

## CNS LAB

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**BATCH: B1**

### Experiment No. 08

**Title – Implement the Diffie-Hellman Key Exchange algorithm for a given**

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problem.

#### **Objectives:**

To implement the **Diffie-Hellman Key Exchange Algorithm** to enable two parties to securely share a secret key over an insecure communication channel. This shared key can then be used for symmetric encryption or decryption in secure communication.

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#### **Problem Statement:**

In secure communications, two parties (commonly referred to as Alice and Bob) need to agree on a secret key that can be used for encrypting and decrypting messages. However, they must do this over a public channel where an attacker (Eve) might be listening.

Implement the **Diffie-Hellman Key Exchange Algorithm**, which allows Alice and Bob to securely compute a shared secret key without directly transmitting it over the insecure channel. The algorithm should:

1. Accept a large prime number  $p$  and a primitive root modulo  $p$ ,  $g$ .
2. Allow each party to select a private key ( $a$  for Alice,  $b$  for Bob).
3. Compute the corresponding public keys:
  - o Alice computes  $A = g^a \bmod p$
  - o Bob computes  $B = g^b \bmod p$
4. Exchange public keys between Alice and Bob.
5. Compute the shared secret key:
  - o Alice computes  $K = B^a \bmod p$
  - o Bob computes  $K = A^b \bmod p$
6. Validate that both computed secret keys are equal, i.e.,  $K_{\text{Alice}} == K_{\text{Bob}}$ .

Additionally, demonstrate the correctness of the algorithm with an example and optionally simulate an attacker attempting to derive the secret key without access to the private keys.

## Equipment / Tools

- Java Development Kit (JDK) 8 or later (for BigInteger and SecureRandom).
  - A code editor (VS Code, IntelliJ, Notepad++) or terminal editor (vim, nano).
  - Terminal / Command Prompt to compile and run Java.
  - (Optional) Internet to look up recommended prime sizes (e.g., 2048-bit) or libraries for production usage.
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## Theory (Detailed)

### Basic idea

Diffie–Hellman allows two parties to agree on a shared secret  $K$  using public values and their private secrets, without sending  $K$  over the channel.

Public parameters:

- $p$  — a large prime
- $g$  — a generator (primitive root) modulo  $p$  (i.e.,  $g$  generates a large subgroup mod  $p$ )

Private keys:

- Alice chooses a secret  $a$  (private)
- Bob chooses a secret  $b$  (private)

Public keys:

- Alice computes  $A = g^a \bmod p$  and sends  $A$  to Bob.
- Bob computes  $B = g^b \bmod p$  and sends  $B$  to Alice.

Shared secret:

- Alice computes  $K_A = B^a \bmod p = (g^b)^a \bmod p = g^{(ab)} \bmod p$
- Bob computes  $K_B = A^b \bmod p = (g^a)^b \bmod p = g^{(ab)} \bmod p$

Thus  $K_A == K_B == g^{(ab)} \bmod p$ .

## Why it is secure (informal)

- Given  $g, p, A = g^a \bmod p$ , recovering  $a$  is the **discrete logarithm problem (DLP)**, believed to be hard for appropriately large  $p$  and generator  $g$ .
- An eavesdropper who sees  $g, p, A, B$  would need to solve DLP to learn  $a$  or  $b$  and then compute  $g^{(ab)}$ . For properly chosen parameters (e.g., 2048-bit  $p$  or elliptic curve variants), this is computationally infeasible.

## Practical considerations

- **Prime choice:** Use safe primes or standardized parameters (RFCs). For production, use 2048-bit or larger prime groups or elliptic-curve Diffie–Hellman (ECDH).
- **Authentication:** DH by itself does not authenticate peers — vulnerable to man-in-the-middle (MitM). Use digital signatures, certificates, or authenticated variants (e.g., TLS) to prevent MitM.
- **Ephemeral DH:** Use ephemeral keys (generate fresh  $a, b$  per session) for forward secrecy.
- **Modular exponentiation:** Implemented efficiently via square-and-multiply; `BigInteger.modPow` in Java does that.

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## Procedure (Step-by-step)

1. Choose a large prime  $p$  and a generator  $g$  modulo  $p$ . (Publicly known.)
2. Alice chooses a private random integer  $a$  such that  $1 \leq a \leq p-2$ .
3. Bob chooses a private random integer  $b$  such that  $1 \leq b \leq p-2$ .
4. Alice computes public  $A = g^a \bmod p$  and sends  $A$  to Bob over the insecure channel.
5. Bob computes public  $B = g^b \bmod p$  and sends  $B$  to Alice.
6. Alice computes shared secret  $K_A = B^a \bmod p$ .
7. Bob computes shared secret  $K_B = A^b \bmod p$ .
8. Verify  $K_A == K_B$ . Use the resulting  $K$  (or a key derived from it

via a key-derivation function) as the symmetric encryption key.

9. (Optional demonstrate attacker) Attempt to compute  $a$  from  $A$  by brute force for small prime  $p$  to show infeasibility for large  $p$ .

CODE:

```
import java.math.BigInteger;
import java.security.SecureRandom;
import java.util.Scanner;

/**
 * Diffie-Hellman demonstration in Java using BigInteger.
 *
 * Features:
 * - Interactive: accepts p, g, and private keys (or 'r' for random).
 * - Uses BigInteger.modPow for modular exponentiation.
 * - Optional brute-force discrete-log solver for small p (to simulate Eve).
 *
 * Note: For production use, use standardized primes and authenticated DH
 (e.g., within TLS).
 */
public class DiffieHellman {
    private static final SecureRandom random = new SecureRandom();

    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        System.out.println("Diffie-Hellman Key Exchange Demo (Java)");
        System.out.println("-----");

        System.out.print("Enter prime p (decimal): ");
        BigInteger p = new BigInteger(sc.next());

        if (!p.isProbablePrime(20)) {
            System.out.println("Warning: p is not strongly prime according to
quick test (probablePrime).");
        }

        System.out.print("Enter generator g (decimal): ");
        BigInteger g = new BigInteger(sc.next());

        BigInteger a = readPrivate("Alice", sc, p);
        BigInteger b = readPrivate("Bob", sc, p);
    }
}
```

```

// Compute public keys
BigInteger A = g.modPow(a, p); // Alice's public
BigInteger B = g.modPow(b, p); // Bob's public

System.out.println("\nPublic values:");
System.out.println("A (Alice's public key) =  $g^a \bmod p$  = " + A);
System.out.println("B (Bob's public key) =  $g^b \bmod p$  = " + B);

// Compute shared secrets
BigInteger K_A = B.modPow(a, p);
BigInteger K_B = A.modPow(b, p);

System.out.println("\nShared secret computed by Alice: " + K_A);
System.out.println("Shared secret computed by Bob: " + K_B);
System.out.println("Shared keys equal? " + K_A.equals(K_B));

// Optional: attempt brute force discrete log for small p
if (p.bitLength() <= 20) { // small p -> feasible to brute force
    System.out.println("\nEve (attacker) simulation: trying to recover
Alice's private key a by brute force...");
    BigInteger found = bruteForcePrivateKey(A, g, p);
    if (found != null) {
        System.out.println("Eve found a = " + found + " (verifies:  $g^a \bmod p$  = " + g.modPow(found, p) + ")");
        System.out.println("Eve can compute shared secret  $K = g^{ab} \bmod p$  = " + g.modPow(found.multiply(b), p).mod(p));
    } else {
        System.out.println("Eve failed to find a (unexpected).");
    }
} else {
    System.out.println("\nEve simulation skipped: p is too large for
brute-force in this demo.");
    System.out.println("For real-world p (e.g., 2048-bit), discrete
log brute force is infeasible.");
}

sc.close();
}

private static BigInteger readPrivate(String who, Scanner sc, BigInteger
p) {
    while (true) {
        System.out.print("Enter " + who + "'s private key (decimal) or 'r'
for random: ");
        String s = sc.next();
        if (s.equalsIgnoreCase("r")) {
            // generate random  $1 \leq x \leq p-2$ 
            BigInteger max = p.subtract(BigInteger.valueOf(2));

```

```

        BigInteger x;
        do {
            x = new BigInteger(max.bitLength(), random);
        } while (x.compareTo(BigInteger.ONE) < 0 || x.compareTo(max) >
0);

        System.out.println(who + " private key (randomly chosen): " +
x);

        return x;
    } else {
        try {
            BigInteger x = new BigInteger(s);
            if (x.compareTo(BigInteger.ONE) < 0 ||
x.compareTo(p.subtract(BigInteger.ONE)) > 0) {
                System.out.println("Private key must satisfy  $1 \leq \text{key} \leq p-2$ . Try again.");
                continue;
            }
            return x;
        } catch (NumberFormatException ex) {
            System.out.println("Invalid number. Try again.");
        }
    }
}

// Brute force discrete log: find x such that  $g^x \bmod p == \text{publicKey}$ .
// Only feasible for small p; used for demonstration of insecurity with
small primes.
private static BigInteger bruteForcePrivateKey(BigInteger publicKey,
BigInteger g, BigInteger p) {
    BigInteger x = BigInteger.ZERO;
    BigInteger limit = p; // search 0..p-1
    while (x.compareTo(limit) < 0) {
        if (g.modPow(x, p).equals(publicKey)) {
            return x;
        }
        x = x.add(BigInteger.ONE);
    }
    return null;
}
}

```

## RESULTS:

```
PS C:\Users\ASUS> cd C:\Users\ASUS\Desktop\CNSL\22510076_CNS_A8
PS C:\Users\ASUS\Desktop\CNSL\22510076_CNS_A8> javac DiffieHellman.java
>>
PS C:\Users\ASUS\Desktop\CNSL\22510076_CNS_A8> java DiffieHellman
>>
Diffie-Hellman Key Exchange Demo (Java)
-----
Enter prime p (decimal): 23
Enter generator g (decimal): 5
Enter Alice's private key (decimal) or 'r' for random: 6
Enter Bob's private key (decimal) or 'r' for random: 15

Public values:
A (Alice's public key) =  $g^a \bmod p = 8$ 
B (Bob's public key)   =  $g^b \bmod p = 19$ 

Shared secret computed by Alice: 2
Shared secret computed by Bob:   2
Shared keys equal? true

Eve (attacker) simulation: trying to recover Alice's private key a by brute force...
Eve found a = 6 (verifies:  $g^a \bmod p = 8$ )
Eve can compute shared secret  $K = g^{(ab)} \bmod p = 2$ 
PS C:\Users\ASUS\Desktop\CNSL\22510076_CNS_A8> █
```

### Steps (what you will show in assignment)

1. State public parameters  $p$  and  $g$ .
2. Show chosen private keys  $a$  and  $b$  (or show they are randomly generated).
3. Show public keys  $A = g^a \bmod p$  and  $B = g^b \bmod p$ .
4. Show Alice's computation  $K_A = B^a \bmod p$ .
5. Show Bob's computation  $K_B = A^b \bmod p$ .
6. Verify  $K_A == K_B$ . Print the value of  $K$ .
7. Optionally, run Eve's brute-force search (only for small  $p$ ) and show she recovers  $a$  and reconstructs  $K$ .

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### Observations and Conclusion

#### Observations

- For the example ( $p=23$ ,  $g=5$ ,  $a=6$ ,  $b=15$ ) both parties computed the

same shared secret  $K = 2$  using the exchanged public keys — which demonstrates correctness.

- The computation uses modular exponentiation and relies on properties of exponents:  $(g^a)^b = g^{(ab)} = (g^b)^a$ .
- When  $p$  is small, a brute-force attacker can recover private keys quickly — demonstration shows this.
- For large primes ( $p$  with hundreds or thousands of bits) and good generators, discrete log algorithms become infeasible, so DH is secure in practice (assuming proper parameter choice).
- DH does **not** provide authentication: an active MitM attacker who can intercept and substitute public keys can perform two separate DH exchanges and decrypt communications unless authentication is added (e.g., signatures, certificates, pre-shared keys).

## Conclusion

- The implemented Diffie–Hellman algorithm correctly allows two parties to compute a shared secret without sending the secret directly.
- Security depends entirely on parameter choice (large safe primes / standardized groups) and on protecting against active MitM attackers via authentication.
- For production systems, prefer using standardized groups, ephemeral keys for forward secrecy, and authenticated key exchange protocols (e.g., TLS/ECDHE).
- The brute-force demonstration shows why small primes are insecure — always use large primes or switch to elliptic-curve DH (ECDH) for better performance/security.