Two-Link Planar Robot Manipulator with Distributed Mass

Clear all data

```
clc;
clear all;
close all;
clf; %Clear all figures
```

Basic Definition

```
bar_mass = [1; 1]; % mass of bar in kg
bar_length = [1; 1]; %Length of bar in m
center_of_mass  = [0.45; 0.45];
moment_of_inertia = [0.084; 0.084];
initial_parameters = [30; 0; 45; 0]; % Initial degree (theta1, theta_dot_1, theta2, the
initial_parameters = [deg2rad(initial_parameters(1,1)); deg2rad(initial_parameters(2,1)
applied_torque = [0; 0]; % Applied torque to the bar (T)
% where T is applied wrt to +ve x axis
gravitational_acceleration = 9.81; %m/s2
graph_time = 10; % Time frame for plotting
```

Symbolic Definition

```
syms g m1 m2 h1 h2 v1 v2 I1 I2 w1 w2 l1 l2 r1 r2 x1(t) x2(t) y1(t) y2(t) theta1(t) thet
```

```
Warning: Can only make assumptions on variable names, not 'x1(t)'.
Warning: Can only make assumptions on variable names, not 'x2(t)'.
Warning: Can only make assumptions on variable names, not 'y1(t)'.
Warning: Can only make assumptions on variable names, not 'y2(t)'.
Warning: Can only make assumptions on variable names, not 'theta1(t)'.
Warning: Can only make assumptions on variable names, not 'theta2(t)'.
```

```
sympref('AbbreviateOutput',false);
sympref('MatrixWithSquareBrackets',true);
sympref('PolynomialDisplayStyle','ascend');
```

Define Generalised Coordinates

```
q = sym('q',[2,1]);
q(1) = theta1;
q(2) = theta2;
q
```

q =

$$\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix}$$

```
dq = diff(q,t)
```

dq =

$$\begin{bmatrix} \frac{\partial}{\partial t}\ \theta_1(t) \\ \frac{\partial}{\partial t}\ \theta_2(t) \end{bmatrix}$$

Defining Generalised Inputs

```
u = sym('u',[2,1]);
u(1) = T1;
u(2) = T2;
u
```

u =

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

Derive Lagrangian Function

```
PE = m1*g*h1 + m2*g*h2;
PE = subs(PE,[h1, h2],[r1*cos(theta1), l1*cos(theta1) + r2*cos(theta1 + theta2)])
```

PE = $(\cos(\theta_1(t)+\theta_2(t))\, r_2 + \cos(\theta_1(t))\, l_1)\, g\, m_2 + \cos(\theta_1(t))\, g\, m_1\, r_1$

```
KE = (m1*(v1^2)/2) + (I1*(w1^2)/2) + (m2*(v2^2)/2) + (I2*(w2^2)/2);
KE = subs(KE,[v1, v2],[sqrt(diff(x1,t)^2 + diff(y1,t)^2), sqrt(diff(x2,t)^2 + diff(y2,t
KE = subs(KE,[x1,y1,w1,x2,y2,w2],[r1*sin(theta1), r1*cos(theta1), diff(theta1,t), l1*si
```

KE =

$$\frac{m_2\left(\left(r_2\cos(\theta_1(t)+\theta_2(t))\left(\frac{\partial}{\partial t}\ \theta_1(t)+\frac{\partial}{\partial t}\ \theta_2(t)\right)+l_1\cos(\theta_1(t))\frac{\partial}{\partial t}\ \theta_1(t)\right)^2+\left(r_2\sin(\theta_1(t)+\theta_2(t))\left(\frac{\partial}{\partial t}\ \theta_1(t)\cdot\right.\right.\right.}{2}$$

```
LE = KE - PE;
LE = simplify(LE)
```

LE =

$$\frac{I_1\left(\frac{\partial}{\partial t}\ \theta_1(t)\right)^2}{2}+\frac{I_2\left(\frac{\partial}{\partial t}\ \theta_1(t)\right)^2}{2}+\frac{I_2\left(\frac{\partial}{\partial t}\ \theta_2(t)\right)^2}{2}+I_2\frac{\partial}{\partial t}\ \theta_2(t)\frac{\partial}{\partial t}\ \theta_1(t)+\frac{l_1{}^2\, m_2\left(\frac{\partial}{\partial t}\ \theta_1(t)\right)^2}{2}+\frac{m_1\, r_1{}^2\left(\frac{\partial}{\partial t}\ \theta_1(t\right.}{2}$$

Derive Euler Lagrangian Equations

```
dLE_dq = jacobian(LE,q);
dLE_ddq = jacobian(KE,dq);
d_dLE_dtDdq = diff(dLE_ddq,t);
EU_eq1 = d_dLE_dtDdq(1,1) - dLE_dq(1,1) - u(1);
EU_eq2 = d_dLE_dtDdq(1,2) - dLE_dq(1,2) - u(2);
EU_eq1 = simplify(EU_eq1);
```

```
EU_eq2 = simplify(EU_eq2);
syms thethaddot1 thethaddot2 'real'
EU_eq1 = subs(EU_eq1,[diff(theta1,t,2), diff(theta2,t,2)],[thethaddot1, thethaddot2]);
EU_eq2 = subs(EU_eq2,[diff(theta1,t,2), diff(theta2,t,2)],[thethaddot1, thethaddot2]);
```

Symbolic Solution to EL Equation

```
[thethaddot1, thethaddot2]=solve([EU_eq1, EU_eq2],[thethaddot1, thethaddot2])
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in
the solution, specify the 'ReturnConditions' value as 'true'.
thethaddot1 =
```

$$\frac{I_2\,T_1 - I_2\,T_2 + T_1\,m_2\,{r_2}^2 - T_2\,m_2\,{r_2}^2 + g\,l_1\,{m_2}^2\,{r_2}^2\,\sin(\theta_1(t)) + I_2\,g\,l_1\,m_2\,\sin(\theta_1(t)) + I_2\,g\,m_1\,r_1\,\sin(\theta_1(t)) - T_2\,l}{}$$

```
thethaddot2 =
```

$$-\frac{I_2\,T_1 - I_1\,T_2 - I_2\,T_2 - T_2\,{l_1}^2\,m_2 - T_2\,m_1\,{r_1}^2 + T_1\,m_2\,{r_2}^2 - T_2\,m_2\,{r_2}^2 + g\,l_1\,{m_2}^2\,{r_2}^2\,\sin(\theta_1(t)) + I_2\,g\,l_1\,m_2\,\sin(\theta}{}$$

Defining State Space Matrix

```
syms x1(t) x2(t) x3(t) x4(t)
x = [x1;x2;x3;x4]
```

```
x(t) =
```

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

```
%x(1) = theta1;
%x(2) = diff(theta1,t);
%x(3) = theta2;
%x(4) = diff(theta2,t);
thethaddot1 = subs(thethaddot1,[g, T1, m1, l1, r1, I1, T2, m2, l2, r2, I2,theta1, theta
```

```
thethaddot1 =
```

$$-\frac{\frac{16301277\,\sin(x_1(t))}{4000000} + \frac{5157\,\sin(x_3(t))\,x_2(t)^2}{40000} + \frac{5157\,\sin(x_3(t))\,x_4(t)^2}{40000} - \frac{79461\,\cos(x_3(t))\,\sin(x_1(t) + x_3(t))}{40000} +}{\frac{81\,\cos(x_3(t))^2}{400} - \frac{1474329}{4000000}}$$

```
thethaddot2 = subs(thethaddot2,[g, T1, m1, l1, r1, I1, T2, m2, l2, r2, I2,theta1, theta
```

```
thethaddot2 =
```

$$\frac{\frac{16301277 \sin(x_1(t))}{4000000} - \frac{22717017 \sin(x_1(t) + x_3(t))}{4000000} + \frac{14157 \sin(x_3(t))\, x_2(t)^2}{20000} + \frac{5157 \sin(x_3(t))\, x_4(t)^2}{40000} + \frac{256041}{}}{}$$

```
dx = [x2;
    thethaddot1;
    x4;
    thethaddot2]
```

```
dx(t) =
```

$$\left[\begin{array}{c} \\ -\frac{\frac{16301277 \sin(x_1(t))}{4000000} + \frac{5157 \sin(x_3(t))\, x_2(t}{40000}}{} \\ \\ \frac{\frac{16301277 \sin(x_1(t))}{4000000} - \frac{22717017 \sin(x_1(t) + x_3(t))}{4000000} + \frac{14157 \sin(x_3(t))\, x_2(t)^2}{20000} + \frac{5157 \sin(x_3(t))\, x_4(t)^2}{40000} + \frac{25604}{}}{} \\ \end{array}\right.$$

```
vars = [x1(t) x2(t) x3(t) x4(t)];
function_converter = odeFunction(dx, vars); % Converts symbolic function in Function
func_dx = @(t,x)function_converter(t,x);
```

Solving Differential Equation to get thetha

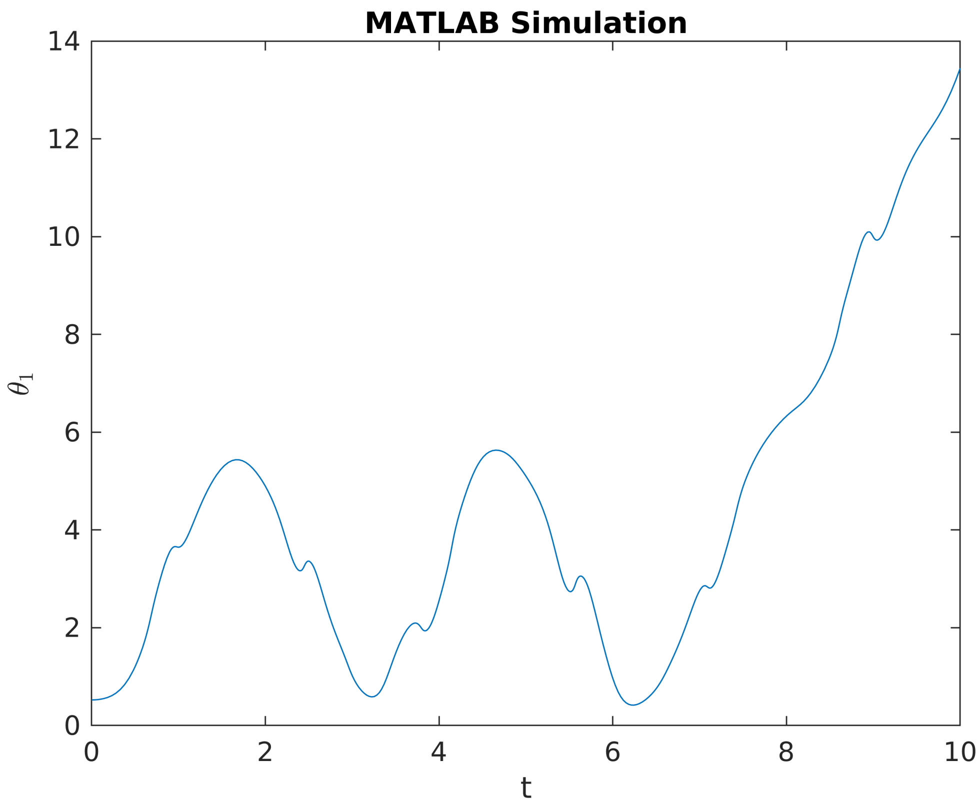
```
tspan =[0 graph_time];
[t,state_space_matrix] = ode45(func_dx,tspan,initial_parameters);
```

Calculating Link Positions

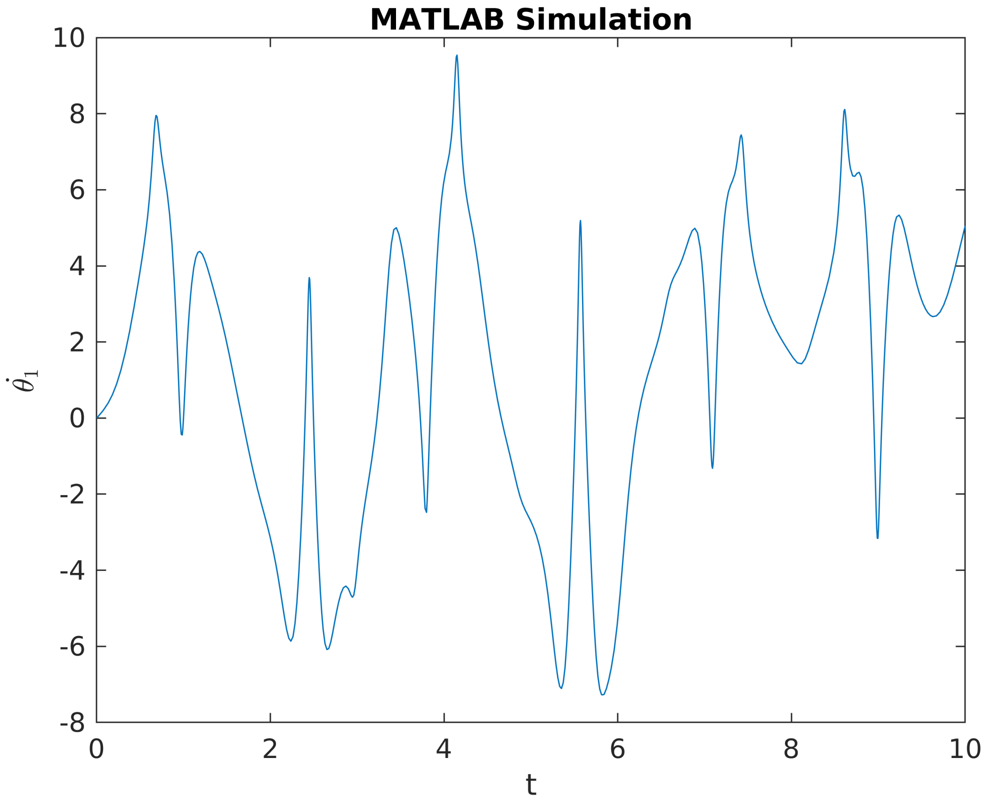
```
r_size = size(t);
position = zeros(r_size(1,1),4);
for i = 1:r_size(1,1)
    position(i,1) = bar_length(1,1)*sin(state_space_matrix(i,1));
    position(i,2) = bar_length(1,1)*cos(state_space_matrix(i,1));
    position(i,3) = position(i,1) + bar_length(2,1)*sin(state_space_matrix(i,1) + state
    position(i,4) = position(i,2) + bar_length(2,1)*cos(state_space_matrix(i,1) + state
    com_coordinates(i,1) = center_of_mass(1,1)*sin(state_space_matrix(i,1));
    com_coordinates(i,2) = center_of_mass(1,1)*cos(state_space_matrix(i,1));
    com_coordinates(i,3) = position(i,1) + center_of_mass(2,1)*sin(state_space_matrix(i
    com_coordinates(i,4) = position(i,2) + center_of_mass(2,1)*cos(state_space_matrix(i

end
```

Animation

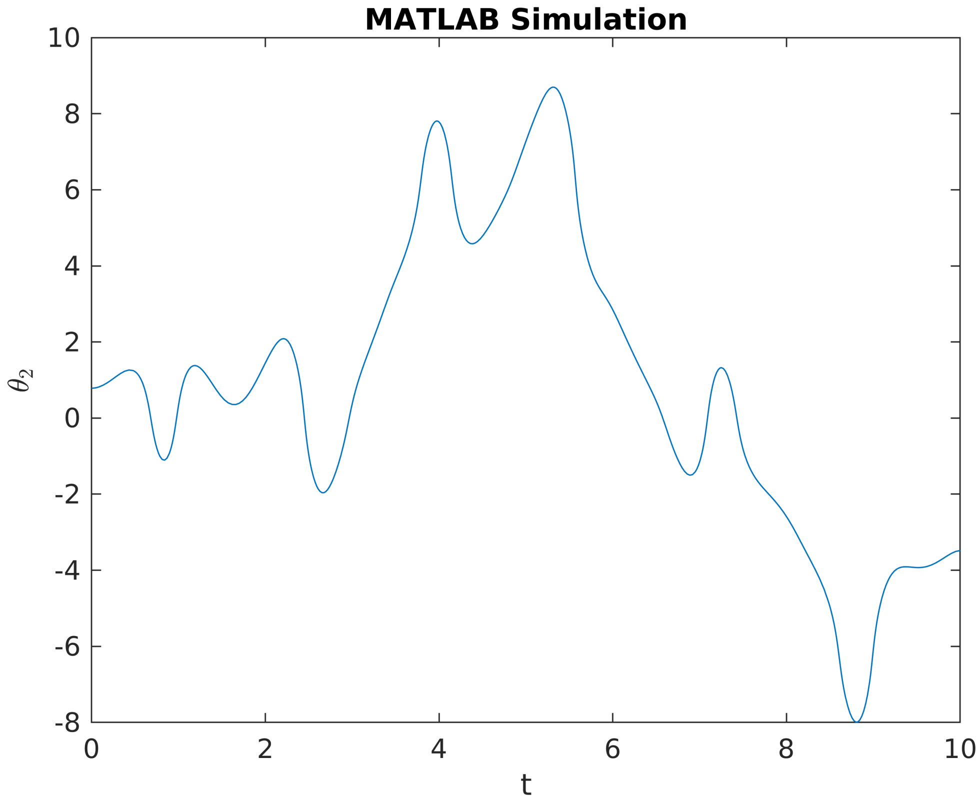
```
xAxisArrayXCoordinates = [-2 2];
xAxisArrayYCoordinates = [0 0];
yAxisArrayXCoordinates = [0 0];
yAxisArrayYCoordinates = [-2 2];
th = 0:pi/50:2*pi;
xunit = (bar_length(1,1) + bar_length(2,1)) * cos(th);
yunit = (bar_length(1,1) + bar_length(2,1)) * sin(th);
plot(t,state_space_matrix(:,1))
xlabel('t')
ylabel('${\theta_1}$', 'Interpreter','latex')
title('MATLAB Simulation')
saveas(gcf,'theta1.jpg')
```



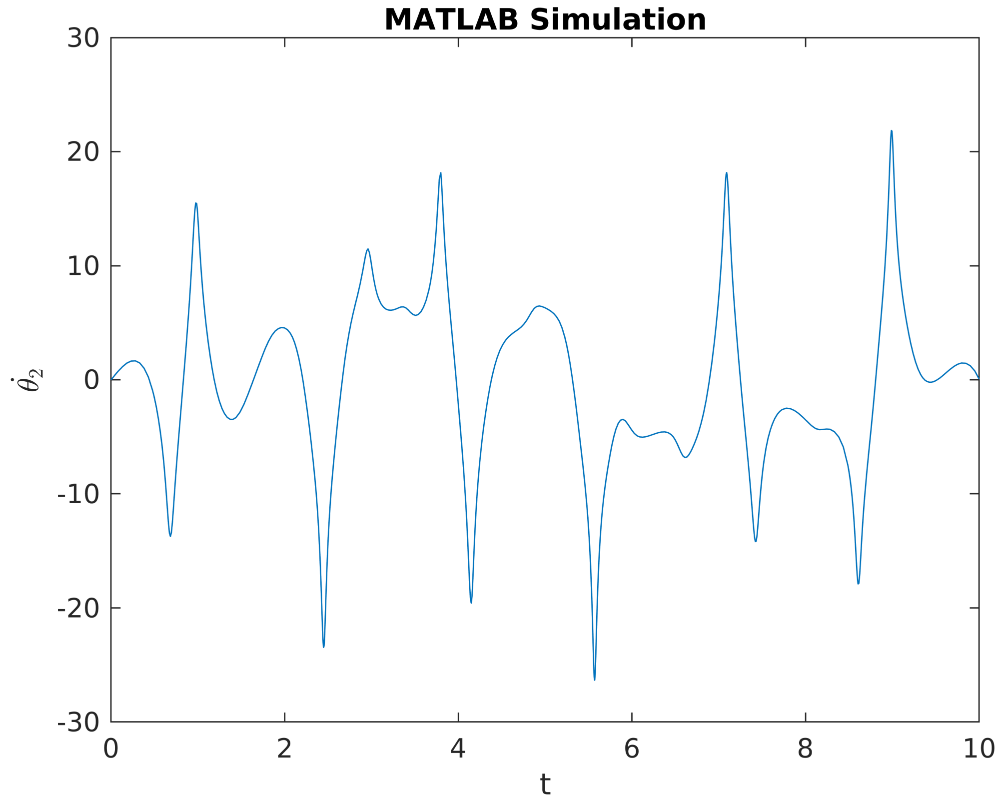

```
plot(t,state_space_matrix(:,2))
xlabel('t')
ylabel('$\dot{\theta_1}$', 'Interpreter','latex')
title('MATLAB Simulation')
saveas(gcf,'theta_dot_1.jpg')
```




```
plot(t,state_space_matrix(:,3))
xlabel('t')
ylabel('${\theta_2}$', 'Interpreter','latex')
title('MATLAB Simulation')
saveas(gcf,'theta2.jpg')
```



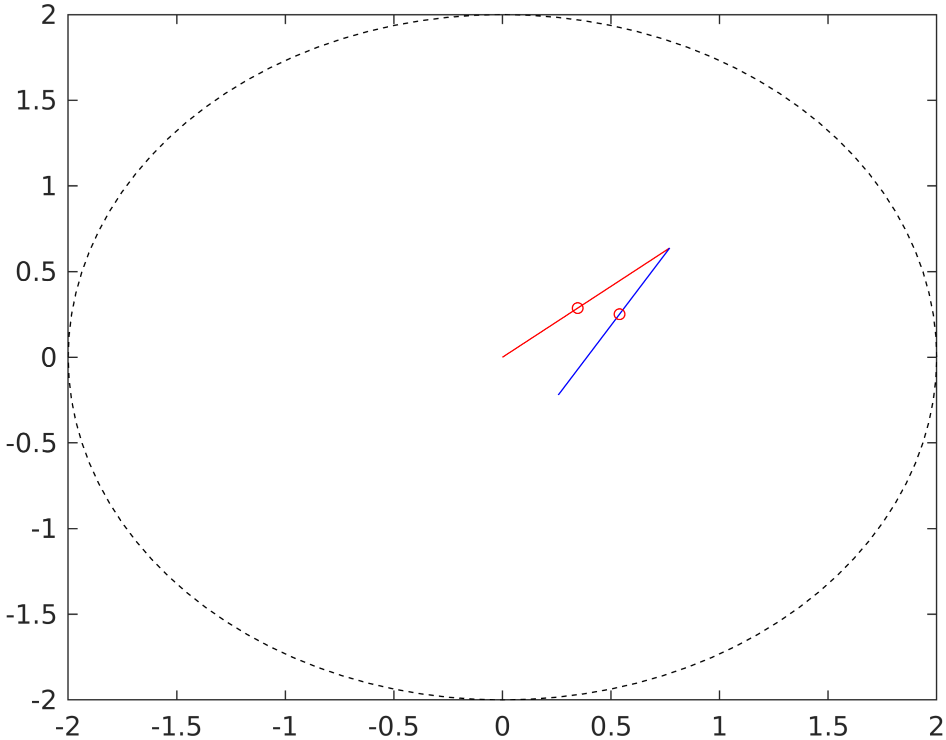

```
plot(t,state_space_matrix(:,4))
xlabel('t')
ylabel('$\dot{\theta_2}$', 'Interpreter','latex')
title('MATLAB Simulation')
saveas(gcf,'theta_dot_2.jpg')
```




```
delete('MATLAB Simulation.avi');
```

```
Warning: File 'MATLAB Simulation.avi' not found.
```

```
animation = VideoWriter('MATLAB Simulation.avi');
open(animation);
for i = 1:1:r_size(1,1)
    %Plotting Graph
    link1XCoordinates = [0 position(i,1)];
    link1YCoordinates = [0 position(i,2)];
    link2XCoordinates = [position(i,1) position(i,3)];
    link2YCoordinates = [position(i,2) position(i,4)];
    plot(xunit, yunit,'k','LineStyle','--'); % Draw Circular Axes
    hold on;
    plot(link1XCoordinates, link1YCoordinates, 'red');
    plot(com_coordinates(i,1),com_coordinates(i,2),'-ro','MarkerSize',4);
    plot(link2XCoordinates, link2YCoordinates, 'blue');
    plot(com_coordinates(i,3),com_coordinates(i,4),'-ro','MarkerSize',4);
    frame = getframe(gcf);
    writeVideo(animation,frame);
    pause(0.1); % pause to see realtime animation. Given in seconds
    hold off;
end
```




```
close(animation);
```