ME 206 – Statics and Dynamics

Experiment 2 - Measurement of Position, Velocity, and Acceleration of a Particle in 2D in a rotating frame of reference

Experiment Topic - Tracking the trajectory of a volleyball just dropped under gravity, with a rotating frame of reference.

Group 12

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Introduction:

In engineering and physics, understanding the motion of objects as seen from different reference frames is essential for analysing real-world systems. Many natural and mechanical processes take place in frames that are themselves in motion — for example, the Earth's rotation affects weather patterns, and rotating machinery parts experience additional apparent forces. To describe such systems accurately, it becomes necessary to study relative motion — how the observed position, velocity, and acceleration of a particle depend on the motion of the observer's frame.

In this experiment, we aim to observe and analyse the motion of a freely falling ball from a rotating reference frame. The motion of the ball, when seen from an inertial (non-rotating) frame, is a simple vertical free-fall under gravity. However, when the same motion is viewed from a rotating frame, it appears modified due to the presence of additional apparent accelerations. These apparent effects arise purely due to the rotation of the observer's frame and include phenomena such as the centrifugal and Coriolis accelerations. Through this experiment, we attempt to visualise how a simple vertical fall is perceived differently in a rotating coordinate system and how the measured values of velocity and acceleration deviate from their theoretical inertial counterparts.

To achieve this, a custom 3D-printed rotating tripod setup was designed and fabricated. The tripod holds a smartphone camera, which captures the free-fall of a small ball while rotating at an approximately constant angular velocity of $\Omega = 11.519$ rad/s about the z-axis. The rotation is powered by a DC motor driven by a 9 V battery. The setup is lightweight and portable, and the phone is held firmly to prevent unwanted translational motion or inclination during recording. The fall of the ball is recorded from a fixed height of $h = 5$ m, and the resulting video provides the motion data of the particle as viewed from the rotating frame.

Using this data, the position of the ball is extracted frame by frame with the help of motion-tracking software. From the position information, the velocity and acceleration of the particle are determined as a function of time. The results are then compared with the theoretical expectations for a freely falling body in an inertial frame of reference. The primary objective is to understand how rotation affects the observed motion and how the theoretical and experimental values of velocity and acceleration differ due to the effects of the rotating reference frame.

This study not only reinforces the concept of reference frames but also illustrates the practical challenges in measurement and analysis when the observer's frame is non-inertial. Through this simple yet insightful setup, students can directly visualise how rotation influences motion and gain an intuitive understanding of relative motion, fictitious forces, and the importance of frame selection in dynamics.

Experimental Design:

Aim of the Experiment:

To measure and analyse the position, velocity and acceleration of a particle in free-fall as observed from a rotating frame of reference with approximately constant angular speed Ω, and to identify the Coriolis and centrifugal contributions in the measured accelerations.

Experimental Setup:

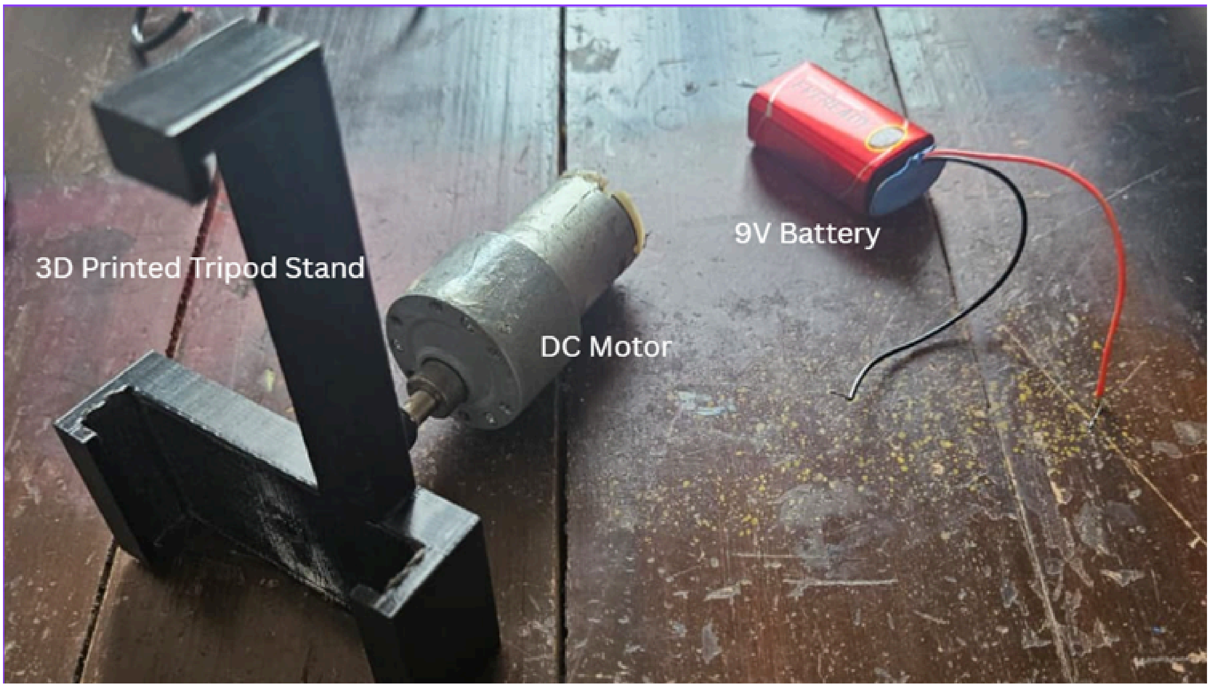
To study the motion of volleyball, we created a controlled environment where the ball was served and tracked as it followed a curved path influenced by gravity and air resistance.

Apparatus/Equipment

- Volley Ball
- 3D-printed rotating tripod
- Motor controller to set approximately constant angular speed (Ω)
- Calibration scale (ruler or marked stick) placed in the plane of the drop and visible in the video
- Computer with Tracker and Python (NumPy, Pandas, SciPy, Matplotlib) for processing

Setup Description:

- The rotating mount holds the camera (phone) and rotates about a vertical axis with angular velocity $\boldsymbol{\Omega} = \Omega\,\hat{\mathbf{k}}$. The drop region is placed in front of the rotating camera so the ball falls within the camera's field of view throughout the drop.
- Align the camera so its optical axis points roughly radially inward/outward — keep the motion approximately in a plane perpendicular to the camera's axis to reduce parallax. Place a calibration scale in the same plane as the fall.
- Measure and log the motor angular speed Ω. If available, record the camera's internal gyroscope for redundancy.
- Motion was recorded at 60 frames per second.
- The video was analysed frame by frame to determine the ball's position at different timestamps.

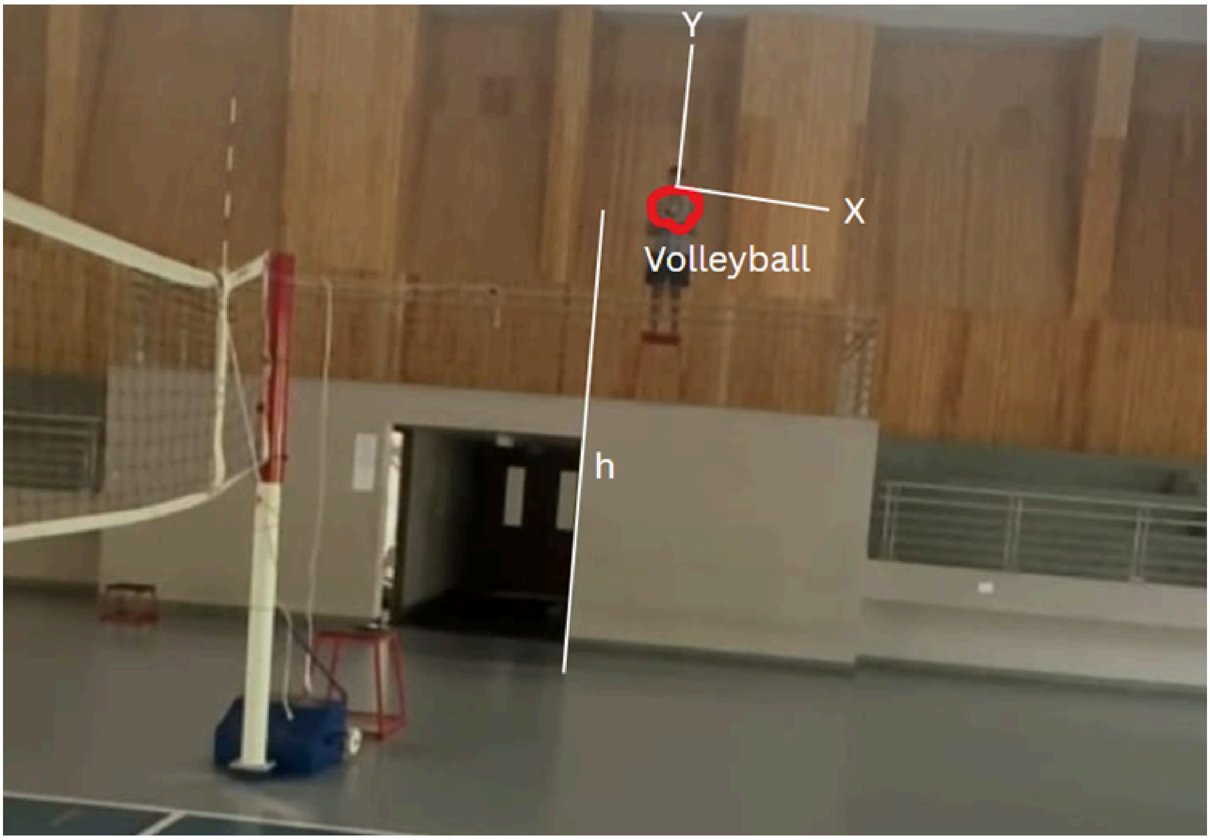


Apparatus/Material required

Fully attached apparatus

Procedure

- Fix the camera on the rotating mount and set the motor to the desired constant Ω. Record the measured Ω.
- Position the calibration scale in the plane of the drop and ensure it remains visible in the video.
- Mark the ball with a high-contrast dot to improve tracking. Place the ball at the fixed height h above the landing surface.
- Start rotation and allow it to reach steady Ω. Start video recording (confirm frame rate).
- Release the ball without imparting horizontal velocity (try to minimize hand motion). Record the entire fall until impact or until it leaves the frame.
- Repeat the drop multiple times for each Ω (e.g., 3–5 trials per Ω) .



Volleyball court where we carried out our experiment (z–axis is the direction vector perpendicular to the screen)

Key Considerations:

- The camera was aligned carefully to reduce perspective distortion.
- Lighting and resolution were controlled to improve accuracy.
- Multiple trials were conducted to ensure consistency.
- The software provided tools to smooth data and correct small tracking errors.

Measurement Techniques:

The primary measurement technique employed in this experiment was Digital Particle Tracking Velocimetry (DPTV)using the open-source video analysis software, ***Video Tracker***.

Methodology

1. **High-Speed Video Capture:** The trajectory of the underhand-served volleyball was recorded using a digital camera operating at 240 frames per second. The camera was rotated with the setup explained above, with constant Ω and kept parallel to the plane of the trajectory of the ball.

2. **Spatial Calibration:** A bottle with known measurements was placed in front of the camera and in the same plane as the volleyball's trajectory. This allowed for the conversion of pixel coordinates from the video into real-world metric units (meters).

3. **Kinematic Data Extraction:** The recorded video was imported into the Video Tracker software. The position of the volleyball's centre was marked at every frame, generating a time series of Cartesian (x, y) coordinates.

4. **Numerical Differentiation:** The software calculated the instantaneous position, velocity and acceleration components **(**r_{ABx}, r_{ABy}, v_{ABx} , v_{ABy}, a_{ABx}, a_{ABy} **)**. Where r_{AB}, v_{AB}, a_{AB}

is the position of the ball in the rotating frame of reference.

Mathematical and Theoretical Modelling:

1) Position of the particle:

$$r_A = r_B + r_{AB}$$

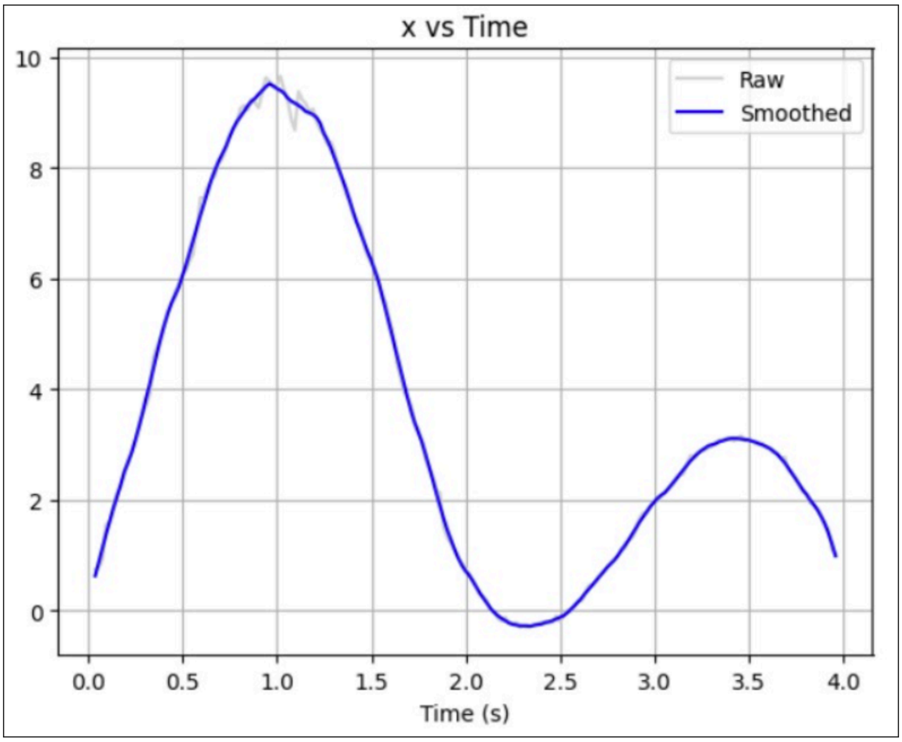
(Here r_A is the position vector to the particle from the fixed frame of reference, r_B is the position vector to the rotating frame, r_{AB} is the position vector to the particle from the rotating frame of reference)

- In our experimental setup, the fixed frame of reference and the rotating frame 0of reference coincide, making $r_B = 0$.
- The experiment only measures the position of the particle in one plane, that is, in x and y coordinates; thus, we can express r_{AB} it in terms of i and j vector components.

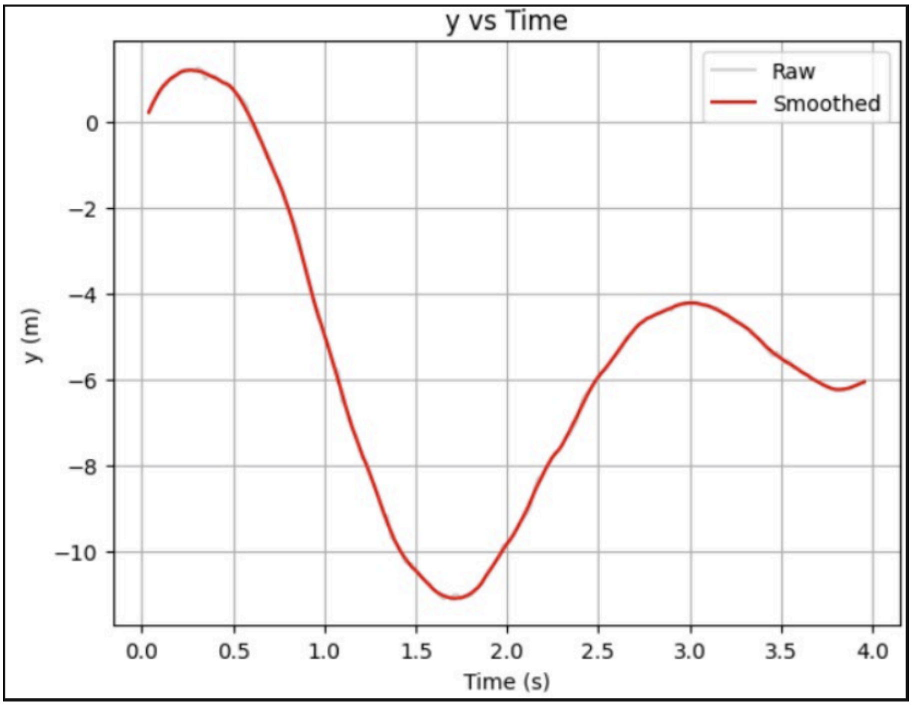
$$\Rightarrow \ r_{AB} = r_{ABx}\,\mathbf{i} + r_{ABy}\,\mathbf{j}$$

$$\therefore \ r_A = r_{ABx}\,\mathbf{i} + r_{ABy}\,\mathbf{j}$$

- From the software, we got every x and y coordinate of the particle, which will be $r_{ABx}\,\mathbf{i}$ and $r_{ABy}\,\mathbf{j}$.



$(r_{ABx}\ V/S\ time\ graph)$



$(r_{ABy}\ V/S\ time\ graph)$

2) Velocity of particle:

$$\frac{d\,r_A}{dt} = \frac{d\,r_{AB}}{dt} + \mathbf{\Omega} \times r_{AB}$$

- The software we used to measure the free-falling particle gave the instantaneous velocity in x and y coordinates, which can express $\frac{d\,r_{AB}}{dt}$ in v_{ABx} **i** and v_{ABy} **j.**
- From the experimental setup, **Ω** was in the z direction, making **Ω** = |**Ω**| **k,** where the magnitude of |**Ω**| was 110 rpm, equivalent to 11.519 rad/s**.**

$$\Rightarrow \frac{d\,r_{AB}}{dt} = v_{ABx}\mathbf{i} + v_{ABy}\,\mathbf{j}$$

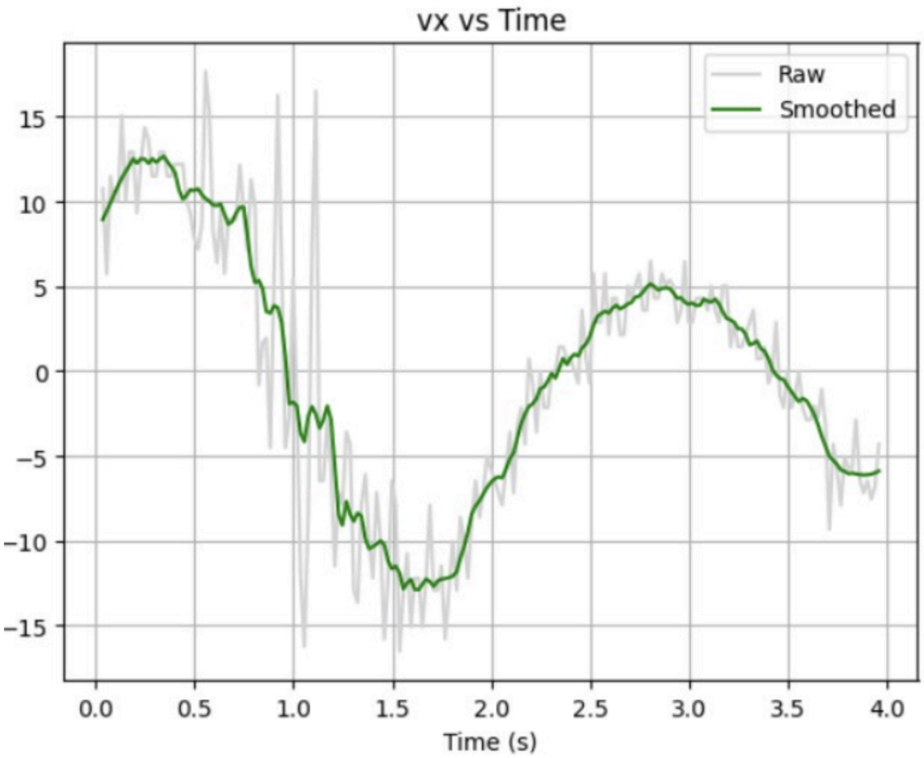
$$\Rightarrow \mathbf{\Omega = 11.519\ k\ rad/s}$$

$$\therefore \frac{d\,r_A}{dt} = v_{ABx}\mathbf{i} + v_{ABy}\,\mathbf{j} + \mathbf{\Omega} \times (r_{ABx}\,\mathbf{i} + r_{ABy}\,\mathbf{j})$$

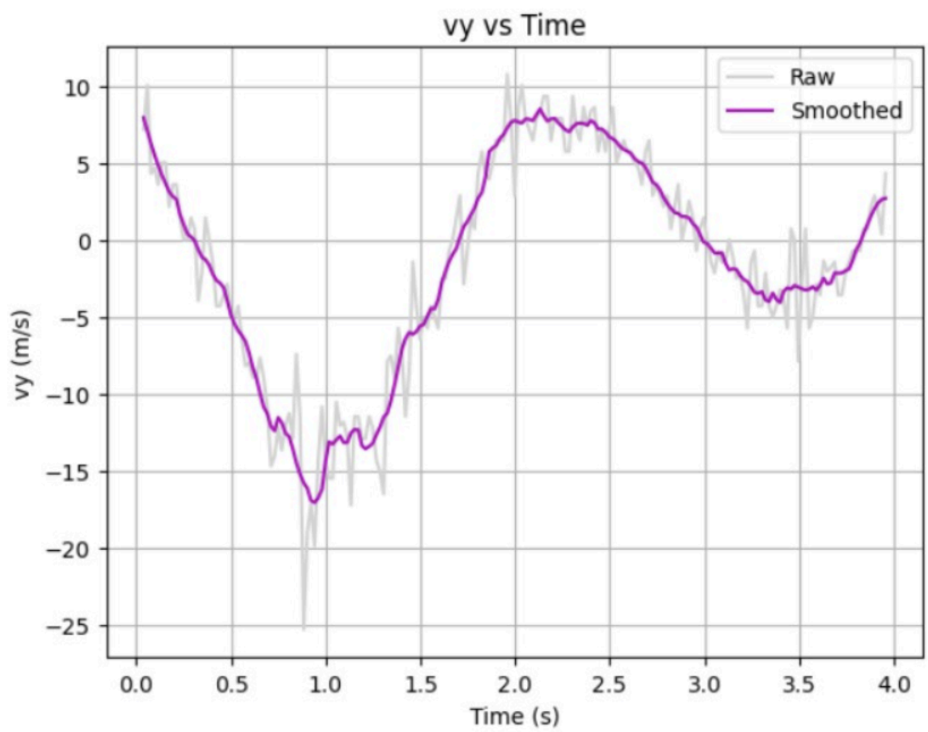
$$\therefore \frac{d\,r_A}{dt} = v_{ABx}\mathbf{i} + v_{ABy}\,\mathbf{j} + \mathbf{\Omega} r_{ABx}\mathbf{j} - \mathbf{\Omega} r_{ABy}\mathbf{i}$$

$$\therefore \frac{d\,r_A}{dt} = (v_{ABx} - \mathbf{\Omega} r_{ABy})\mathbf{i} + (v_{ABy} + \mathbf{\Omega} r_{ABx})\mathbf{j}$$

- From the software, we got every x and y velocity of the particle, which will be v_{ABx} **i** and v_{ABy} **j**.



$(v_{ABx}\ V/S\ time\ graph)$



$(v_{ABy}\ V/S\ time\ graph)$

3) **Acceleration of the particle:**

$$\frac{d}{dt}\left(\frac{d\,r_A}{dt}\right) = \frac{d^2\,r_{AB}}{dt^2} + 2\boldsymbol{\Omega} \times \frac{d\,r_{AB}}{dt} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times r_{AB})$$

- The software we used to measure the free-falling particle gave the instantaneous acceleration in x and y coordinates, which can express $\frac{d^2\,r_{AB}}{dt^2}$ in $a_{ABx}\,\mathbf{i}$ and $a_{ABy}\,\mathbf{j}$.

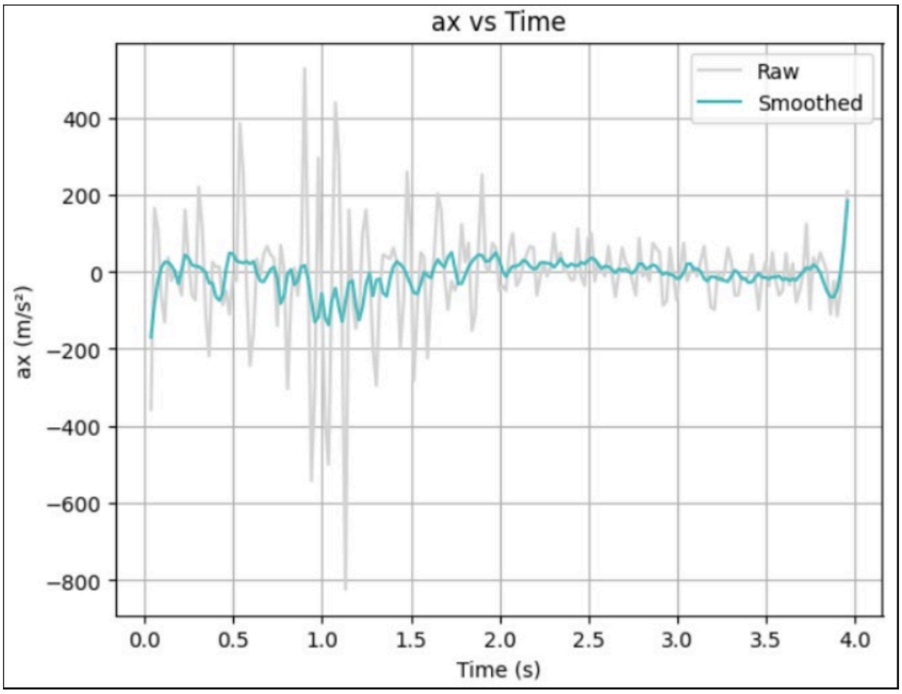
$$\Rightarrow \frac{d^2\,r_{AB}}{dt^2} = a_{ABx}\,\mathbf{i} + a_{ABy}\,\mathbf{j}$$

$$\therefore \frac{d}{dt}\left(\frac{d\,r_A}{dt}\right) = (a_{ABx}i + a_{ABy}j) + 2\boldsymbol{\Omega} \times (v_{ABx}\,\mathbf{i} + v_{ABy}\,\mathbf{j}) + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times (r_{ABx}\,\mathbf{i} + r_{ABy}\,\mathbf{j}))$$

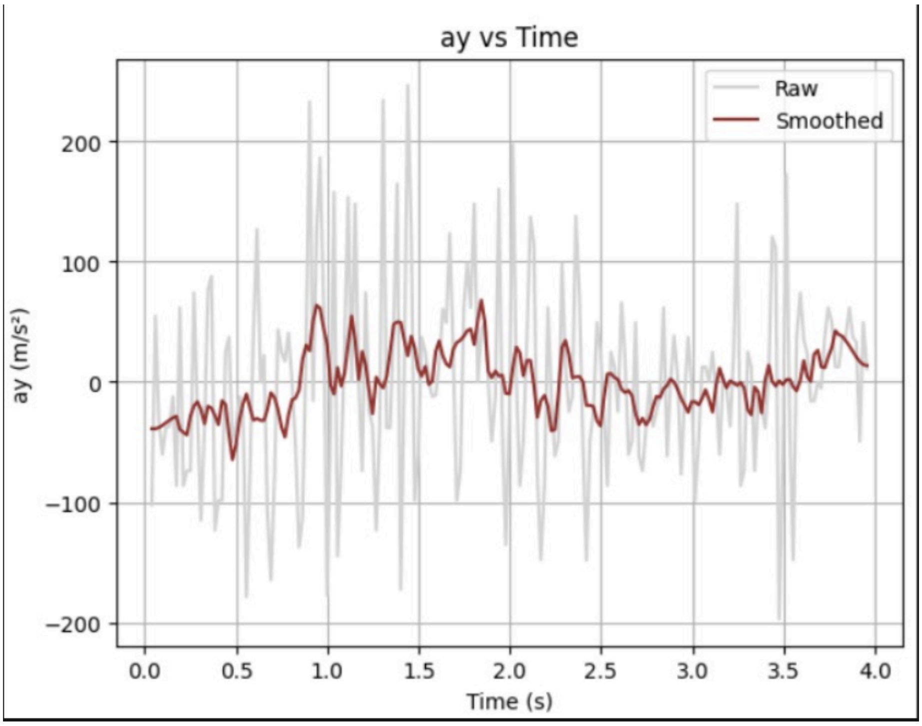
$$\therefore \frac{d}{dt}\left(\frac{d\,r_A}{dt}\right) = (a_{ABx}i + a_{ABy}j) + \boldsymbol{\Omega} v_{ABx}\mathbf{j} - \boldsymbol{\Omega} v_{ABy}\mathbf{i} - \Omega^2 r_{ABx}\mathbf{i} - \Omega^2 r_{ABy}\mathbf{j}$$

$$\therefore \frac{d}{dt}\left(\frac{d\,r_A}{dt}\right) = (a_{ABx} - \boldsymbol{\Omega} v_{ABy} - \Omega^2 r_{ABx})\mathbf{i} + (a_{ABy} + \boldsymbol{\Omega} v_{ABx} - \Omega^2 r_{ABy})\mathbf{j}$$

- From the software, we got every x and y velocity of the particle, which will be $a_{ABx}\,\mathbf{i}$ and $a_{ABy}\,\mathbf{j}$.



$(a_{ABx}\; V/S\; time\; graph)$



$(a_{ABy}\ V/S\ time\ graph)$

Result and Discussion:

The objective of this experiment was to derive the relationships that describe how the velocity and acceleration of a particle, as observed from a rotating frame of reference, relate to those measured in a stationary (inertial) frame.

Through vector differentiation and the use of rotational kinematics, the following results were obtained:

$$\frac{d\,r_A}{dt} = \frac{d\,r_{AB}}{dt} + \mathbf{\Omega} \times r_{AB}$$

And

$$\frac{d}{dt}\left(\frac{d\,r_A}{dt}\right) = \frac{d^2\,r_{AB}}{dt^2} + 2\mathbf{\Omega} \times \frac{d\,r_{AB}}{dt} + \mathbf{\Omega} \times (\mathbf{\Omega} \times r_{AB})$$

These are the fundamental transformation equations connecting velocity and acceleration in a rotating frame to those in an inertial frame.

Interpretation of the Result

- The first term in each equation represents the actual rate of change (velocity or acceleration) measured within the rotating frame itself.

- The second term in the velocity equation, $\mathbf{\Omega} \times r_{AB}$, accounts for the apparent velocity due to the rotation of the frame.

- In the acceleration equation, the second term, $\mathbf{2\Omega} \times \frac{d\,r_{AB}}{dt}$, represents the Coriolis acceleration, which appears only in rotating systems.

- The third term, $\mathbf{\Omega} \times (\mathbf{\Omega} \times r_{AB})$, corresponds to the centrifugal acceleration, directed outward from the axis of rotation.

These results show that motion observed from a rotating frame is influenced not only by the true motion of the particle but also by additional fictitious accelerations arising due to the rotation of the frame itself.

In practical terms, such corrections are essential for accurately analysing motion on the rotating Earth, in rotating machinery, or in aerospace systems. For example, the Coriolis term explains the deflection of projectiles and atmospheric winds, while the centrifugal term affects objects moving along curved paths on rotating platforms.

Thus, the experiment successfully demonstrated how mathematical differentiation in a rotating frame introduces extra terms that have clear physical interpretations, confirming the theoretical foundation of relative motion in non-inertial reference frames.

- **Source of Discrepancy:**
    1. Measurement and Tracking Errors:

        - Limited camera resolution and frame rate caused small errors in determining the exact position of the volleyball at each instant.

        - Manual or software-based marking of the ball's location introduced pixel-level inaccuracies that accumulated over time.

    2. Assumption of Ideal Conditions:

        - The theoretical derivation assumes uniform rotation (constant angular velocity) and neglects variations due to vibration or instability in the rotating frame.

        - Air resistance and drag forces acting on the ball were not considered in the analytical model, which had a slight impact on the actual trajectory.

3. Synchronization Issues:

    - In practical measurements, slight misalignment between the rotational reference frame and the camera's coordinate system can cause discrepancies when transforming coordinates.

4. Environmental Factors:

    - Variations in lighting, air currents, or uneven floor surfaces can slightly alter the free-fall path of the ball, making it deviate from the theoretical trajectory predicted by pure rotational dynamics.

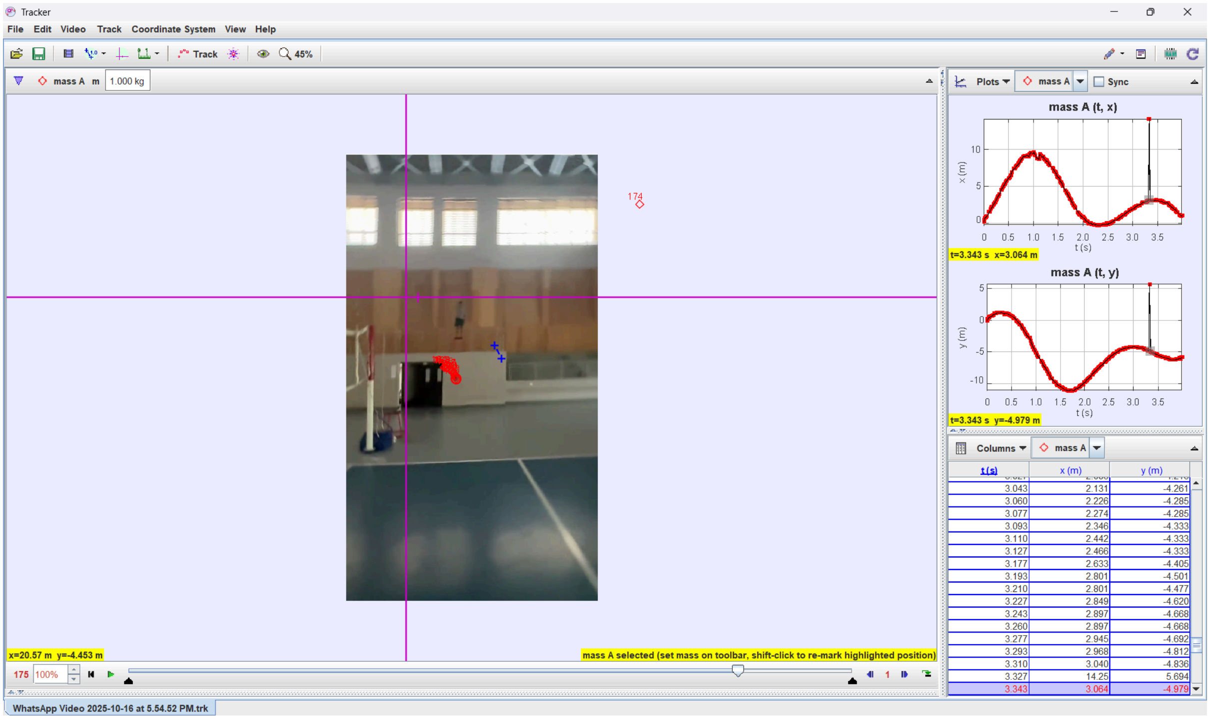
Scope of Improvement;

- Use a higher-resolution, high-frame-rate camera for more accurate motion tracking.
- Utilise automated image-processing software to minimise manual marking and reduce human error.
- Ensure precise alignment between the camera and the rotating frame axes.
- Include air resistance and drag effects in the mathematical model for better real-world correlation.
- Maintain a stable rotation speed using a motorised setup with controlled speed.

Software/ tools Used:

1) **Tracker (Open-Source Video Analysis Tool):**

The experimental tracking of particle motion was carried out using *Tracker*, an open-source video analysis and modelling software. Tracker allowed us to import motion videos, mark particle positions frame by frame, and automatically extract kinematic data, including displacement, velocity, and acceleration. This tool provided an effective way to bridge the gap between experimental observation and theoretical motion models.



2) Python Libraries for Data Processing and Visualisation:

- NumPy: Used for numerical computations such as handling arrays, calculating velocity and acceleration from position data, and performing mathematical operations efficiently.

- Pandas: Utilised for organising, cleaning, and analysing the experimental data in tabular form. Pandas enabled smooth handling of large datasets and facilitated structured data analysis.

- Matplotlib: Applied for graphical analysis and visualisation. It was used to plot position, velocity, and acceleration graphs, compare experimental data with theoretical predictions, and highlight key deviations caused by external factors (e.g., air resistance).

Acknowledgement:

We thank those who assisted and participated in completing our experiment 1 for the Statics and Dynamics course. Most of all, we sincerely thank Professor K.R. Jayaprakash for his remarkable mentorship and expert guidance. His clear explanations and considerate assistance have significantly enhanced our understanding of the subject and helped us overcome the experiment's intricacies. We especially want to thank the teaching assistants, whose valuable advice and insightful observations contributed significantly to the smooth running of our work. Moreover, we thank the Indian Institute of Technology Gandhinagar for providing the opportunities that enabled us to perform this experiment. Lastly, we would like to thank our fellow students and colleagues for their cooperation, feedback, and support, which contributed to the experiment's accomplishment.