
- You may work in groups of **up to three** students, and you must submit a *single* solution in a PDF file named **a2.pdf**, submitted to MarkUs. Submissions must be **typed**. Remember to write the *full name* and *MarkUs username* of *every* member of your group prominently on your submission.
- Please refer to the course website for the **late submission policy**.
- Please read and understand the policy on Academic Integrity given on the course website. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes). For example, indicate clearly the **name** of every student from another group with whom you had discussions, the **title and sections** of every textbook you consulted (including the course textbook), the **source** of every web document you used (including documents from the course webpage), etc.
- For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks will be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions. In particular, keep in mind that one of the objectives of this assignment is to get you to practice writing proofs. This means that a significant number of marks will be allocated to the structure of your proofs and how they were written up.

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1. In *dyadic* notation, natural numbers are represented in base 2 using the digits 1 and 2 (instead of 0 and 1). For example, 18 is represented by the dyadic string "1122" because $18 = 1 \cdot 8 + 1 \cdot 4 + 2 \cdot 2 + 2 \cdot 1$. Note that each integer has a unique dyadic representation, because there cannot be any leading digits. This is unlike binary notation, where each integer has infinitely many representations (e.g., 18 is represented by each of the infinitely many binary strings 10010, 010010, 0010010, ...). In particular, the unique dyadic representation of 0 is the empty string (ε). For any dyadic string $w \in \{1,2\}^*$, $\mathrm{val}(w)$ represents the decimal value of w (e.g., $\mathrm{val}(122) = 10$ and $\mathrm{val}(\varepsilon) = 0$).

Construct a DFA that accepts exactly those dyadic strings whose value is a multiple of 5.

Provide a *brief* justification that your DFA is correct, e.g., by giving appropriate state invariants for your DFA (no proof required).

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2. Give an **NFA** and a **regex** for the following language

$$L = \Big\{ w \in \{a,b\}^* : \text{either all the symbols in odd positions within } w \text{ are } a\text{'s, or all the symbols in even positions within } w \text{ are } a\text{'s, or both} \Big\}$$

For example, if $w = s_1 s_2 s_3 s_4 s_5$, where each $s_i \in \{a,b\}$, then $w \in L$ iff $s_1 = s_3 = s_5 = a$, or $s_2 = s_4 = a$, or $s_1 = s_2 = s_3 = s_4 = s_5 = a$.

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3. Let R_1, R_2 and R_3 be regular expressions. Prove or disprove the following statement.

$$\text{If } \mathcal{L}(R_1) \subseteq \mathcal{L}(R_2) \subseteq \mathcal{L}(R_3), \text{ then } (R_1^* + R_3)^* \equiv (R_2^* + R_3^*).$$

You may use the following facts in your answer (no need to prove them, just use them to draw conclusions): for all languages L_1, L_2,

- if $L_1 \subseteq L_2$, then $L_1^{\circledast} \subseteq L_2^{\circledast}$;
- $(L_1^{\circledast})^{\circledast} = L_1^{\circledast}$.

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4. Let Σ be an arbitrary alphabet. For every $w \in \Sigma^*$,

$$\rho(w) = \begin{cases} \varepsilon & \text{if } w = \varepsilon, \\ cx & \text{if } w = xc \text{ for some } x \in \Sigma^* \text{ and } c \in \Sigma. \end{cases}$$

For every language $L \subseteq \Sigma^*$, we define

$$R(L) = \{x \in \Sigma^* : x = \rho(y) \text{ for some } y \in L\}.$$

Prove that if L is a regular language, then so is $R(L)$.
More specifically, you must show the **construction of a DFA** for $R(L)$ based on a DFA for L.
Present your DFA by giving its **formal definition**, NOT diagrams.

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