

Assignment 3

Voting

Voting systems based on majority rule are susceptible to strategic agenda-setting. Let's explore how one might do this on some basic examples.

Q.1) Suppose there are four alternatives, named A, B, C, and D. There are three voters who have the following individual rankings:

$$B \succ_1 C \succ_1 D \succ_1 A$$

$$C \succ_2 D \succ_2 A \succ_2 B$$

$$D \succ_3 A \succ_3 B \succ_3 C$$

You're in charge of designing an agenda for considering the alternatives in pairs and eliminating them using majority vote, via an elimination tournament in the style of the examples shown in Slide 14 (at PDF page 19) of Lecture 11.

The odds at this point are:

- $A \succ B \rightarrow \text{Votes } 2$
- $B \succ C \rightarrow \text{Votes } 2$
- $B \succ D \rightarrow \text{Votes } 2$
- $C \succ A \rightarrow \text{Votes } 2$
- $C \succ D \rightarrow \text{Votes } 2$
- $D \succ A \rightarrow \text{Votes } 2$

1. You would like alternative A to win. Can you design an agenda (i.e., an elimination tournament) in which A wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) To design an agenda in which alternative A wins, we can structure the tournament as follows:

- Round 1: C (2V), D (1V) \rightarrow Winner: C
- Round 2: B (2V), C (1V) \rightarrow Winner: B
- Round 3: A (2V), B (1V) \rightarrow Winner: A

2. You would like alternative B to win. Can you design an agenda (i.e., an elimination tournament) in which B wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) To design an agenda in which alternative B wins, we can structure the tournament as follows:

- Round 1: D (2V), A (1V) \rightarrow Winner: D
- Round 2: C (2V), D (1V) \rightarrow Winner: C
- Round 3: B (2V), C (1V) \rightarrow Winner: B

3. You would like alternative C to win. Can you design an agenda (i.e., an elimination tournament) in which C wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) To design an agenda in which alternative C wins, we can structure the tournament as follows:

- Round 1: A (2V), B (1V) \rightarrow Winner: A
- Round 2: D (2V), A (1V) \rightarrow Winner: D
- Round 3: C (2V), D (1V) \rightarrow Winner: C

4. You would like alternative D to win. Can you design an agenda (i.e., an elimination tournament) in which D wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) To design an agenda in which alternative D wins, we can structure the tournament as follows:

- Round 1: B (2V), C (1V) \rightarrow Winner: B
- Round 2: A (2V), B (1V) \rightarrow Winner: A
- Round 3: D (2V), A (1V) \rightarrow Winner: D

Q.2) Now, consider the same question, but for a slightly different set of individual rankings in which the last two positions in voter 3's ranking have been swapped. That is, we have:

$$B \succ_1 C \succ_1 D \succ_1 A$$

$$C \succ_2 D \succ_2 A \succ_2 B$$

$$D \succ_3 A \succ_3 C \succ_3 B$$

A) The odds at this point are:

- $A \succ B \rightarrow \text{Votes } 2$
- $C \succ A \rightarrow \text{Votes } 2$
- $C \succ B \rightarrow \text{Votes } 2$
- $C \succ D \rightarrow \text{Votes } 2$
- $D \succ A \rightarrow \text{Votes } 3$
- $D \succ B \rightarrow \text{Votes } 2$

1. Can you design an agenda in which A wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) Player A cannot win with the current odds. Round 3 needs Player A to win, So:

- Round 3: A (2V), B (1V) \rightarrow Winner: A
- Round 2: Not possible as B has no odds to win against any other player.

Thus, there is no way for Player A to win.

2. Can you design an agenda in which B wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) Player B cannot win with the current odds. No match shows that it has odds to win against any other Rival.

3. Can you design an agenda in which C wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) To design an agenda in which alternative C wins, we can structure the tournament as follows:

- Round 1: A (2V), B (1V) \rightarrow Winner: A
- Round 2: D (3V), A (0V) \rightarrow Winner: D
- Round 3: C (2V), D (1V) \rightarrow Winner: C

4. Can you design an agenda in which D wins? If so, describe how you would structure it; if not, explain why it is not possible.

A) Player D cannot win with the current odds. Round 3 needs Player D to win, So, there are two options:

- Round 3: D (3V), A (0V) \rightarrow Winner: D
 - Round 2: A (2V), B (1V) \rightarrow Winner: A
 - Round 1: Not possible as B has no odds to win against C
- Round 3: D (2V), B (1V) \rightarrow Winner: D
 - Round 2: Not possible as B has no odds to win against C

Thus, there is no way for Player D to win.