# Controller design

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$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ \frac{1}{m}(\cos(x_2)\cdot\cos(x_3))U_1 - g \\ \frac{1}{I_x}(x_7 \cdot x_8(I_y - I_z) - I_p \Omega x_7 + U_2) \\ \frac{1}{I_y}(x_6 \cdot x_8(I_z - I_x) + I_p \Omega x_6 + U_3) \\ \frac{1}{I_z}(x_6 \cdot x_7(I_x - I_y) + U_4) \end{bmatrix}$$

## Designing $z$ :-

$$S_1 = \dot{e} + \lambda_1 e = (\dot{z} - \dot{z}_d) + \lambda(z - z_d) \qquad z_d = z \text{ desired}$$
$$= (x_5 - \dot{z}_d) + \lambda_1(x_1 - z_d)$$

$$\dot{S}_1 = \dot{x}_5 - \ddot{z}_d + \lambda_1(\dot{x}_1 - \dot{z}_d)$$
$$= \frac{1}{m}\left\{\cos(x_2)\cdot\cos(x_3)U_1 - g\right\} - \ddot{z}_d + \lambda_1(x_5 - \dot{z}_d)$$

$$S\dot{S}_1 = S_1 \cdot \left\{ \frac{\cos(x_2)\cdot\cos(x_3)}{m} \left\{ \frac{m(\lambda_1(x_5 - \dot{z}_d) - \ddot{z}_d) - g}{\cos x_2 \cdot \cos x_3} + U_1 \right\} \right\}$$

$$\boxed{U_1 = -\left\{ \frac{m(\lambda_1(x_5 - \dot{z}_d) - \ddot{z}_d) - g}{\cos(x_2)\cos(x_3)} + K_1 \right\} sat(S_1)}$$

$sat(S_1)$
↳ $saturation(S_1)$

## Designing $\phi$ :-

$$S_2 = \dot{e} + \lambda_2 e = (\dot{\phi} - \dot{\phi}_d) + \lambda_2(\phi - \phi_d)$$
$$\dot{S}_2 = \ddot{e} + \lambda_2 \dot{e} = (\ddot{\phi} - \cancelto{0}{\ddot{\phi}_d}) + \lambda_2(\dot{\phi} - \cancelto{0}{\dot{\phi}_d})$$

$$\dot{S}_2 = \frac{1}{I_x}\left\{(x_7 x_8(I_y - I_z) - I_p \cdot \Omega \cdot x_7 + U_2) + \lambda_2 I_x (x_6)\right\}$$

$$S_2\dot{S}_2 = \frac{S_2}{I_x}\left\{(x_7 x_8(I_y - I_z) - I_p \Omega x_7 + \lambda_2 I_x x_6) + U_2\right\}$$

$$\boxed{U_2 = -\left\{x_7 x_8(I_y - I_z) - I_p \cdot \Omega \cdot x_7 + \lambda_2 \cdot I_x \cdot x_6 + K_2\right\} \cdot sat(S_2)} \implies saturation(S_2)$$

## Designing $\theta$ :-

$$S_3 = \dot{e} + \lambda_3 e = (\dot{\theta} - \dot{\theta}_d) + \lambda_3(\theta - \theta_d)$$
$$\dot{S}_3 = \ddot{e} + \lambda_3 \ddot{e} = (\ddot{\theta} - \ddot{\theta}_d) + \lambda_3(\dot{\theta} - \dot{\theta}_d)$$
$$\dot{S}_3 = \frac{1}{I_y}\left\{X_6 X_8(I_z - I_x) + I_p \cdot \Omega \cdot X_6 + U_3\right\} + \lambda_3 X_7$$

$$S_3\dot{S}_3 = \frac{S_3}{I_y}\left\{(X_6 X_8(I_z - I_x) + I_p \cdot \Omega \cdot X_6 + \lambda_3 I_y X_7) + U_3\right\}$$

$$\boxed{U_3 = -(X_6 \cdot X_8(I_z - I_x) + I_p \Omega X_6 + \lambda_3 I_y X_7 + K_3) \cdot sat(S_3)}$$

Shall we choose $\Omega = W_1 - W_2 + W_3 - W_4$

$Max(\Omega) = max(W_1 + W_3) - min(W_2 + W_4)$

## Designing $\psi$ :-

$$S_4 = \dot{e} + \lambda_4 e = (\dot{\psi} - \dot{\psi}_d) + \lambda_4(\psi - \psi_d)$$

$$\dot{S}_4 = \ddot{e} + \lambda_4 \dot{e} = (\ddot{\psi} - \cancelto{0}{\ddot{\psi}_d}) + \lambda_4(\dot{\psi} - \cancelto{0}{\dot{\psi}_d})$$

$$\dot{S}_4 = \frac{1}{I_z}\left\{X_6 X_7(I_x - I_y) + U_4\right\} + \lambda_4 X_8$$

$$S_4\dot{S}_4 = S_4 \cdot \frac{1}{I_z}\left\{X_6 X_7(I_x - I_y) + \lambda_4 I_z X_8 + U_4\right\}$$

$$\boxed{U_4 = -\left(X_6 X_7(I_x - I_y) + \lambda_4 I_z X_8 + K_4\right) \cdot sat(S_4)}$$