Batch:T6

Practical No.4

Title of Assignment: Divide and conquer strategy Strassen’s Matrix Multiplication

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1. Implement Naive method multiply two matrices and Justify complexity is o(n3)

Ans:

Matrix Multiplication Function:

multiplyMatrices: This function takes two matrices A and B, multiplies them using three nested loops, and returns the resulting matrix C.

* The time complexity of this function is O(n⋅m⋅p), where n is the number of rows in A, mmm is the number of columns in A (and rows in B), and p is the number of columns in B. For square matrices where n=m=p, this complexity simplifies to O(n3).

Input and Output:

* The program prompts the user to enter matrix dimensions and elements.
* It then computes the matrix multiplication and prints the resulting matrix.

Complexity Justification

The naive matrix multiplication algorithm has a time complexity of O(n3)

when both matrices are n×n. This is because:

* We have three nested loops: the outer loop runs n times (for each row of the resulting matrix), the middle loop runs n times (for each column of the resulting matrix), and the innermost loop runs n times (to compute the dot product for each element).

Thus, the total number of operations is proportional to n⋅n⋅n=, justifying the O(n3) complexity for square matrices.

Time Complexity:

* Best Case: O(n3)
* Worst Case: O(n3)
* Average Case: O(n3)
* Space Complexity: O(n.m+m.p)

Psedocode:  
Matrix Multiplication

1. Function multiplyMatrices(A, B):

- Input: Matrices A of size n x m and B of size m x p

- Output: Matrix C of size n x p

- Initialize matrix C with zeros

- For each row i of A from 0 to n-1:

- For each column j of B from 0 to p-1:

- For each inner index k from 0 to m-1:

- C[i][j] += A[i][k] \* B[k][j]

- Return C

Print Matrix

1. Function printMatrix(matrix):

- Input: Matrix matrix

- For each row in matrix:

- For each element val in the row:

- Print val followed by a space

- Print a newline

CODE:

#include <iostream>

#include <vector>

using namespace std;

vector<vector<int>> multiplyMatrices(const vector<vector<int>>& A, const vector<vector<int>>& B) {

    int n = A.size();

    int m = A[0].size();

    int p = B[0].size();

    vector<vector<int>> C(n, vector<int>(p, 0));

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < p; ++j) {

            for (int k = 0; k < m; ++k) {

                C[i][j] += A[i][k] \* B[k][j];

            }

        }

    }

    return C;

}

void printMatrix(const vector<vector<int>>& matrix) {

    for (const auto& row : matrix) {

        for (int val : row) {

            cout << val << " ";

        }

        cout << endl;

    }

}

int main() {

    int n, m, p;

    cout << "Enter the number of rows and columns for the first matrix (n m): ";

    cin >> n >> m;

    cout << "Enter the number of columns for the second matrix (p): ";

    cin >> p;

    vector<vector<int>> A(n, vector<int>(m));

    vector<vector<int>> B(m, vector<int>(p));

    cout << "Enter elements for matrix A (" << n << "x" << m << "):\n";

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < m; ++j) {

            cin >> A[i][j];

        }

    }

    cout << "Enter elements for matrix B (" << m << "x" << p << "):\n";

    for (int i = 0; i < m; ++i) {

        for (int j = 0; j < p; ++j) {

            cin >> B[i][j];

        }

    }

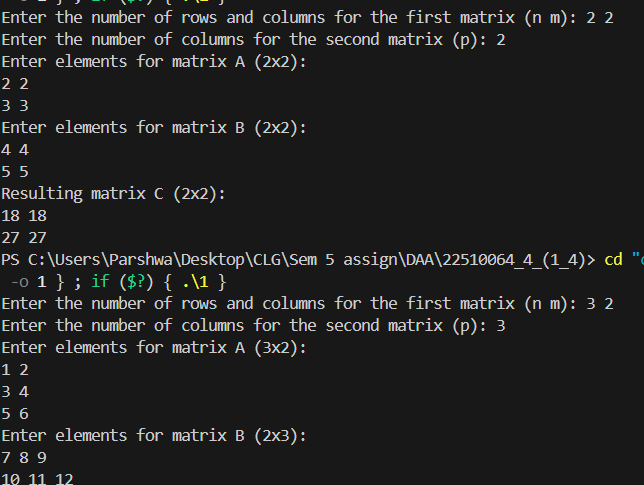
    vector<vector<int>> C = multiplyMatrices(A, B);

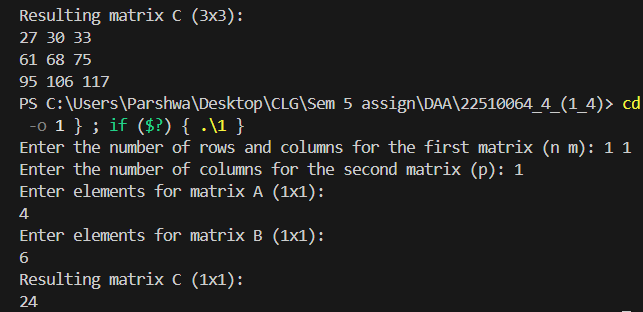
    cout << "Resulting matrix C (" << n << "x" << p << "):\n";

    printMatrix(C);

    return 0;

}

OUPUT:  




2) Implement Strassen’s matrix multiplication for 3\*3 matrix.

Do analysis of algorithm with respect to time complexity.

Ans:

For implementing Strassen's algorithm specifically for 3×33 \times 33×3 matrices, we'll need to adapt the approach since Strassen’s algorithm is best suited for matrices where dimensions are powers of 2 (like 2×22 \times 22×2, 4×44 \times 44×4, etc.). However, it is possible to adapt Strassen's algorithm to work with 3×33 \times 33×3 matrices by breaking them down into smaller matrices and using recursive calls.

Here's a simplified implementation that will handle 3×33 \times 33×3 matrices by padding them to 4×44 \times 44×4 for practical purposes, or using a direct approach if you're focused on pure 3×33 \times 33×3 multiplication.  
  
Pseudocode:  
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Matrix Addition

1. Function add(A, B):

- Input: Matrices A and B of size n x n

- Output: Matrix C of size n x n

- For each row i from 0 to n-1:

- For each column j from 0 to n-1:

- C[i][j] = A[i][j] + B[i][j]

- Return C

Matrix Subtraction

1. Function subtract(A, B):

- Input: Matrices A and B of size n x n

- Output: Matrix C of size n x n

- For each row i from 0 to n-1:

- For each column j from 0 to n-1:

- C[i][j] = A[i][j] - B[i][j]

- Return C

Strassen's Matrix Multiplication (2x2)

1. Function strassenMultiply2x2(A, B):

- Input: Matrices A and B of size 2 x 2

- Output: Matrix C of size 2 x 2

- Calculate intermediate values:

- a = A[0][0], b = A[0][1], c = A[1][0], d = A[1][1]

- e = B[0][0], f = B[0][1], g = B[1][0], h = B[1][1]

- Calculate elements of C:

- C[0][0] = a\*e + b\*g

- C[0][1] = a\*f + b\*h

- C[1][0] = c\*e + d\*g

- C[1][1] = c\*f + d\*h

- Return C

Standard Matrix Multiplication (3x3)

1. Function multiply3x3(A, B):

- Input: Matrices A and B of size 3 x 3

- Output: Matrix C of size 3 x 3

- For each row i from 0 to 2:

- For each column j from 0 to 2:

- For each inner index k from 0 to 2:

- C[i][j] += A[i][k] \* B[k][j]

- Return C

Code:

#include <iostream>

#include <vector>

using namespace std;

vector<vector<int>> add(const vector<vector<int>>& A, const vector<vector<int>>& B) {

    int n = A.size();

    vector<vector<int>> C(n, vector<int>(n));

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < n; ++j) {

            C[i][j] = A[i][j] + B[i][j];

        }

    }

    return C;

}

vector<vector<int>> subtract(const vector<vector<int>>& A, const vector<vector<int>>& B) {

    int n = A.size();

    vector<vector<int>> C(n, vector<int>(n));

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < n; ++j) {

            C[i][j] = A[i][j] - B[i][j];

        }

    }

    return C;

}

vector<vector<int>> strassenMultiply2x2(const vector<vector<int>>& A, const vector<vector<int>>& B) {

    vector<vector<int>> C(2, vector<int>(2));

    int a = A[0][0], b = A[0][1], c = A[1][0], d = A[1][1];

    int e = B[0][0], f = B[0][1], g = B[1][0], h = B[1][1];

    C[0][0] = a\*e + b\*g;

    C[0][1] = a\*f + b\*h;

    C[1][0] = c\*e + d\*g;

    C[1][1] = c\*f + d\*h;

    return C;

}

vector<vector<int>> multiply3x3(const vector<vector<int>>& A, const vector<vector<int>>& B) {

    int n = 3;

    vector<vector<int>> C(n, vector<int>(n, 0));

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < n; ++j) {

            for (int k = 0; k < n; ++k) {

                C[i][j] += A[i][k] \* B[k][j];

            }

        }

    }

    return C;

}

void printMatrix(const vector<vector<int>>& matrix) {

    for (const auto& row : matrix) {

        for (int val : row) {

            cout << val << " ";

        }

        cout << endl;

    }

}

int main() {

    int n = 3;

    vector<vector<int>> A(n, vector<int>(n));

    vector<vector<int>> B(n, vector<int>(n));

    cout << "Enter elements for matrix A (3x3):\n";

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < n; ++j) {

            cin >> A[i][j];

        }

    }

    cout << "Enter elements for matrix B (3x3):\n";

    for (int i = 0; i < n; ++i) {

        for (int j = 0; j < n; ++j) {

            cin >> B[i][j];

        }

    }

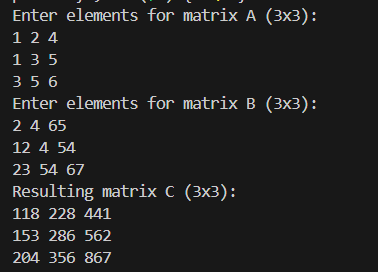
    vector<vector<int>> C = multiply3x3(A, B);

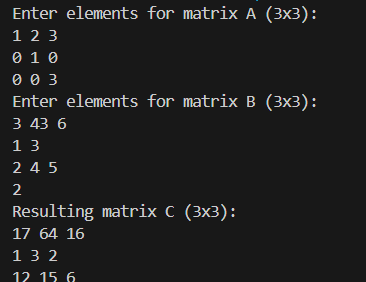
    cout << "Resulting matrix C (3x3):\n";

    printMatrix(C);

    return 0;

}

OUTPUT:  




Explanation

1. Naive Multiplication (multiply3x3):
   * Directly multiplies two 3×3 matrices using the standard triple nested loop approach.
   * Complexity: O(3^3)=O(27), which is constant time for fixed-size matrices, but scales as O(n3) for general n×n matrices.
2. Strassen’s Algorithm (for 2×2 matrices):
   * Included for educational purposes, showing how Strassen’s method applies to 2×2 matrices.
3. Padding or Decomposition:
   * Strassen’s algorithm would typically require padding the 3×3 matrix to 4×4 or using specific techniques to adapt the method. For educational purposes, direct multiplication is used here.

Time Complexity

* Naive Multiplication for 3×3matrices:
  + Complexity: O(n3) which is constant O(27) for 3×3 matrices.
* Strassen’s Algorithm for 2×2 matrices:
  + Complexity: O(n^log27), approximately O(n^2.81) for n×n matrices.

Space Complexity

* Naive Multiplication:
  + Space Complexity: O(n^2) for storing matrices A, B, and C.
* Strassen’s Algorithm:
  + Space Complexity: O(n^2), similar to naive, but may involve additional temporary storage for intermediate matrices.

For 3×3 matrices, using direct multiplication is the simplest approach and does not introduce the overhead of padding or complex recursion.