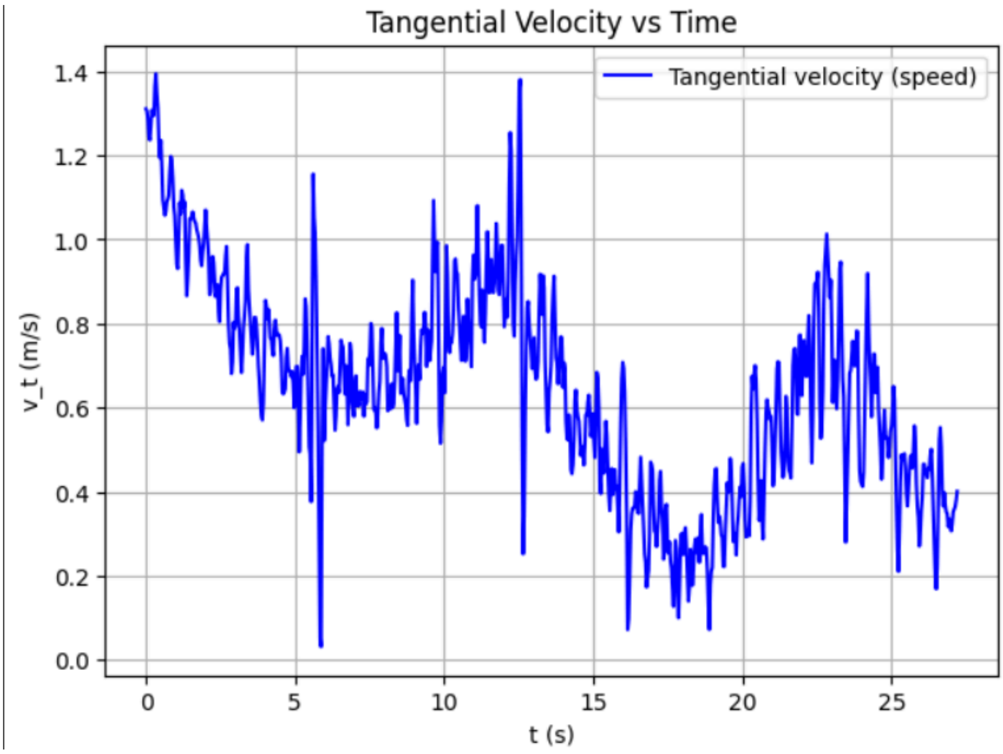
- In the T–N system, the motion of a particle is described along the instantaneous path (tangent) and perpendicular to the path (normal).
- The position vector itself is still expressed in Cartesian or polar form, but velocity and acceleration are projected into the tangent and normal directions.

Velocity Analysis:

- The velocity is:

$$v\ (t)=v\ e_t$$

- The direction of velocity is always tangent to the trajectory.



Acceleration Analysis:

- The total acceleration has two components:

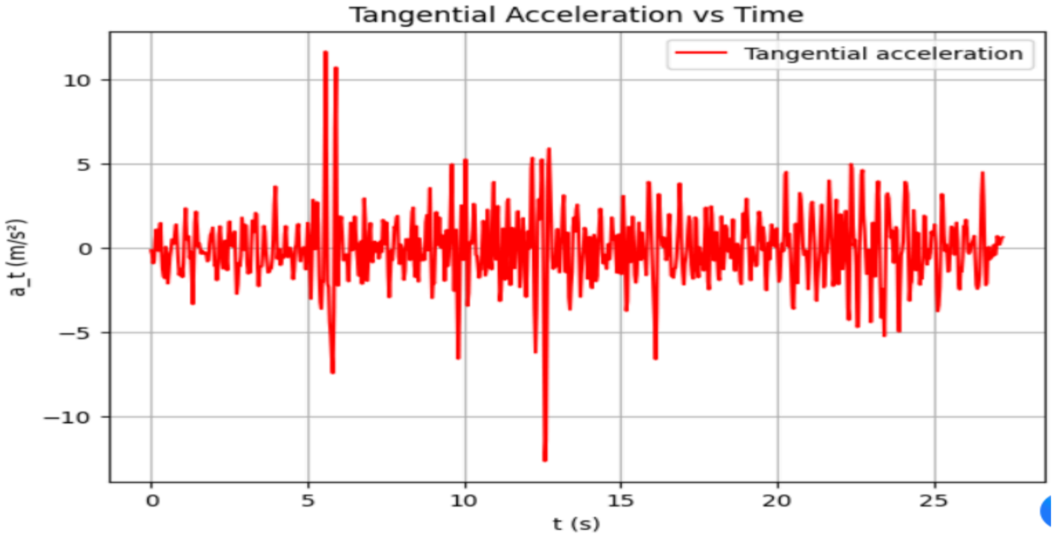
$$a(t)=\frac{dv}{dt}e_t+\frac{v^2}{R}\ e_n$$

where

- **Tangential acceleration**:

  $$a = \frac{dv}{dt} e_t$$

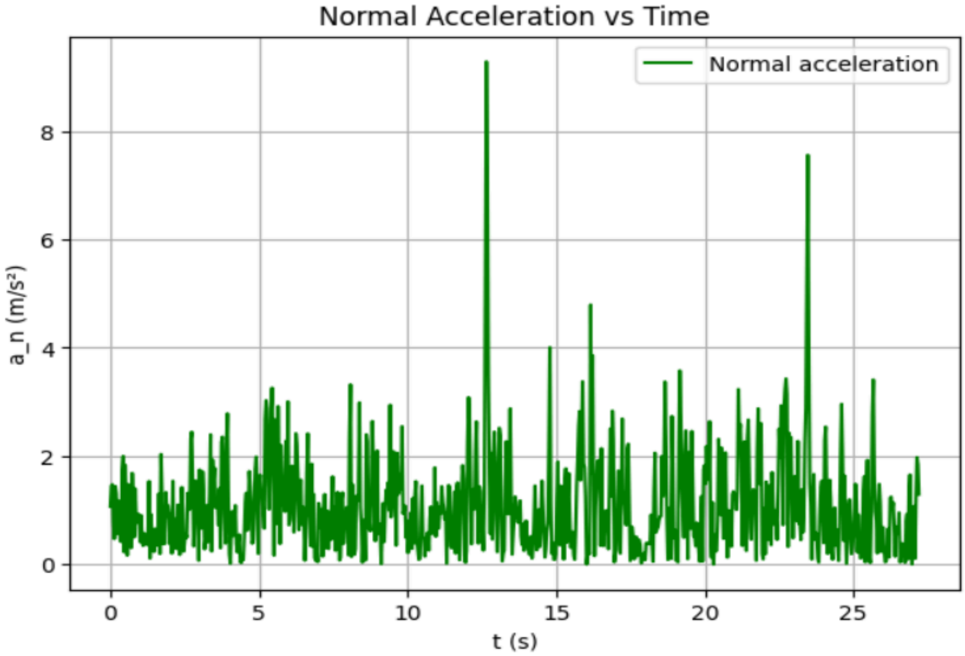
  Represents the rate of change of speed (magnitude of velocity).



- **Normal acceleration**:

  $$a = \frac{v^2}{R} e_n$$

  where R is the radius of curvature. This represents the change of direction of the velocity vector.



3) **Polar Coordinate System:**

Position Analysis:

- In plane motion, the position of the particle is expressed as:

$$r(t) = r\, e_r$$

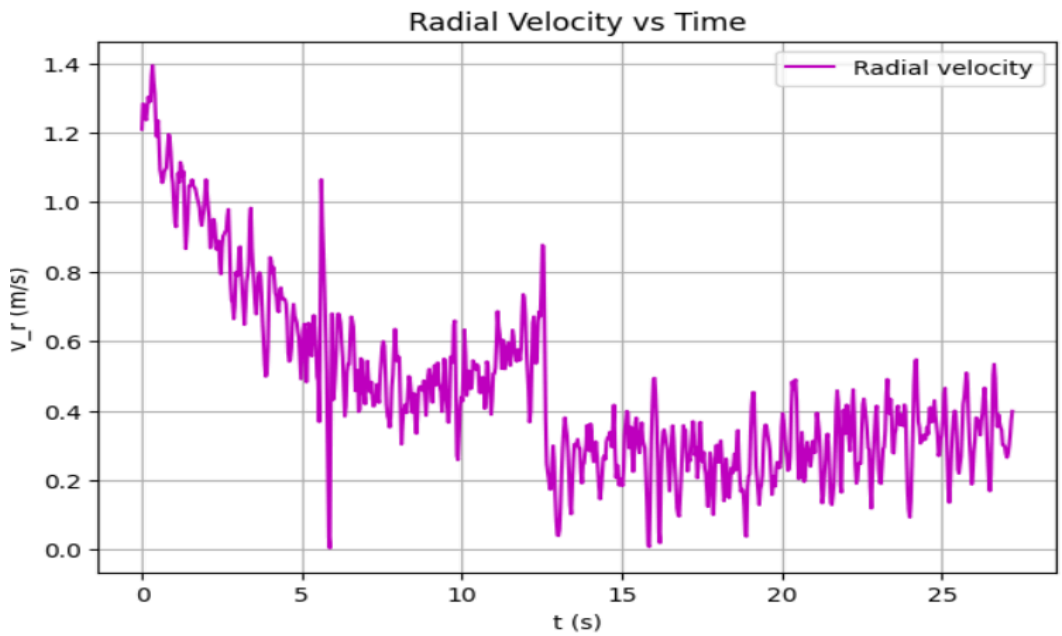
where r is the radial distance from the origin, and e_r is the radial unit vector.

Velocity Analysis:

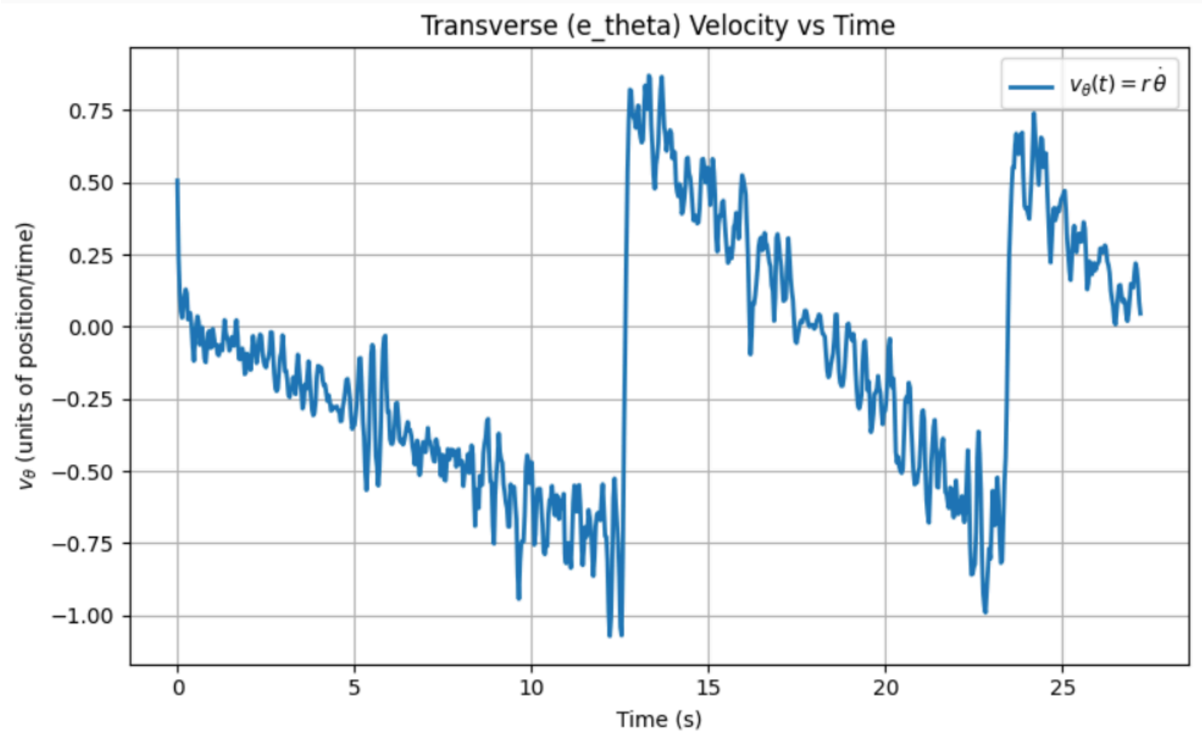
- Velocity in polar form has both radial and transverse components:

$$v(t)= \frac{dr}{dt} e_r + r\frac{d\theta}{dt}\, e_\theta$$

  - Radial component: $\frac{dr}{dt}$ (change in distance).



- Transverse component: $r\frac{d\theta}{dt}$ (angular motion contribution).



Acceleration Analysis:

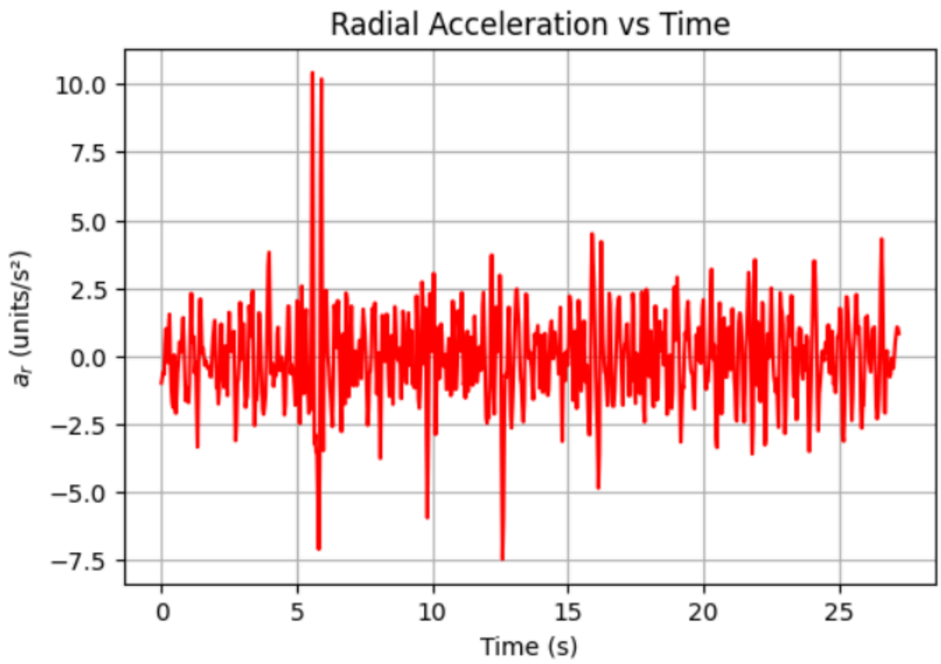
- Acceleration in polar form is:

$$a(t) = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)e_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)e_\theta$$

- Radial acceleration:

$$a_r = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)$$

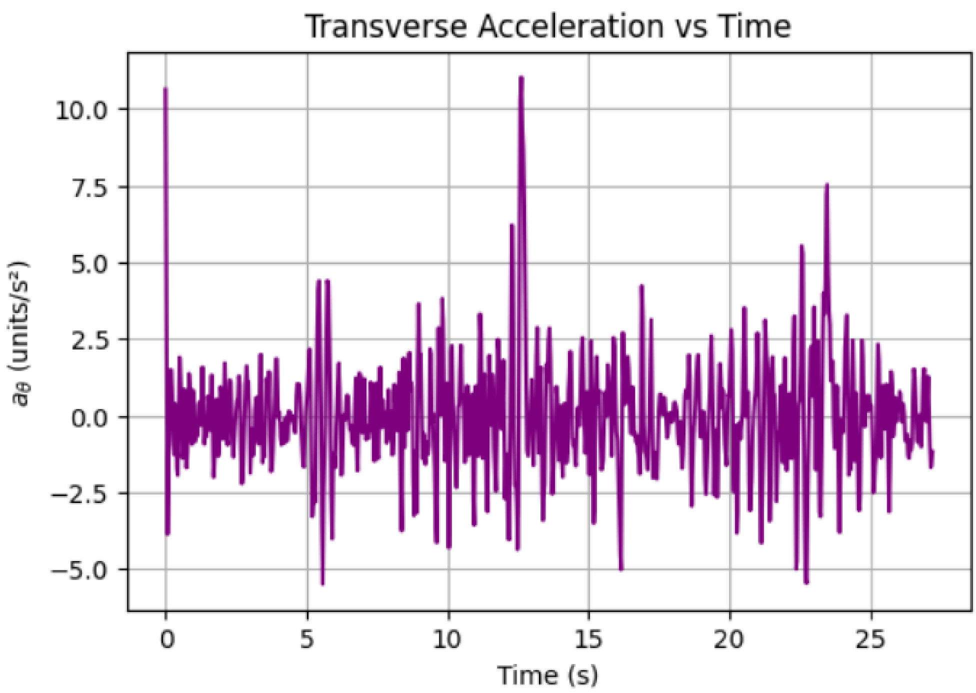
Combines radial change and centripetal effect.



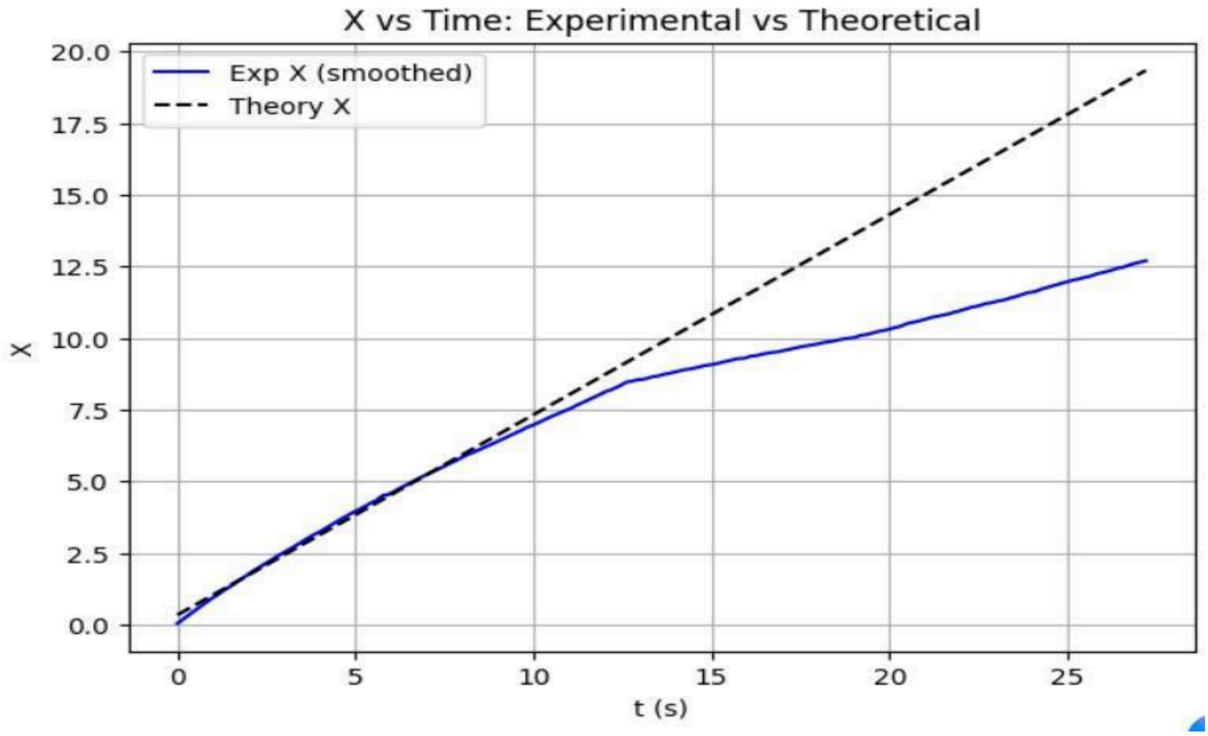
- Transverse acceleration:

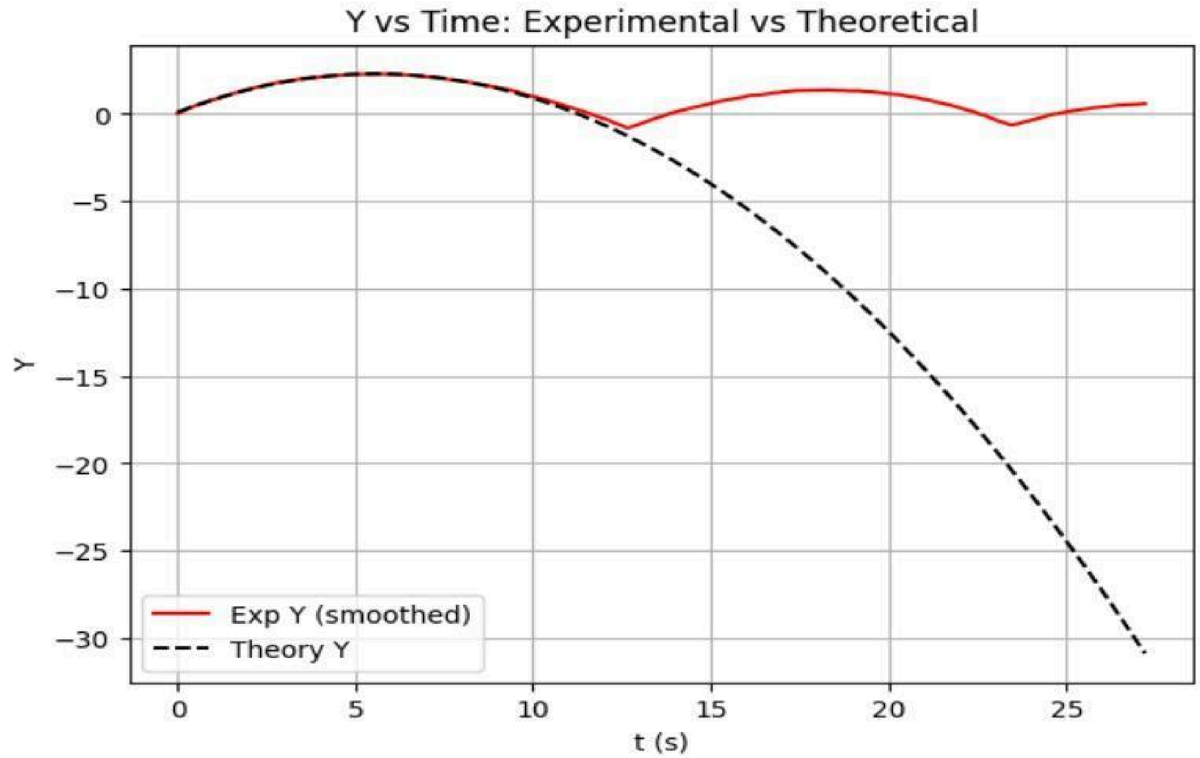
$$a_\theta = \left(r\frac{d^2\theta}{dt^2} \; + \; 2\frac{dr}{dt}\frac{d\theta}{dt} \right)$$

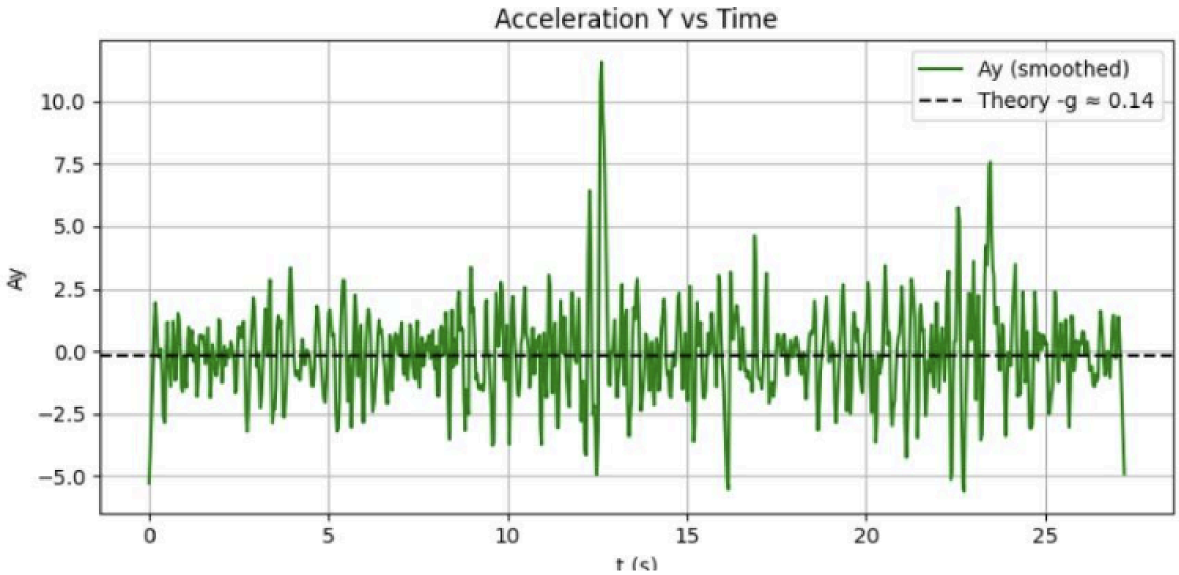
Includes angular acceleration and Coriolis-like term.



Experimental Graph V/S Theoretical Graph :
(Cartesian Coordinate System)







Tracker (and your code) compute velocities and accelerations from pixel positions (or calibrated coordinates). If the scale calibration was not set correctly, the conversion from "pixels per frame²" → "m/s²" will be wrong.

Specifically, if your calibration stick (the reference length you gave Tracker) was too large compared to the real scale, then each pixel corresponds to an exaggerated physical length. That reduces the calculated accelerations drastically.

$$a_y(measured) = \ a_y(true) \text{ x L(pixel calibration) / L(true)}$$

$$a_y(measured) = -0.14$$

Theoretical VS Experimental Deviation Reasons:

1) Physics effects (true dynamics that theory ignores):

- Initial impulse (finite contact time with the hand)

Initially, at t = 0, the hand exerts a force instantaneously for a short time. This generates a short-duration upward acceleration a_y that opposes gravity while the ball is still in the hand. Early-time a_y more positive (or less negative) than -g; a short region where v_y rapidly rises.

- Air resistance (drag)

Effect on motion:

- Downward acceleration magnitude is reduced (effective ($a_y > -g$) in magnitude); trajectory is flatter than a no-drag parabola.
- Horizontal speed decays slowly: ($v_x(t)$) is not strictly constant.

Why fitted (g) drops: when you fit a no-drag parabola to data influenced by drag, the fitter compensates by reducing (g) so the quadratic term is smaller—a systematic bias.

- Bounces/impacts (inelastic collisions)

Contact with the ground produces a large, short impulse upward.

Signatures:

  - Sudden sign flip in (v_y), sharp spikes in (a_y) (very large but short).
  - Step-like reductions in horizontal speed if tangential impulse or friction acts during collision.
  - Rotation/spin and deformation

Rotational motion and contact friction can transfer translational energy into rotation and vice versa, further reducing post-collision (v).

Ball deforming stores energy temporarily and dissipates it as heat/sound → loss in rebound height.

2) Measurement & recording errors

- Pixel → meter scaling & perspective

If the calibration stick is not exactly in the same plane or the camera shows perspective, the scale along (x, y) is biased.

Effect: systematic scaling error in x,y → proportional scaling of (v) and (a) (so fitted (g) will scale incorrectly).

- Out-of-plane motion/parallax

If the ball moves toward/away from the camera, pixel scale changes over time → nonlinear distortion of measured positions.

Effect: artificial curvature or sudden changes in measured (x, y).

- Frame rate (temporal sampling)

Impacts are very short; if frame rate is low, you sample only a few frames during impact → you underestimate peak (a) and mis-estimate pre/post velocities → restitution error and smearing of spike shape (aliasing).

Effect: acceleration spikes are reduced in amplitude and smeared; restitution measured from the nearest frames becomes inaccurate.

- Tracker noise & tracking jitter

Incorrect point detection, flicker, or mis-tracking causes small random jumps in coordinates.

Effect: derivative amplification—noise becomes large in (v) and especially in (a).

3) Data processing & modelling issues

- Smoothing (Savitzky–Golay or moving average)

Smoothing reduces noise but also reduces curvature (second derivatives), which lowers apparent acceleration magnitudes, particularly across short events (impacts) and near the apex.

Effect: fitted (g) from smoothed (y(t)) tends to be smaller; (a_y) spikes attenuated; restitution estimates reduced.

- Numerical differentiation

Differentiation amplifies noise. Central difference formulas are fine, but only after smoothing. Non-uniform (Δt) must be handled properly.

Effect: raw (a) very noisy; smoothing helps but trades fidelity.

4) How do these differences manifest in your signals

For each signal, listed typical deviations and the physics/measurement cause(s) are listed:

(x(t)): theory linear; experiment slightly curved or with slope drops after bounce.

- Causes: horizontal friction at bounce, parallax errors, drag in (x), and scaling error.

(y(t)): theory perfect parabola; experiment: flatter parabola (reduced curvature), apex shifted, smaller rebound peaks.

- Causes: drag (flattens curve), smoothing (reduces curvature), scale error, and rebound energy loss.

($v_{x(t)}$): theory constant (v_{0x}); experiment slowly decaying and step changes at bounce.

- Causes: horizontal drag/friction, tangential impulse on bounce, tracking noise.

($v_{y(t)}$): theory linear (-g); experiment: initial quick rise (hand impulse), roughly linear between events but with slope slightly shallower than (-g), instantaneous sign changes at bounces.

- Causes: hand impulse, drag, bounce impulse, and smoothing blunting slope.

($a_{y(t)}$): theory constant (-g); experiment: early-time positive/less-negative segment (hand impulse), then values of magnitude smaller than (g) (drag), and very large short upward spikes at bounces.

- Causes: initial impulse, drag, impact spikes, and smoothing attenuating spikes.

Fitted (g): theory fixed 9.81; experiment (g_{fit} 9.81) usually.

- Causes: drag (main), smoothing bias, fitting to a limited/biased segment, and calibration error.

Coefficient of restitution (e): theory (elastic) (e = 1) if ideal; experiment (e 1).

- Causes: inelastic deformation, energy dissipation, rotational energy, and measurement sampling error.

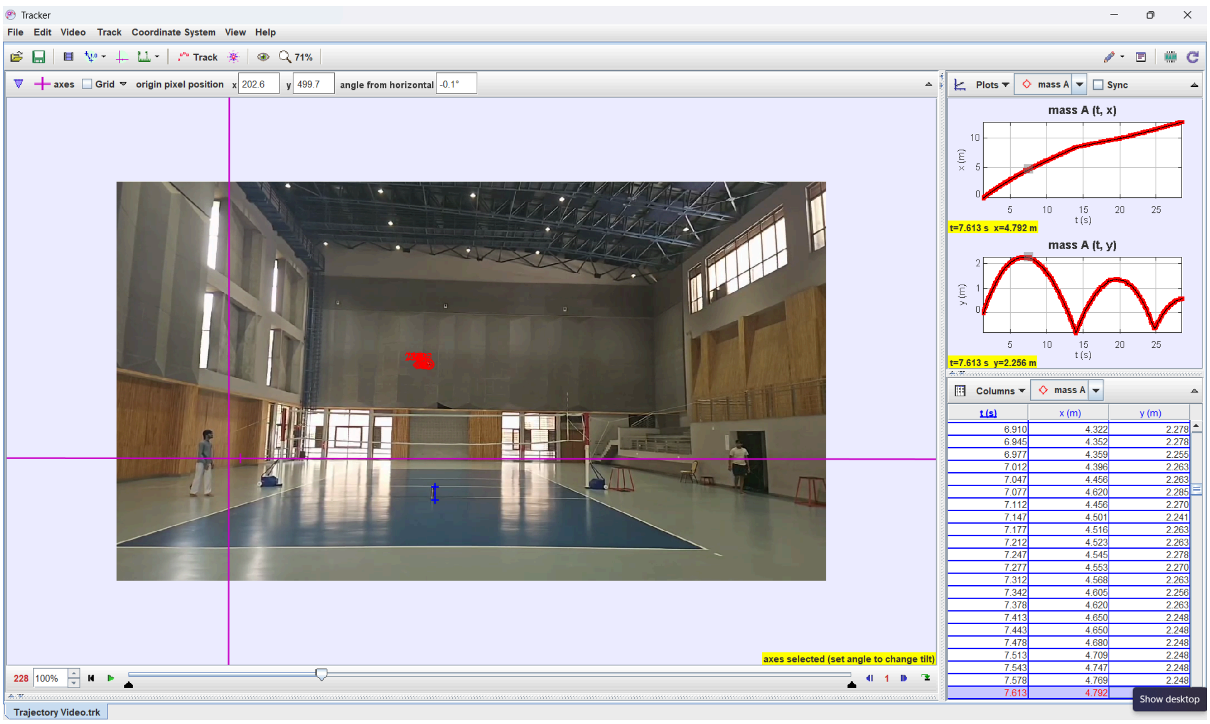
Remedies/improvements:

- Use a high-frame-rate camera to resolve impulses and spikes.
- Place a calibration stick in the same plane as the motion (reduce pixel → meter bias).
- Keep the motion in one plane and the camera orthogonal to that plane (minimise parallax).
- Mark the ball with a high-contrast marker; use automatic tracking.
- Increase the trajectory arc recorded (longer flight) so quadratic curvature is more prominent and less dominated by noise.

Software/ tools Used:

1) Tracker (Open-Source Video Analysis Tool):

The experimental tracking of particle motion was carried out using *Tracker*, an open-source video analysis and modelling software. Tracker allowed us to import motion videos, mark particle positions frame by frame, and automatically extract kinematic data such as displacement, velocity, and acceleration. This tool provided an effective way to bridge the gap between experimental observation and theoretical motion models.



2) **Python Libraries for Data Processing and Visualisation:**

- NumPy: Used for numerical computations such as handling arrays, calculating velocity and acceleration from position data, and performing mathematical operations efficiently.

- Pandas: Utilised for organising, cleaning, and analysing the experimental data in tabular form. Pandas enabled smooth handling of large datasets and facilitated structured data analysis.

- Matplotlib: Applied for graphical analysis and visualisation. It was used to plot position, velocity, and acceleration graphs, compare experimental data with theoretical predictions, and highlight key deviations caused by external factors (e.g., air resistance).

Acknowledgement:

We thank those who assisted and participated in completing our experiment 1 for the Statics and Dynamics course. Most of all, we sincerely thank Professor K.R. Jayaprakash for his remarkable mentorship and expert guidance. His clear expositions and considerate assistance have incredibly enhanced our control over the subject and helped us overcome the intricacies of the experiment. We especially want to thank the teaching assistants, whose valuable advice and insightful observations contributed very much toward the smooth running of our work . Moreover, we thank the Indian Institute of Technology Gandhinagar for providing the opportunities we could use to perform this experiment. Lastly, we would like to thank our fellow students and colleagues for their cooperation, feedback, and support, which contributed to the experiment's accomplishment.