Exercise 1: Information Cascades [40 points]

In this problem we will ask whether an information cascade can occur if each individual sees only the choice of his immediate neighbor rather than the choices of all those who have chosen previously. Let's keep the same setup as in the Information Cascades lecture (Lecture 7), except that, when individual i chooses, they consider only their own color and the choice of individual $i-1$.

1. Briefly explain why the decision problems faced by individuals 1 and 2 are unchanged by this modification to the information network.
2. Individual 3 observes the choices of individual 2, but not the choice of individual 1. What can 3 infer about 2's color from 2's choice?
3. Can 3 infer anything about 1's color from 2's choice? Explain.
4. What should 3 do if he picks a red ball and he knows that 2 declared Red? What if 3's picked ball was blue and 2 declared Red?
5. Do you think that a cascade can form in this world? Explain why or why not. A formal proof is not necessary, a brief argument is sufficient.

① Decision problem for 1 & 2 is unchanged because they only share the true info w/ any "influence from the previous agents.

② 3 can infer that 2 is probably true in stating the true color & hence shape the majority.

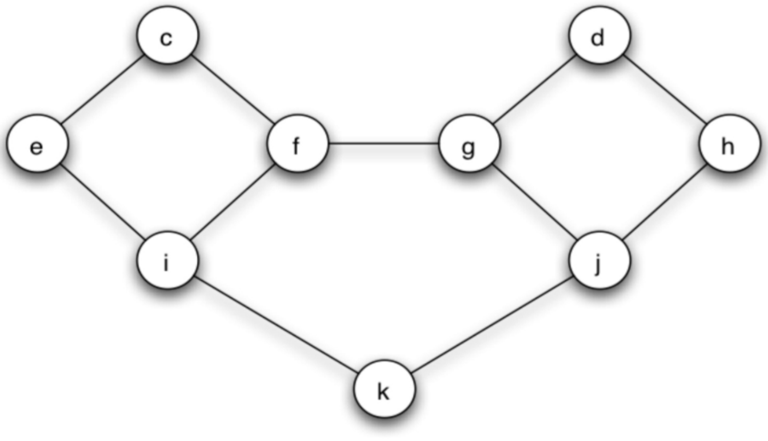
③ Yes, 3 can infer about 1's color being opposite if 2^{nd} & 3^{rd}'s colors are same or the other way around.

④ 3 should say 'Red' if his is Red.
3 should say 'Red' if 2^{nd} picked 'Red' & his is Blue & there are only 3 balls.

⑤ No, because of mere 3 balls & 3 picks, if 1 & 2 are honest, the 3^{rd} can decide the truth from his pick.

Exercise 2: Information Cascades [30 points]

Consider the network depicted in Figure; suppose that each node starts with the behavior B, and each node has a threshold of $q = \frac{1}{2}$ for switching to behavior A.



1. Let e and f form a two-node set S of initial adopters of behavior A. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A?
2. Find a cluster of density greater than $1 - q = \frac{1}{2}$ in the part of the graph outside S that blocks behavior A from spreading to all nodes, starting from S, at threshold q.

for $q > 1/2$

(1) → 'c' has $2/2$ neighbours w/ behaviour 'a' so 'c' will change

→ 'i' has $2/3$ neighbours w/ behaviour 'a' so 'i' will change

→ 'k' will have $1/2$ neighbours of b. 'a' so it will not change

→ 'g' has $1/3$ neighbours of b. 'a' so it will not change.

∴ Changed nodes = $\{c, i\}$

(2) The cluster outside 's' that keeps it from spreading is:

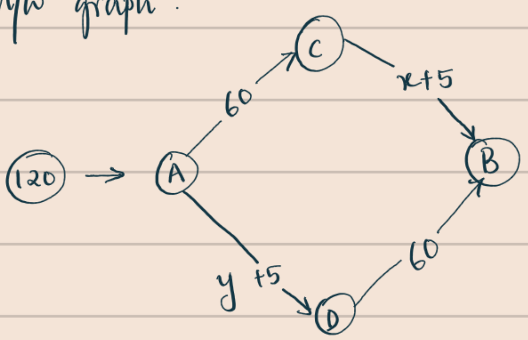
$S' = \{D, G, H, J, K\}$

Exercise 3: Traffic Flow [30 points]

There are two cities A and B joined by two routes. There are 120 travelers who begin in city A and must travel to city B. There are two routes between A and B. Route I begins with a highway leaving city A, this highway takes one hour of travel time regardless of how many travelers use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 5 plus the number of travelers who use the street. Route II begins with a local street leaving city A, which requires a travel time in minutes equal to 5 plus the number of travelers who use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of travelers who use this highway.

1. Draw the network described above and label the edges with the travel time needed to move along the edge. Let x be the number of travelers who use Route I. The network should be a directed graph as all roads are one-way.
2. Travelers simultaneously chose which route to use. Find the Nash equilibrium value of x.
3. Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. This adds two new routes. One new route consists of the local street leaving city A (on Route II), the new road and the local street into city B (on Route I). The second new route consists of the highway leaving city A (on Route I), the new road and the highway leading into city B (on Route II). The new road is very short and takes no travel time. Find the new Nash equilibrium.

1.) n/w graph:



x → no. of cars via path C

y → no. of cars via path D

2.) → Assume all 120 people choose path C:

∴ $t = 60 + (120+5) = 185$ mins.

→ Assume all 120 people choose path D:

∴ $t = (120+5) + 60 = 185$ mins.

This shows that there's no clear dominant Nash Equilibrium.

∴ for a non dominant strategy, the no. of cars via C & D should sum be same & travel time be equal.

i.e. $x + y = 120$

& $(x+5) + \cancel{60} = (y+5) + \cancel{60}$

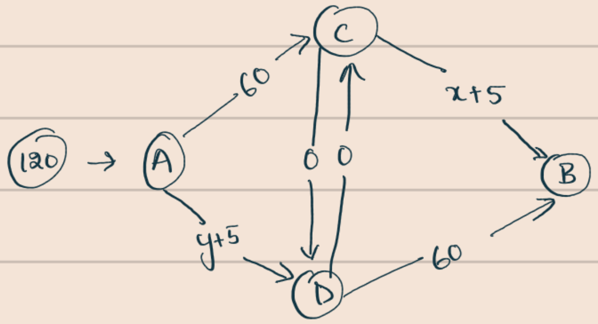
$\therefore x = y$

& $x + y = 120 \Rightarrow x = y = 60.$

$\therefore$ 60 people $A \to C \to B \to t = 125$ mins

60 people $A \to D \to B \to t = 125$ mins

(3)



with the new path added, people have 4 options:

$A \to C \to B$: $t = 185$ mins

$A \to C \to D \to B$: $t = 120$ mins

$A \to D \to C \to B$: $t = 250$ mins

$A \to D \to B$: $t = 185$ mins.

$\therefore$ A new Nash Equ^{m} is found which is better than before:

$A \to C \to D \to B$ for all 120 people w/ $t = 120$ mins.