Assignment 1

Friday, January 13, 2023 2:22 PM

Exercise 1 (Geometry)

Calculate the rotation of vector $(4, 7)$ by $60\,^\circ$.

$\vec{v} = 4\hat{i} + 7\hat{j}$

$|\vec{v}| = \sqrt{4^2 + 7^2} = \sqrt{16+49} = \sqrt{65}$

$\angle\vec{v} = \tan^{-1}(7/4) = 60.25^\circ \cong 60^\circ$

$\vec{v} = \sqrt{65}\, e^{j\alpha/3}$

After Rotation, $\vec{v}_n = \sqrt{65}\, e^{j\alpha/3}$

$\therefore \vec{v} = -4 + 9\hat{j}$

Exercise 2 (Probability)

Your robot is misbehaving, and you're trying to find out why. You narrow it down to three possible software bugs, b_1, b_2 and b_3. You're not sure which bug it is, but you try the same experiment on a brand new robot to make sure the issue is not hardware-related. You estimate that the anomalous behavior will appear with probability 0.7 if it's due to bug b_1, 0.5 if it's due to bug b_2, and 0.6 if it's due to bug b_3. Given that the new test shows the anomalous behavior, what probabilities should you assign to the three possible bugs?

Hints:

- Consider the event A that an anomaly occurs, and the events $B1$, $B2$, and $B3$ that bugs b_1, b_2, and b_3 occur; then, $P(A|B1) = 0.7$, $P(A|B2) = 0.5$, and $P(A|B3) = 0.6$.
- The problem is asking you the probabilities $P(B1|A)$, $P(B2|A)$, and $P(B3|A)$. Which formula do we use to calculate this?
- The problem does not give you $P(A)$. Which formula do we use to calculate this?
- The problem does not give you $P(B1)$, $P(B2)$, and $P(B3)$. Think about what they mean: they are the probability that a bug occurs, without information on which specific anomaly occurred. What reason would there be to decide that a bug is more probable than another, if no information is available? You can safely say that all these probabilities can be set to the same constant K. Since you have no value for K, maybe there's a hope that in the final formula it will cancel out...

Exercise 3 (Calculus)

Given

$$f(x) = \epsilon\left(\left(\frac{\alpha}{x}\right)^{12} - 2\left(\frac{\alpha}{x}\right)^{6}\right),$$

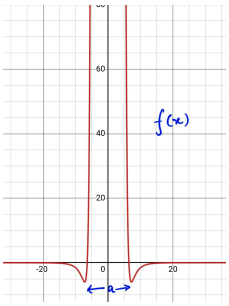
1. Draw the graph of $f(x)$;
2. Calculate and draw the graph of $g(x) = -\frac{\mathrm{d}}{\mathrm{d}x} f(x)$.

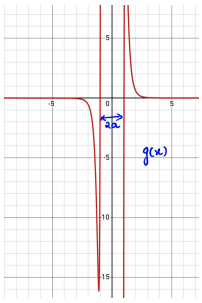
To draw the graphs, you can use programs such as Matlab or GNUPlot, or do it by hand.

$f(x) = 6\left[\left(\frac{\alpha}{x}\right)^{12} - 2\left(\frac{\alpha}{x}\right)^{6}\right]$

$g(x) = -f'(x) = -6\left[12\left(\frac{\alpha}{x}\right)^{11}\cdot\frac{\alpha}{x^2} - 2\cdot 6\left(\frac{\alpha}{x}\right)^{5}\frac{\alpha}{x^2}\right]$

$\therefore g(x) = \frac{-72}{x}\left[\left(\frac{\alpha}{x}\right)^{12} - \left(\frac{\alpha}{x}\right)^{6}\right]$





Exercise 4 (Differential Equations)

Find all the fixed points for $\dot{x} = x^2 - 25$ and classify their stability.

Fixed pts. : $\dot{x} = x^2 - 25 = (x-5)(x+5)$

for finding fixed pts. :

$\dot{x} = 0$

$\therefore (x-5)(x+5) = 0$

$\therefore x^* = \pm 5$

Stability : $f(x) = \dot{x} = x^2 - 25$

$\therefore f'(x) = 2x$

$\forall\ x = +5 \rightarrow f'(x^*) = 10 > 0 \longrightarrow$ Unstable

$\forall\ x = -5 \rightarrow f'(x^*) = -10 \longrightarrow$ Stable.

$\therefore x = +5 \longrightarrow$ Fixed / Unstable

$x = -5 \rightarrow$ Fixed / Stable.