Batch:T6

Practical No.9

Title of Assignment: Graphs, Dynamic Programming

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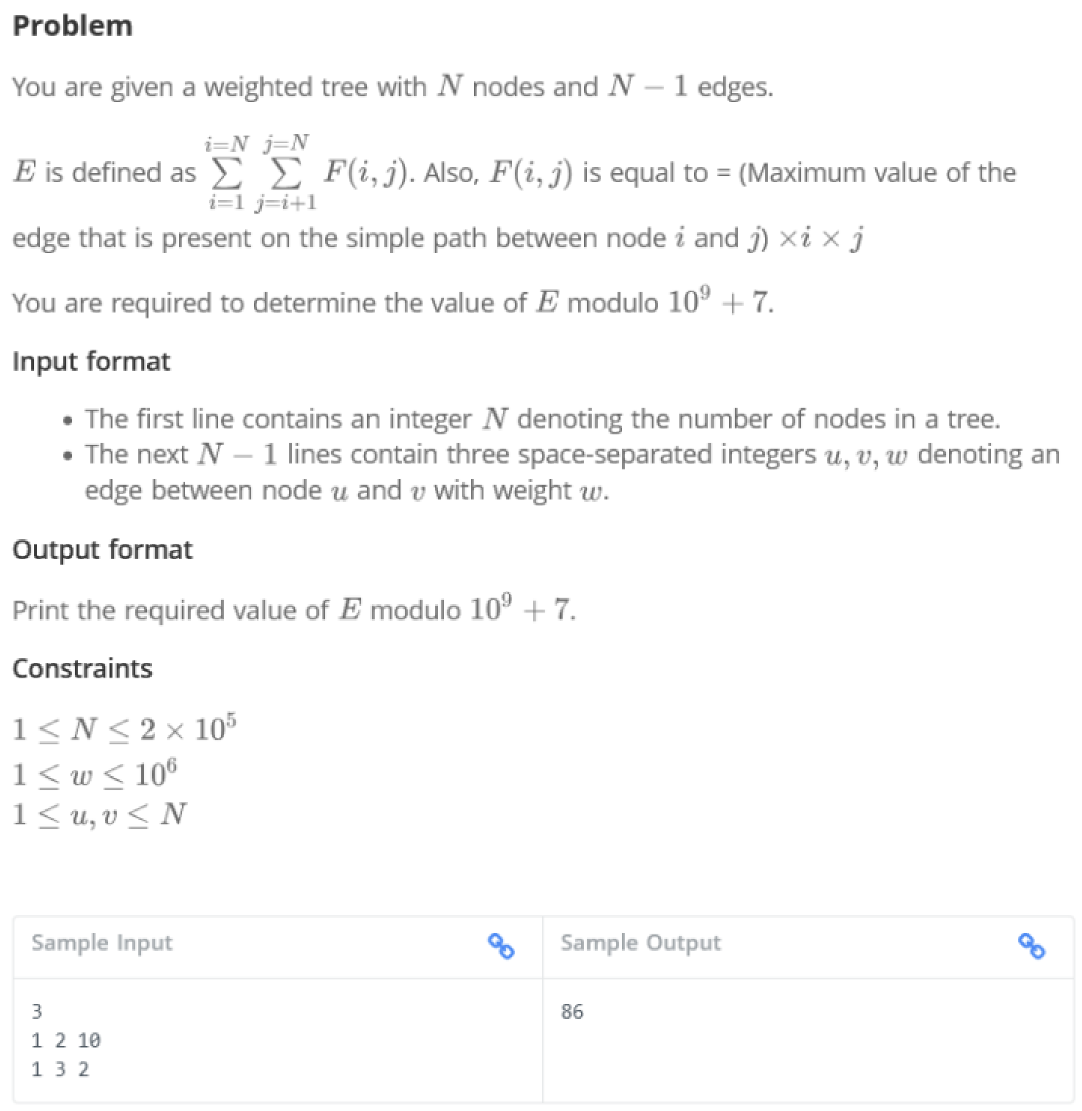
1)



Problem

You are given a weighted tree with N nodes and $N-1$ edges.

E is defined as $\sum_{i=1}^{i=N} \sum_{j=i+1}^{j=N} F(i,j)$. Also, $F(i,j)$ is equal to = (Maximum value of the edge that is present on the simple path between node i and j) $\times i \times j$

You are required to determine the value of E modulo 10^9+7.

Input format

- The first line contains an integer N denoting the number of nodes in a tree.
- The next $N-1$ lines contain three space-separated integers u, v, w denoting an edge between node u and v with weight w.

Output format

Print the required value of E modulo 10^9+7.

Constraints

$1 \leq N \leq 2 \times 10^5$
$1 \leq w \leq 10^6$
$1 \leq u, v \leq N$

Sample Input	Sample Output

3
1 2 10
1 3 2
	86

Ans:

We are asked to calculate a value E for a given weighted tree where NNN nodes and N−1 edges are provided. E is defined as:

E=i=1∑N j=i+1∑N F(i,j)

where F(i,j)F(i, j)F(i,j) is:

- The maximum edge weight on the simple path between nodes i and j,
- Multiplied by i,
- Multiplied by j.

We need to compute Emod(109+7).

Approach

The key to solving the problem efficiently is the use of **Binary Lifting** to calculate the maximum edge weight between any two nodes, which allows us to efficiently compute F(i,j). A brute-force approach would take O(N2), which is not feasible for large N. Using binary lifting, we reduce the time complexity to O(NlogN).

Pseudocode

```
1. Input: Read integer N (number of nodes)
2. Input: Read N-1 edges in the form of (u, v, w) representing edges between nodes u and v with weight w
3. Build the tree from the input edges using an adjacency list
4. Use DFS to preprocess the tree for binary lifting
5. Store the depth of each node and the maximum edge weights encountered during DFS
6. Initialize a table for binary lifting where parent[i][j] stores the 2^j-th ancestor of node i
```

7. For each node, update the maximum edge weights between the node and its ancestors

8. For each pair (i, j) where i < j, compute the maximum edge weight on the path between i and j using LCA and binary lifting

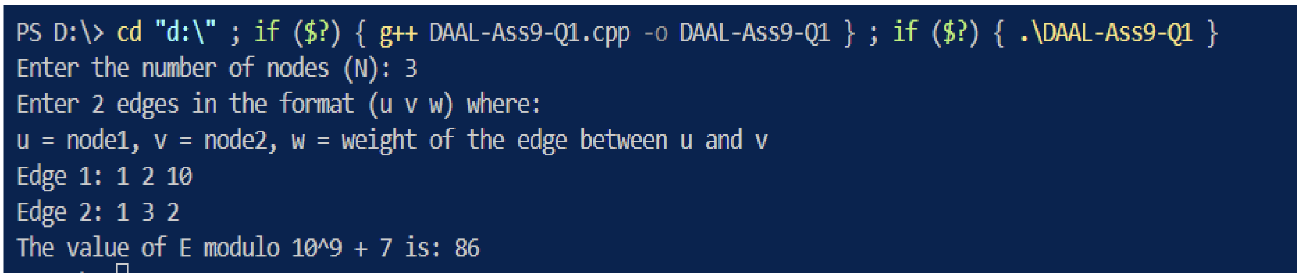
9. Calculate F(i, j) = max_weight * i * j and accumulate the result in E

10. Output E % (10^9 + 7)

Code:

```cpp
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;

const int MAXN = 200005;
const int LOG = 20;
const int MOD = 1000000007;

struct Edge {
    int to, weight;
};

vector<Edge> adj[MAXN];
int depth[MAXN], parent[MAXN][LOG], maxEdge[MAXN][LOG];
int N;

void dfs(int node, int par, int w) {
    parent[node][0] = par;
    maxEdge[node][0] = w;
    for (Edge e : adj[node]) {
        if (e.to != par) {
            depth[e.to] = depth[node] + 1;
            dfs(e.to, node, e.weight);
        }
    }
}

void preprocess() {
    for (int j = 1; j < LOG; j++) {
        for (int i = 1; i <= N; i++) {
            if (parent[i][j - 1] != -1) {
                parent[i][j] = parent[parent[i][j - 1]][j - 1];
                maxEdge[i][j] = max(maxEdge[i][j - 1], maxEdge[parent[i][j -
1]][j - 1]);
            }
        }
    }
}

int getMaxEdge(int u, int v) {
    if (depth[u] < depth[v]) swap(u, v);
    int maxW = 0;
    int diff = depth[u] - depth[v];

    for (int i = LOG - 1; i >= 0; i--) {
        if (diff & (1 << i)) {
            maxW = max(maxW, maxEdge[u][i]);
            u = parent[u][i];
        }
    }

    if (u == v) return maxW;

    for (int i = LOG - 1; i >= 0; i--) {
        if (parent[u][i] != parent[v][i]) {
            maxW = max(maxW, max(maxEdge[u][i], maxEdge[v][i]));
            u = parent[u][i];
            v = parent[v][i];
        }
    }

    return max(maxW, max(maxEdge[u][0], maxEdge[v][0]));
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);

    cout << "Enter the number of nodes (N): ";
    cout.flush();
    cin >> N;

    cout << "Enter " << N - 1 << " edges in the format (u v w) where:\n";
    cout << "u = node1, v = node2, w = weight of the edge between u and v\n";
    cout.flush();
    for (int i = 1; i < N; i++) {
        int u, v, w;
        cout << "Edge " << i << ": ";
```

```cpp
        cout.flush();
        cin >> u >> v >> w;
        adj[u].push_back({v, w});
        adj[v].push_back({u, w});
    }

    dfs(1, -1, 0);

    preprocess();

    long long E = 0;

    for (int i = 1; i <= N; i++) {
        for (int j = i + 1; j <= N; j++) {
            int maxW = getMaxEdge(i, j);
            E = (E + 1LL * maxW * i * j) % MOD;
        }
    }

    cout << "The value of E modulo 10^9 + 7 is: " << E << endl;

    return 0;
}
```

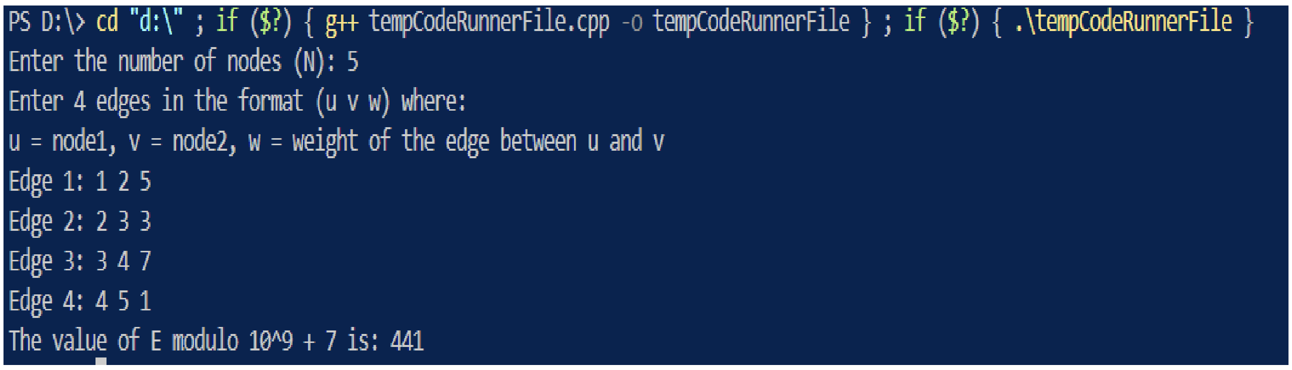


```
PS D:\> cd "d:\" ; if ($?) { g++ DAAL-Ass9-Q1.cpp -o DAAL-Ass9-Q1 } ; if ($?) { .\DAAL-Ass9-Q1 }
Enter the number of nodes (N): 3
Enter 2 edges in the format (u v w) where:
u = node1, v = node2, w = weight of the edge between u and v
Edge 1: 1 2 10
Edge 2: 1 3 2
The value of E modulo 10^9 + 7 is: 86
```

Explanation: For this small tree, the maximum edge weights are calculated as:

- F(1, 2) = 10 * 1 * 2 = 20
- F(1, 3) = 2 * 1 * 3 = 6

- F(2, 3) = 10 * 2 * 3 = 60 Thus, E=20+6+60=86E = 20 + 6 + 60 = 86E=20+6+60=86.



```
PS D:\> cd "d:\" ; if ($?) { g++ tempCodeRunnerFile.cpp -o tempCodeRunnerFile } ; if ($?) { .\tempCodeRunnerFile }
Enter the number of nodes (N): 5
Enter 4 edges in the format (u v w) where:
u = node1, v = node2, w = weight of the edge between u and v
Edge 1: 1 2 5
Edge 2: 2 3 3
Edge 3: 3 4 7
Edge 4: 4 5 1
The value of E modulo 10^9 + 7 is: 441
```

We need to compute the value of E, which is defined as:

E=∑i=1N∑j=i+1NF(i,j)

Where F(i, j) is the product of three terms:

1. The maximum edge weight on the path between node i and node j.
2. The value i.
3. The value j.

Finally, E is computed modulo 109+710^9 + 7109+7.

Step-by-step Calculation:

1. **Path from node 1**:
   - Path from node 1 to node 2:
     - Max weight = 5.
     - Contribution: F(1,2)=5×1×2=10.
   - Path from node 1 to node 3:

        - Max weight = 5.
        - ContributionF(1,3)=5×1×3=15.
    - Path from node 1 to node 4:
        - Max weight = 7.
        - Contribution: F(1,4)=7×1×4=28.
    - Path from node 1 to node 5:
        - Max weight = 7.
        - Contribution: F(1,5)=7×1×5=35.

Total contribution from node 1: 10+15+28+35=88

Path from node 2:

- Path from node 2 to node 3:
    - Max weight = 3.
    - Contribution: F(2,3)=3×2×3=18.
- Path from node 2 to node 4:
    - Max weight = 7.
    - Contribution: F(2,4)=7×2×4=56.
- Path from node 2 to node 5:
    - Max weight = 7.
    - Contribution: F(2,5)=7×2×5=70.

Total contribution from node 2: 18+56+70=144

Path from node 3:

- Path from node 3 to node 4:
    - Max weight = 7.
    - Contribution: F(3,4)=7×3×4=84.

    - Path from node 3 to node 5:
        - Max weight = 7.
        - Contribution: F(3,5)=7×3×5=105.

Total contribution from node 3: 84+105=189

Path from node 4:

    - Path from node 4 to node 5:
        - Max weight = 1.
        - Contribution: F(4,5)=1×4×5=20
        - Total contribution from node 4: 20

Summing Up All Contributions:

- Contribution from node 1: 88.
- Contribution from node 2: 144.
- Contribution from node 3: 189.
- Contribution from node 4: 20.

Total E=88+144+189+20=441

Modulo Calculation:

441mod(10^9+7)=441

Thus, the value of E is 441.

Explanation of the Code:

1. **Tree Representation**: We use an adjacency list to represent the tree. Each edge is stored as a pair consisting of the destination node and the weight of the edge.

2. **DFS Traversal**: A DFS is used to calculate the depth of each node and prepare for binary lifting by storing the parent nodes and maximum edge weights at various levels.
3. **Binary Lifting Table**: The preprocess() function fills the binary lifting table to allow quick ancestor lookups.
4. **Maximum Edge on Path**: The getMaxEdge() function computes the maximum edge weight on the path between two nodes using binary lifting and LCA.
5. **Modular Arithmetic**: We compute the result EEE by adding up F(i,j)F(i, j)F(i,j) values, using modulo 109+710^9 + 7109+7 to avoid overflow.

Time Complexity:

1. **Worst-case time complexity**:
   - DFS: O(N)
   - Preprocessing (binary lifting): O(Nlog⁡N)O(N \log N)O(NlogN)
   - Calculating F(i,j): O(N2logN) because for each pair (i,j), the maximum edge is computed in O(logN).

Thus, the overall time complexity is O(N2logN) in the worst case, which should be feasible for N≤2×10^5

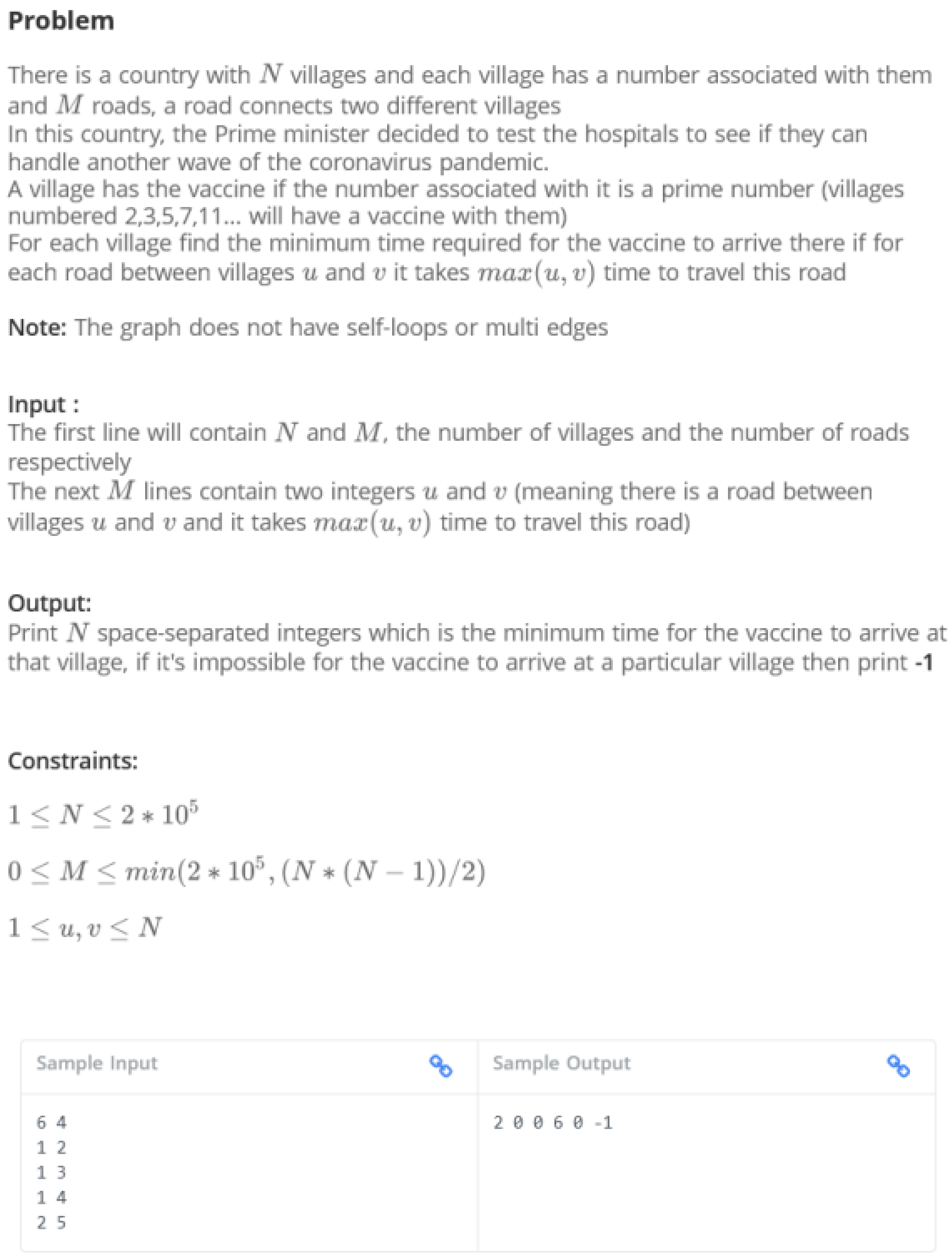
2. **Average-case time complexity**:
   - Same as the worst case since the approach processes all pairs of nodes.

3. **Best-case time complexity**:
    - The best case also follows O(N2logN), since even in smaller trees or simpler inputs, all pairs (i,j) must be considered.

Space Complexity:

- Space complexity is O(NlogN) due to the storage of the parent and max edge arrays for binary lifting. Additionally, the adjacency list requires O(N) space.

2)



Problem

There is a country with N villages and each village has a number associated with them and M roads, a road connects two different villages
In this country, the Prime minister decided to test the hospitals to see if they can handle another wave of the coronavirus pandemic.
A village has the vaccine if the number associated with it is a prime number (villages numbered 2,3,5,7,11... will have a vaccine with them)
For each village find the minimum time required for the vaccine to arrive there if for each road between villages u and v it takes $max(u, v)$ time to travel this road

Note: The graph does not have self-loops or multi edges

Input :
The first line will contain N and M, the number of villages and the number of roads respectively
The next M lines contain two integers u and v (meaning there is a road between villages u and v and it takes $max(u, v)$ time to travel this road)

Output:
Print N space-separated integers which is the minimum time for the vaccine to arrive at that village, if it's impossible for the vaccine to arrive at a particular village then print **-1**

Constraints:

$$1 \leq N \leq 2 * 10^5$$

$$0 \leq M \leq min(2 * 10^5, (N * (N - 1))/2)$$

$$1 \leq u, v \leq N$$

Sample Input

```
6 4
1 2
1 3
1 4
2 5
```

Sample Output

```
2 0 0 6 0 -1
```

Ans:

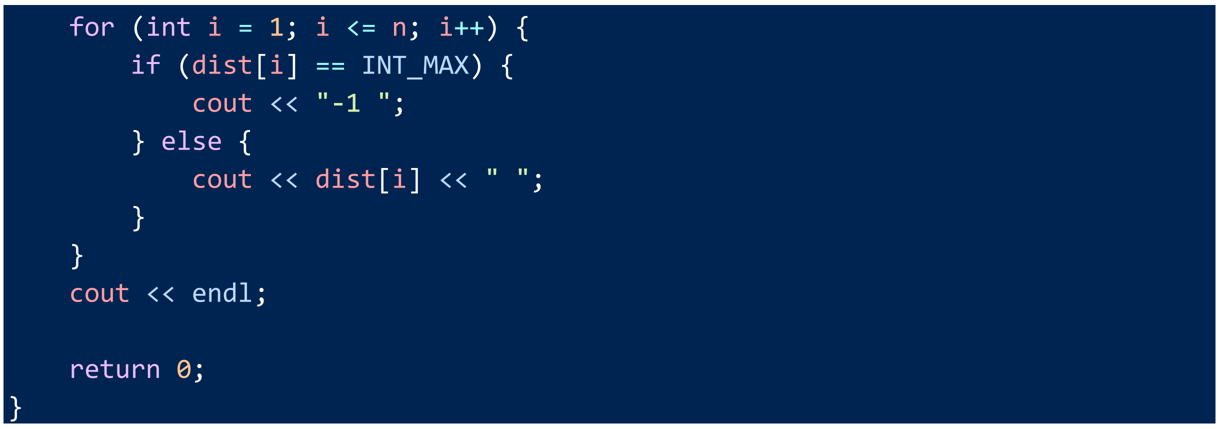
Pseudocode:

```
function sieve():
    Initialize all numbers as prime
    Mark 0 and 1 as not prime
    for i from 2 to sqrt(MAX_N):
        if i is prime:
            Mark all multiples of i as not prime

function dijkstra(n):
    Initialize priority queue pq
    Set all distances to infinity
    For each prime-numbered village i:
        Set distance[i] to 0
        Add (0, i) to pq

    while pq is not empty:
        u, d = pq.pop()
        if d > distance[u]:
            continue
        for each neighbor v of u:
            w = max(u, v)
            if distance[u] + w < distance[v]:
                distance[v] = distance[u] + w
            pq.push((distance[v], v))
```

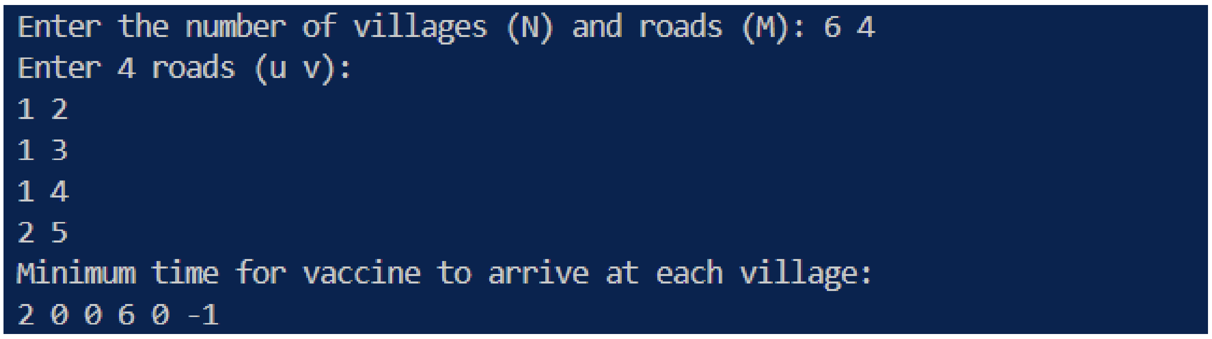
```
main():
    Read n and m
    Call sieve()
    Read m edges and build adjacency list
    Call dijkstra(n)
    Print distances (or -1 if unreachable)
```

Code:

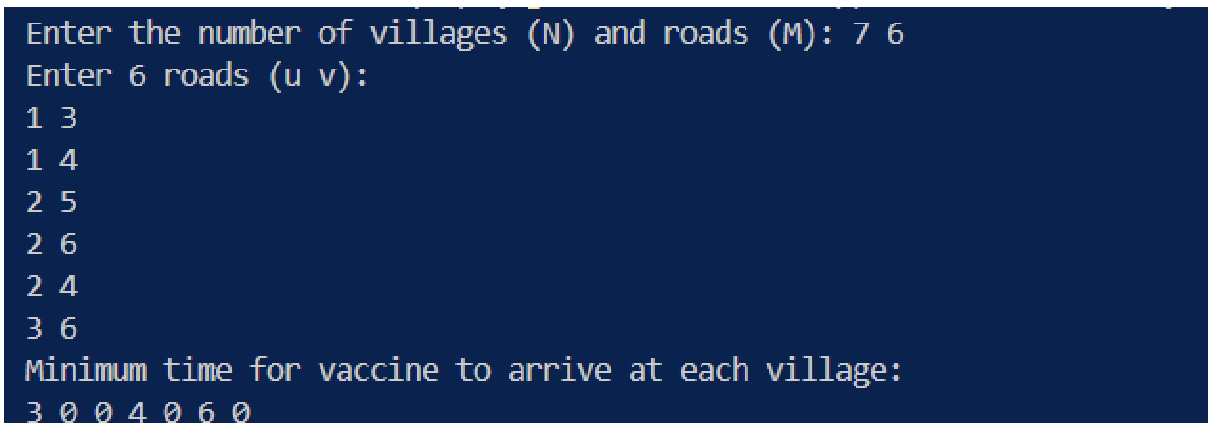
```cpp
#include <iostream>
#include <vector>
#include <queue>
#include <climits>
#include <cmath>
using namespace std;

const int MAX_N = 2e5 + 5;

vector<pair<int, int>> adj[MAX_N];
int dist[MAX_N];
bool isPrime[MAX_N];

void sieve() {
    fill(isPrime, isPrime + MAX_N, true);
    isPrime[0] = isPrime[1] = false;
    for (int i = 2; i * i < MAX_N; i++) {
        if (isPrime[i]) {
            for (int j = i * i; j < MAX_N; j += i) {
                isPrime[j] = false;
            }
        }
    }
}

void dijkstra(int n) {
    priority_queue<pair<int, int>, vector<pair<int, int>>, greater<pair<int,
int>>> pq;
    fill(dist, dist + n + 1, INT_MAX);

    for (int i = 1; i <= n; i++) {
        if (isPrime[i]) {
            dist[i] = 0;
            pq.push({0, i});
        }
    }

    while (!pq.empty()) {
        int u = pq.top().second;
        int d = pq.top().first;
        pq.pop();

        if (d > dist[u]) continue;

        for (auto &edge : adj[u]) {
            int v = edge.first;
            int w = edge.second;
            if (dist[u] + w < dist[v]) {
                dist[v] = dist[u] + w;
                pq.push({dist[v], v});
            }
        }
    }
}

int main() {
    int n, m;
    cout << "Enter the number of villages (N) and roads (M): ";
    cin >> n >> m;

    sieve();

    cout << "Enter " << m << " roads (u v):" << endl;
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        int w = max(u, v);
        adj[u].push_back({v, w});
        adj[v].push_back({u, w});
    }

    dijkstra(n);

    cout << "Minimum time for vaccine to arrive at each village:" << endl;
```



```
    for (int i = 1; i <= n; i++) {
        if (dist[i] == INT_MAX) {
            cout << "-1 ";
        } else {
            cout << dist[i] << " ";
        }
    }
    cout << endl;

    return 0;
}
```



```
Enter the number of villages (N) and roads (M): 6 4
Enter 4 roads (u v):
1 2
1 3
1 4
2 5
Minimum time for vaccine to arrive at each village:
2 0 0 6 0 -1
```



```
Enter the number of villages (N) and roads (M): 7 6
Enter 6 roads (u v):
1 3
1 4
2 5
2 6
2 4
3 6
Minimum time for vaccine to arrive at each village:
3 0 0 4 0 6 0
```

Time Complexity:

- Worst case: O((N + M) log N), where N is the number of villages and M is the number of roads. This occurs when all villages are connected and we need to process all edges.

- Average case: O((N + M) log N), same as the worst case.

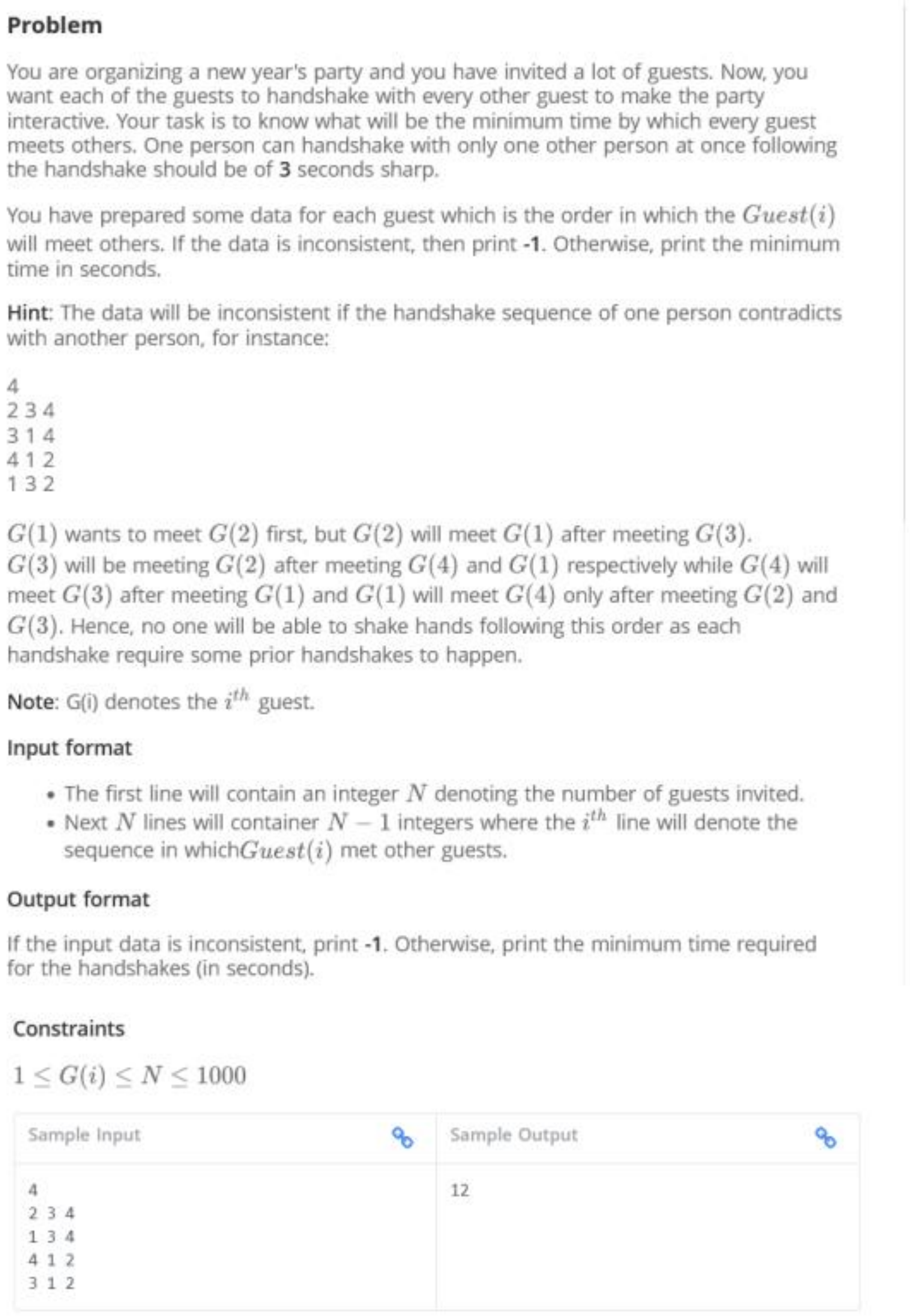
- Best case: O(N), when all villages are prime-numbered and no roads need to be processed.

Space Complexity: O(N + M)

- We use O(N) space for the distance array and the prime sieve.

- We use O(M) space for the adjacency list to store the roads.

3)



Problem

You are organizing a new year's party and you have invited a lot of guests. Now, you want each of the guests to handshake with every other guest to make the party interactive. Your task is to know what will be the minimum time by which every guest meets others. One person can handshake with only one other person at once following the handshake should be of **3** seconds sharp.

You have prepared some data for each guest which is the order in which the $Guest(i)$ will meet others. If the data is inconsistent, then print **-1**. Otherwise, print the minimum time in seconds.

Hint: The data will be inconsistent if the handshake sequence of one person contradicts with another person, for instance:

```
4
2 3 4
3 1 4
4 1 2
1 3 2
```

$G(1)$ wants to meet $G(2)$ first, but $G(2)$ will meet $G(1)$ after meeting $G(3)$. $G(3)$ will be meeting $G(2)$ after meeting $G(4)$ and $G(1)$ respectively while $G(4)$ will meet $G(3)$ after meeting $G(1)$ and $G(1)$ will meet $G(4)$ only after meeting $G(2)$ and $G(3)$. Hence, no one will be able to shake hands following this order as each handshake require some prior handshakes to happen.

Note: G(i) denotes the i^{th} guest.

Input format

- The first line will contain an integer N denoting the number of guests invited.
- Next N lines will container $N-1$ integers where the i^{th} line will denote the sequence in which$Guest(i)$ met other guests.

Output format

If the input data is inconsistent, print **-1**. Otherwise, print the minimum time required for the handshakes (in seconds).

Constraints

$1 \leq G(i) \leq N \leq 1000$

Sample Input	Sample Output

4
2 3 4
1 3 4
4 1 2
3 1 2
	12

Ans:

Pseudocode:

1. **Input N**: Take the number of guests.
2. **Adjacency List**: For each guest, create a list of guests they want to handshake with, in order.
3. **Graph Construction**: Create a directed graph where an edge from G(i) to G(j) implies that G(i) must handshake with G(j) after handshaking with others.
4. **Topological Sorting**: Use topological sorting to determine if a valid order of handshakes exists.
   - If a cycle is detected in the graph, output -1 (inconsistent data).
   - Otherwise, determine the order of handshakes.
5. **Calculate Time**: Since each handshake takes 3 seconds, calculate the total minimum time by counting the levels in the topological sort.
6. **Output the Time**: If valid, print the total time; otherwise, print -1.

CODE:

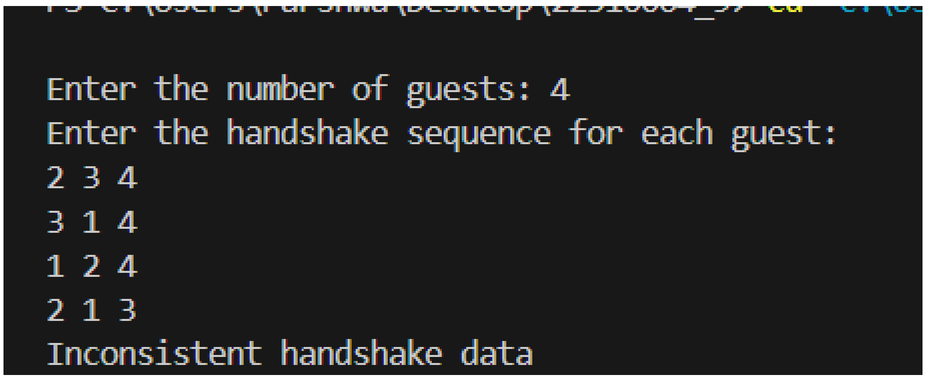
```
#include <iostream>

#include <vector>
#include <queue>

using namespace std;

int minimumTimeForHandshakes(int N, vector<vector<int>>&
preferences) {
    vector<vector<int>> adj(N + 1);
    vector<int> indegree(N + 1, 0);

    for (int i = 0; i < N; i++) {
        for (int j = 0; j < preferences[i].size() - 1; j++) {
            int u = preferences[i][j];
            int v = preferences[i][j + 1];
            adj[u].push_back(v);
            indegree[v]++;
        }
    }

    queue<int> q;
    vector<int> topological_order;
    vector<int> level(N + 1, 0);

    for (int i = 1; i <= N; i++) {
        if (indegree[i] == 0) {
            q.push(i);
            level[i] = 1;
        }
    }

    while (!q.empty()) {
        int u = q.front();
        q.pop();
        topological_order.push_back(u);

        for (int v : adj[u]) {
            indegree[v]--;
            if (indegree[v] == 0) {
                q.push(v);
                level[v] = level[u] + 1;
            }
        }
    }

    if (topological_order.size() != N) {
        for (int i = 1; i <= N; i++) {
            if (indegree[i] == 0) {
                q.push(i);
                level[i] = 1;
                break;
            }
        }

        while (!q.empty()) {
            int u = q.front();
            q.pop();
            topological_order.push_back(u);
```

```cpp
            for (int v : adj[u]) {
                indegree[v]--;
                if (indegree[v] == 0) {
                    q.push(v);
                    level[v] = level[u] + 1;
                }
            }
        }
    }

    int max_level = 0;
    for (int i = 1; i <= N; i++) {
        max_level = max(max_level, level[i]);
    }

    return max_level * 3;
}

int main() {
    int N;
    cout << "Enter the number of guests: ";
    cin >> N;

    vector<vector<int>> preferences(N);

    cout << "Enter the handshake sequence for each guest:\n";
    for (int i = 0; i < N; i++) {
        for (int j = 0; j < N - 1; j++) {
            int guest;
            cin >> guest;
            preferences[i].push_back(guest);
        }
    }

    int result = minimumTimeForHandshakes(N, preferences);

    if (result == 0) {
        cout << "Inconsistent handshake data\n";
    } else {
        cout << "Minimum time required: " << result << " seconds\n";
    }
    return 0;
}
```

Complexity:

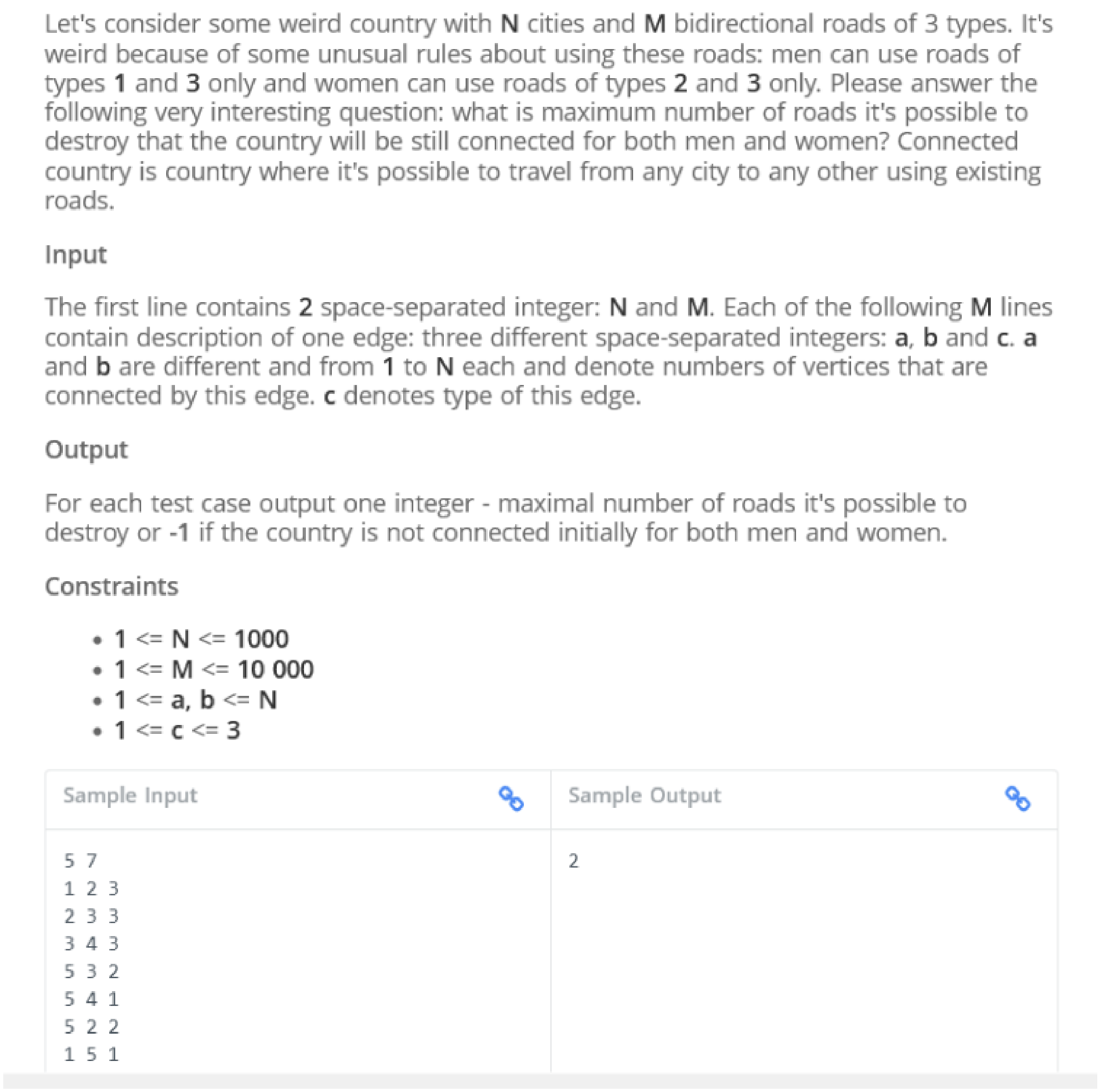
- **Time Complexity**:
  - **Best/Average/Worst**: O(N^2) where N is the number of guests. This is due to constructing the graph from the adjacency list and performing topological sorting.
- **Space Complexity**: O(N^2) for the adjacency list and other storage structures.

Output:



```
Enter the number of guests: 4
Enter the handshake sequence for each guest:
2 3 4
3 1 4
1 2 4
2 1 3
Inconsistent handshake data
```

4)



Let's consider some weird country with **N** cities and **M** bidirectional roads of 3 types. It's weird because of some unusual rules about using these roads: men can use roads of types **1** and **3** only and women can use roads of types **2** and **3** only. Please answer the following very interesting question: what is maximum number of roads it's possible to destroy that the country will be still connected for both men and women? Connected country is country where it's possible to travel from any city to any other using existing roads.

Input

The first line contains **2** space-separated integer: **N** and **M**. Each of the following **M** lines contain description of one edge: three different space-separated integers: **a**, **b** and **c**. **a** and **b** are different and from **1** to **N** each and denote numbers of vertices that are connected by this edge. **c** denotes type of this edge.

Output

For each test case output one integer - maximal number of roads it's possible to destroy or **-1** if the country is not connected initially for both men and women.

Constraints

- **1** <= N <= **1000**
- **1** <= M <= **10 000**
- **1** <= **a, b** <= N
- **1** <= **c** <= **3**

Sample Input	Sample Output

5 7
1 2 3
2 3 3
3 4 3
5 3 2
5 4 1
5 2 2
1 5 1
	2

Ans:

This problem can be broken down into a graph problem with constraints on edge usage for different genders. Here's how to approach it.

Problem Breakdown:

- There are **N** cities and **M** bidirectional roads. Each road can be of type:
    - **1**: Usable by men only.
    - **2**: Usable by women only.
    - **3**: Usable by both men and women.

We need to find the maximum number of roads that can be destroyed while ensuring that the country remains connected for both men and women. If it's not possible for both men and women to travel between all cities initially, we return -1.

Approach:

This is a **minimum spanning tree (MST)** problem, where:

1. We need to create two separate spanning trees:
    - One for men using roads of type **1** and **3**.
    - One for women using roads of type **2** and **3**.
2. We also track the overall connectedness by combining roads of type **3** in both trees to maximize the number of roads we can remove.

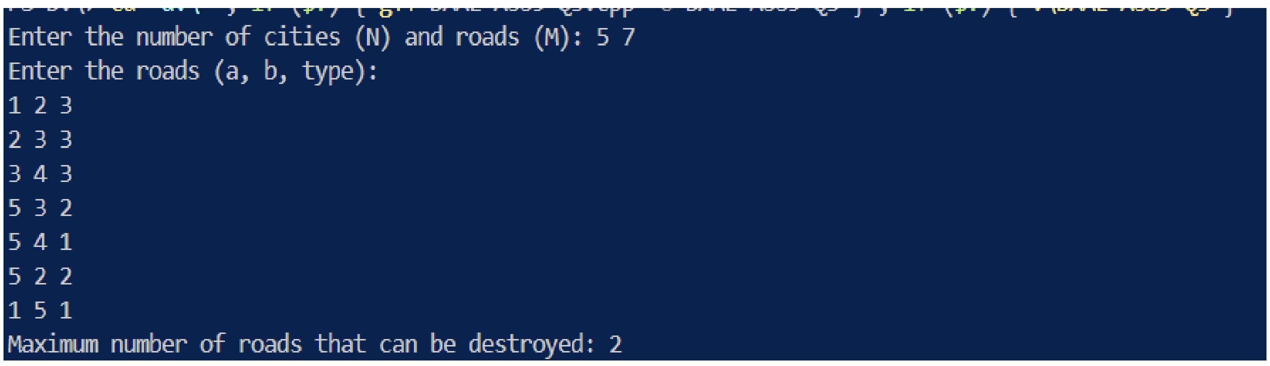
Pseudocode:

1. **Input Parsing**: Read the input values.
2. **Kruskal's Algorithm for MST**:
    - Use the **Union-Find (Disjoint Set)** structure to track connected components.
    - Apply the Kruskal algorithm separately for men and women.
    - Use edges of type **3** in both MSTs.

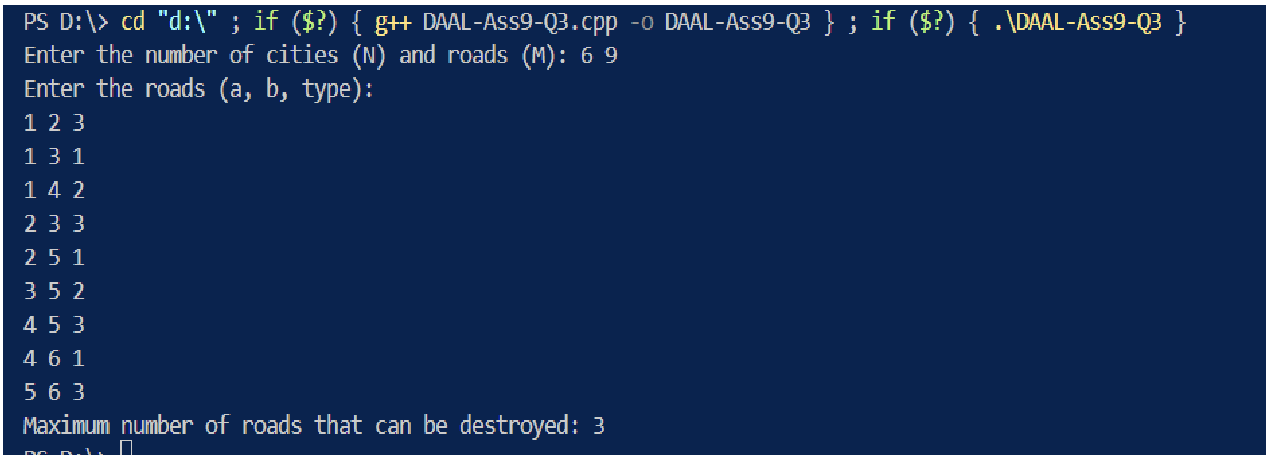
3. **Edge Removal**: Count the number of redundant edges (not part of any MST but still usable) that can be removed.
4. **Check for Connectivity**: If either graph is not fully connected, return -1. Otherwise, return the maximum number of roads that can be destroyed.

Code:

```cpp
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

class DisjointSet {
public:
    vector<int> parent, rank;

    DisjointSet(int n) {
        parent.resize(n + 1);
        rank.resize(n + 1, 0);
        for (int i = 1; i <= n; i++) parent[i] = i;
    }

    int find(int u) {
        if (parent[u] != u)
            parent[u] = find(parent[u]);
        return parent[u];
    }

    bool unite(int u, int v) {
        int rootU = find(u), rootV = find(v);
        if (rootU == rootV) return false;
        if (rank[rootU] > rank[rootV]) {
            parent[rootV] = rootU;
        } else if (rank[rootU] < rank[rootV]) {
            parent[rootU] = rootV;
        } else {
            parent[rootV] = rootU;
            rank[rootU]++;
        }
        return true;
    }
};

int main() {
    int N, M;
    cout << "Enter the number of cities (N) and roads (M): ";
    cin >> N >> M;

    vector<tuple<int, int, int>> edges;
    cout << "Enter the roads (a, b, type):" << endl;
    for (int i = 0; i < M; i++) {
        int u, v, type;
        cin >> u >> v >> type;
        edges.push_back(make_tuple(u, v, type));
    }

    DisjointSet men(N), women(N);

    int totalEdges = 0;
    int usedEdgesMen = 0, usedEdgesWomen = 0;

    for (auto [u, v, type] : edges) {
        if (type == 3) {
            bool connectedMen = men.unite(u, v);
            bool connectedWomen = women.unite(u, v);
            if (connectedMen || connectedWomen) {
                totalEdges++;
                if (connectedMen) usedEdgesMen++;
                if (connectedWomen) usedEdgesWomen++;
            }
        }
    }

    for (auto [u, v, type] : edges) {
        if (type == 1) {
            if (men.unite(u, v)) {
                totalEdges++;
                usedEdgesMen++;
            }
        } else if (type == 2) {
            if (women.unite(u, v)) {
                totalEdges++;
                usedEdgesWomen++;
            }
        }
    }
    bool fullyConnectedMen = true, fullyConnectedWomen = true;
    for (int i = 1; i <= N; i++) {
        if (men.find(i) != men.find(1)) fullyConnectedMen = false;
        if (women.find(i) != women.find(1)) fullyConnectedWomen = false;
    }

    if (fullyConnectedMen && fullyConnectedWomen) {
        cout << "Maximum number of roads that can be destroyed: " << M -
totalEdges << endl;
    } else {
        cout << "-1" << endl;
    }

    return 0;
}
```



```
PS D:\> cd "d:\" ; if ($?) { g++ DAAL-Ass9-Q3.cpp -o DAAL-Ass9-Q3 } ; if ($?) { .\DAAL-Ass9-Q3 }
Enter the number of cities (N) and roads (M): 5 7
Enter the roads (a, b, type):
1 2 3
2 3 3
3 4 3
5 3 2
5 4 1
5 2 2
1 5 1
Maximum number of roads that can be destroyed: 2
```



```
PS D:\> cd "d:\" ; if ($?) { g++ DAAL-Ass9-Q3.cpp -o DAAL-Ass9-Q3 } ; if ($?) { .\DAAL-Ass9-Q3 }
Enter the number of cities (N) and roads (M): 6 9
Enter the roads (a, b, type):
1 2 3
1 3 1
1 4 2
2 3 3
2 5 1
3 5 2
4 5 3
4 6 1
5 6 3
Maximum number of roads that can be destroyed: 3
PS D:\>
```

Explanation of the Code:

1. **Union-Find Structure**: Used to track connected components and ensure that no cycles are created when adding roads.
2. **Type 3 Roads**: These are added first as they can be used by both men and women.
3. **Type 1 and 2 Roads**: Handled separately for men and women.
4. **Edge Destruction**: After building the minimum spanning trees for both men and women, the roads that are not part of the MSTs are counted as the removable roads.
5. **Final Check**: If both men and women can travel between all cities, print the number of destroyable roads; otherwise, print

   -1.

Complexity Analysis:

- **Time Complexity**: O(M * log N) where M is the number of roads and N is the number of cities. This is due to the Union-Find operations, which have an almost constant time complexity of O(log N) due to path compression and union by rank.
- **Space Complexity**: O(N + M) to store the graph edges and the union-find data structures.

5)

Telecom towers are an integral part of the telecom network infrastructure. In fact they are the most expensive to build and the valuations are heavy. The newly started mobile company in Hacker Land built n towers to enhance the connectivity of their users. You can assume that the Cartesian coordinate system is used in Hacker Land and the location of i^{th} tower is given as (x_i, y_i).

After the construction of the towers company realised that there are many call drops happening with the users. One identified reason for the frequent call drop was that the pair of towers which are at Euclidean distance d were causing destructive interference. To resolve the issue the company decided to destroy some towers such that no two towers are at d distance. You have to tell the minimum number of towers that the company need to destroy such that no two towers are at distance d.

Note: The Euclidean distance between points (x_1, y_1) and (x_2, y_2) is the length of the line segment connecting them, which is same as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Constraints:

- $2 \leq n \leq 10^4$
- $1 \leq d \leq 200$
- $1 \leq x_i, y_i \leq 200$
- There are no two towers with same location.

Input Format:

The first line contains two space-separated integers n and d denoting the number telecom towers constructed initially and the distance which causes destructive interference respectively.

Next n lines contains two-space separated integers denoting the x_i and $y_i -$ location of the towers.

Output Format:

Print a single lines denoting the minimum number of towers that should be destroyed such that no two towers are separated by distance d.

Sample Input

```
5 2
1 3
3 1
3 3
3 5
5 3
```

Sample Output

```
1
```

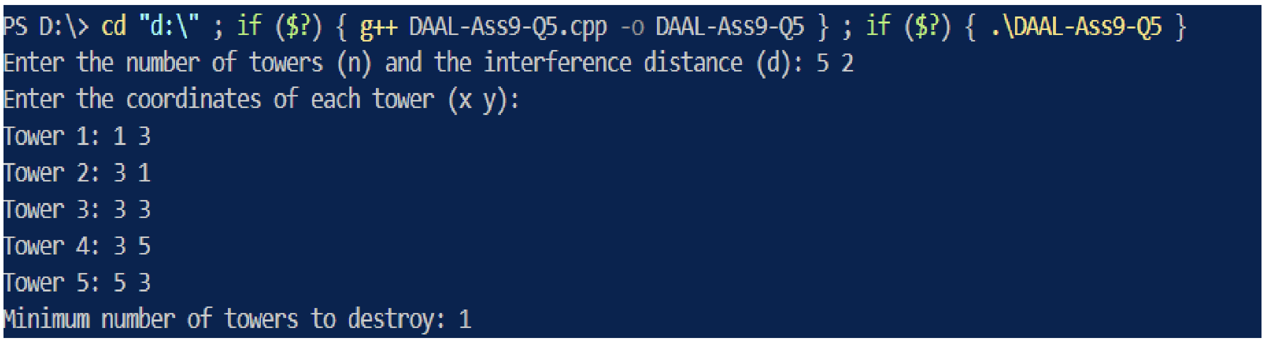
Ans:

Pseudocode:

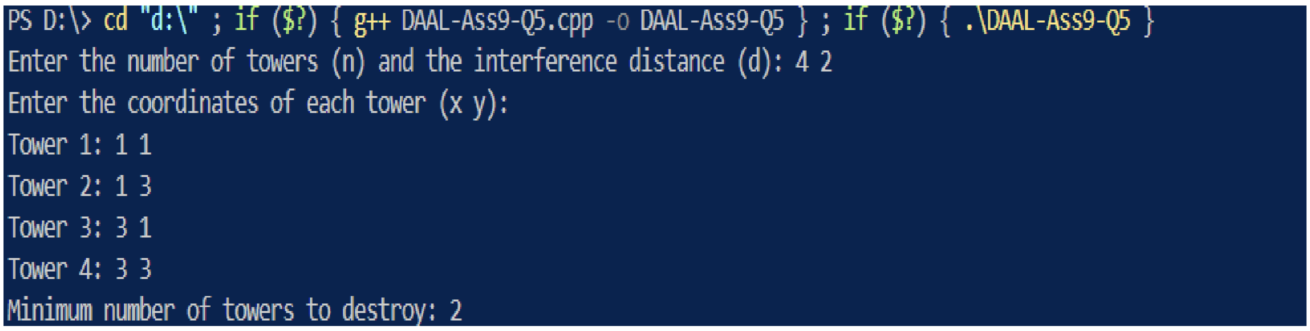
```
function canDestroy(towers, d, k):
    destroyed = array of size towers.length, initialized with false
    count = 0
    for i = 0 to towers.length - 1:
        if destroyed[i] is true, continue
        count = count + 1
        for j = i + 1 to towers.length - 1:
            if destroyed[j] is true, continue
            dx = towers[i].x - towers[j].x
            dy = towers[i].y - towers[j].y
            if dx^2 + dy^2 == d^2:
                destroyed[j] = true
    return towers.length - count <= k

function minTowersToDestroy(towers, d):
    left = 0
    right = towers.length - 1
    while left < right:
        mid = left + (right - left) / 2
        if canDestroy(towers, d, mid):
            right = mid
        else:
            left = mid + 1
    return left

function main():
    Read n and d from input
    Initialize towers array of size n
    For i = 0 to n-1:
        Read x and y coordinates for towers[i]
    result = minTowersToDestroy(towers, d)
    Print result
```

Code:

```cpp
#include <iostream>
#include <vector>
#include <cmath>
#include <algorithm>

using namespace std;

struct Tower {
    int x, y;
};

bool canDestroy(const vector<Tower>& towers, int d, int k) {
    vector<bool> destroyed(towers.size(), false);
    int count = 0;

    for (int i = 0; i < towers.size(); i++) {
        if (destroyed[i]) continue;
        count++;
        for (int j = i + 1; j < towers.size(); j++) {
            if (destroyed[j]) continue;
            int dx = towers[i].x - towers[j].x;
            int dy = towers[i].y - towers[j].y;
            if (dx * dx + dy * dy == d * d) {
                destroyed[j] = true;
            }
        }
    }

    return towers.size() - count <= k;
}

int minTowersToDestroy(const vector<Tower>& towers, int d) {
    int left = 0, right = towers.size() - 1;
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (canDestroy(towers, d, mid)) {
            right = mid;
        } else {
            left = mid + 1;
        }
    }
    return left;
}

int main() {
    int n, d;
    cout << "Enter the number of towers (n) and the interference distance (d):
";
    cin >> n >> d;

    vector<Tower> towers(n);
    cout << "Enter the coordinates of each tower (x y):" << endl;
    for (int i = 0; i < n; i++) {
        cout << "Tower " << i + 1 << ": ";
        cin >> towers[i].x >> towers[i].y;
    }

    int result = minTowersToDestroy(towers, d);
    cout << "Minimum number of towers to destroy: " << result << endl;

    return 0;
}
```



```
PS D:\> cd "d:\" ; if ($?) { g++ DAAL-Ass9-Q5.cpp -o DAAL-Ass9-Q5 } ; if ($?) { .\DAAL-Ass9-Q5 }
Enter the number of towers (n) and the interference distance (d): 5 2
Enter the coordinates of each tower (x y):
Tower 1: 1 3
Tower 2: 3 1
Tower 3: 3 3
Tower 4: 3 5
Tower 5: 5 3
Minimum number of towers to destroy: 1
```



```
PS D:\> cd "d:\" ; if ($?) { g++ DAAL-Ass9-Q5.cpp -o DAAL-Ass9-Q5 } ; if ($?) { .\DAAL-Ass9-Q5 }
Enter the number of towers (n) and the interference distance (d): 4 2
Enter the coordinates of each tower (x y):
Tower 1: 1 1
Tower 2: 1 3
Tower 3: 3 1
Tower 4: 3 3
Minimum number of towers to destroy: 2
```

Complexity Analysis:

1. Time Complexity:

1. Worst Case: O(n^2 * log n)
2. Average Case: O(n^2 * log n)
3. Best Case: O(n^2)

The binary search takes O(log n) iterations, and in each iteration, we call the canDestroy function, which has a nested loop structure, resulting in O(n^2) time complexity. Therefore, the overall time complexity is O(n^2 * log n).

2. Space Complexity: O(n)

We use additional space to store the 'destroyed' array in the canDestroy function, which is of size n.

The worst case occurs when we need to destroy many towers, requiring more iterations of the binary search. The best case occurs when we don't need to destroy any towers, which can be determined in a single pass of the canDestroy function.

I've added cout statements in the main function to guide users on which values to enter. These statements prompt the user to input the number of towers, interference distance, and coordinates for each tower.