Programming Assignment 3

Clear Environment

```
clc;
clear all;
close all;
warning ('off', 'all'); % Stopping all warnings
sympref('AbbreviateOutput', false);
sympref('MatrixWithSquareBrackets', true);
sympref('PolynomialDisplayStyle', 'ascend');
speed_solve = true;
```

System Parameters

```
% Actual Physical Parameters
am1 = 1;am2 = 1;
al1 = 1;al2 = 1;
ar1 = 0.45;ar2 = 0.45;
aI1 = 0.084;aI2 = 0.084;
gravity = 9.81;

% Nomimal Paramters
nm1 = 0.75;nm2 = 0.75;
nI1 = 0.063;nI2 = 0.063;
nr1 = aI1;nr2 = aI2;
```

Part 4.2.1: Initial Setup

a): Trajectory Generation

```
% Joint 1
theta0 = deg2rad(180);
thetaf = 0;
theta0_dot = 0;
thetaf_dot = 0;
t0 = 0;
tf = 10;
trajectory_generation;
q1d = q
```

```
q1d =
```

$$\pi - \frac{3\,\pi\,t^2}{100} + \frac{\pi\,t^3}{500}$$

```
dq1d = diff(q,t)
```

```
dq1d =
```

$$-\frac{3\,\pi\,t}{50} + \frac{3\,\pi\,t^2}{500}$$

```
v1d = diff(q,t,2)
```

v1d =

$$-\frac{3\pi}{50}+\frac{3\pi t}{250}$$

```
clear q

% Joint 2
theta0 = deg2rad(90);
thetaf = 0;
theta0_dot = 0;
thetaf_dot = 0;
t0 = 0;
tf = 10;
trajectory_generation;
q2d = q
```

q2d =

$$\frac{\pi}{2}-\frac{3\pi t^2}{200}+\frac{\pi t^3}{1000}$$

```
dq2d = diff(q,t)
```

dq2d =

$$-\frac{3\pi t}{100}+\frac{3\pi t^2}{1000}$$

```
v2d = diff(q,t,2)
```

v2d =

$$-\frac{3\pi}{100}+\frac{3\pi t}{500}$$

```
clear q
```

b): Manipulator Form

```
syms I1 I2 m1 m2 r1 r2 l1 l2 q1 q2 dq1 dq2 ddq1 ddq2 g U1 U2 v1 v2
a = I1 + I2 + m1*r1^2 + m2*(l1^2 + r2^2);
b = m2*l1*r2;
d = I2 + m2*r2^2;
Mmat= [a+2*b* cos(q2), d+b* cos(q2); d+b* cos(q2), d];
Cmat= [-b* sin(q2)*dq2, -b* sin(q2)*(dq1+dq2); b* sin(q2)*dq1,0];
Gmat= [-m1*g*r1* sin(q1)-m2*g*(l1* sin(q1)+r2* sin(q1+q2)); -m2*g*r2* sin(q1+q2)];
T = simplify(Mmat*[ddq1;ddq2] + Cmat*[dq1;dq2] + Gmat); % Robot Dynamics
T = subs(T,[I1,I2,m1,m2,r1,r2,l1,l2,g],[aI1,aI2,am1,am2,ar1,ar2,al1,al2,gravity])
```

T =

$$\begin{bmatrix} \mathrm{ddq}_1\left(\frac{9\cos(q_2)}{10}+\frac{1573}{1000}\right)-\frac{28449\sin(q_1)}{2000}-\frac{8829\sin(q_2+q_1)}{2000}+\mathrm{ddq}_2\left(\frac{9\cos(q_2)}{20}+\frac{573}{2000}\right)-\frac{9\,\mathrm{dq}_1\,\mathrm{dq}_2\,s}{20} \\ \frac{573\,\mathrm{ddq}_2}{2000}-\frac{8829\sin(q_2+q_1)}{2000}+\frac{9\,\mathrm{dq}_1{}^2\sin(q_2)}{20}+\mathrm{ddq}_1\left(\frac{9\cos(q_2)}{20}+\frac{573}{2000}\right. \end{bmatrix}$$

```
[ddq1, ddq2] = solve(T-[U1;U2],[ddq1,ddq2]);
ddq = [ddq1;ddq2];
control_input = simplify(Mmat*[v1;v2] + Cmat*[dq1;dq2] + Gmat); % Control Input
control_input = simplify(subs(control_input,[I1,I2,m1,m2,r1,r2,l1,l2,g],[nI1,nI2,nm1,nm
```

control_input =

$$\begin{bmatrix} v_1\left(\frac{63\cos(q_2)}{500}+\frac{110823}{125000}\right)-\frac{797553\sin(q_1)}{100000}-\frac{61803\sin(q_2+q_1)}{100000}+v_2\left(\frac{63\cos(q_2)}{1000}+\frac{17073}{250000}\right)-\frac{63\,\mathrm{dq}}{} \\ \frac{17073\,v_2}{250000}-\frac{61803\sin(q_2+q_1)}{100000}+\frac{63\,\mathrm{dq}_1{}^2\sin(q_2)}{1000}+v_1\left(\frac{63\cos(q_2)}{1000}+\frac{1'}{25}\right. \end{bmatrix}$$

4.2.2 Robust Control Design

C): Robust Control Law Design

```
A = [zeros(2) eye(2);zeros(2) zeros(2)] % A = [0 I; 0 0]
```

```
A = 4×4
     0     0     1     0
     0     0     0     1
     0     0     0     0
     0     0     0     0
```

```
B = [zeros(2); eye(2)] % B = [0;I]
```

```
B = 4×2
     0     0
     0     0
     1     0
     0     1
```

```
lambda = [-3, -3, -4, -4]; % Eigen Values of system
K = place(A,B,lambda)
```

```
K = 2×4
    12     0     7     0
     0    12     0     7
```

```
Acl = A - B*K
```

```
Acl = 4×4
     0     0     1     0
     0     0     0     1
   -12     0    -7     0
     0   -12     0    -7
```

```
Q = eye(4)*10
```

```
Q = 4×4
```

```
    10     0     0     0
     0    10     0     0
     0     0    10     0
     0     0     0    10
```

```
P = lyap(Acl',Q)% Acl'*P + P*Acl = -Q
```

```
P = 4×4
   12.2024         0    0.4167         0
         0   12.2024         0    0.4167
    0.4167         0    0.7738         0
         0    0.4167         0    0.7738
```

```
rho = 3.5
```

```
rho = 3.5000
```

E): Simulate the System W/O Boundary Layer

```
phi = 0
```

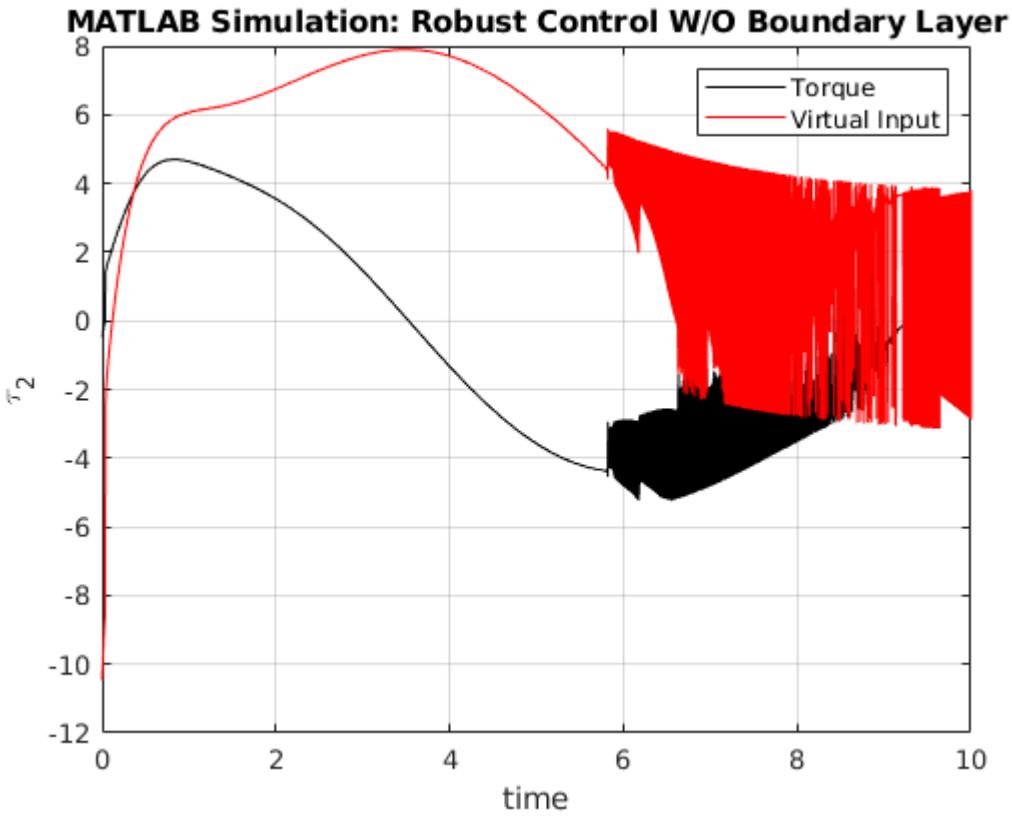
```
phi = 0
```

```
enable_robust_control = true
```

```
enable_robust_control = logical
   1
```

```
system_type = "MSWRCNBL";
graph_title = "MATLAB Simulation: Robust Control W/O Boundary Layer";
simulate_system;
```

```
initial_params = 4×1
    3.4907
    2.1817
         0
         0
Simulating System
t_size = 175713
Calculating System Parameters for Plotting
Plotting System
```




```
x_rcwobl = state_space_matrix;
v_rcwobl = virtual_input;
tau_rcwobl = system_input;
time_rcwobl =time_points;
tau1_max = max(tau_rcwobl(:, 1))
```

```
tau1_max = 6.6485
```

```
tau1_min = min(tau_rcwobl(:, 1))
```

```
tau1_min = -20.1039
```

```
tau2_max = max(tau_rcwobl(:, 2))
```

```
tau2_max = 4.6970
```

```
tau2_min = min(tau_rcwobl(:, 2))
```

```
tau2_min = -5.2293
```

F): Simulate the System W Boundary Layer

```
phi = 0.1
```

```
phi = 0.1000
```

```
enable_robust_control = true
```

```
enable_robust_control = logical
   1
```

```
system_type = "MSWRCWBL"
```

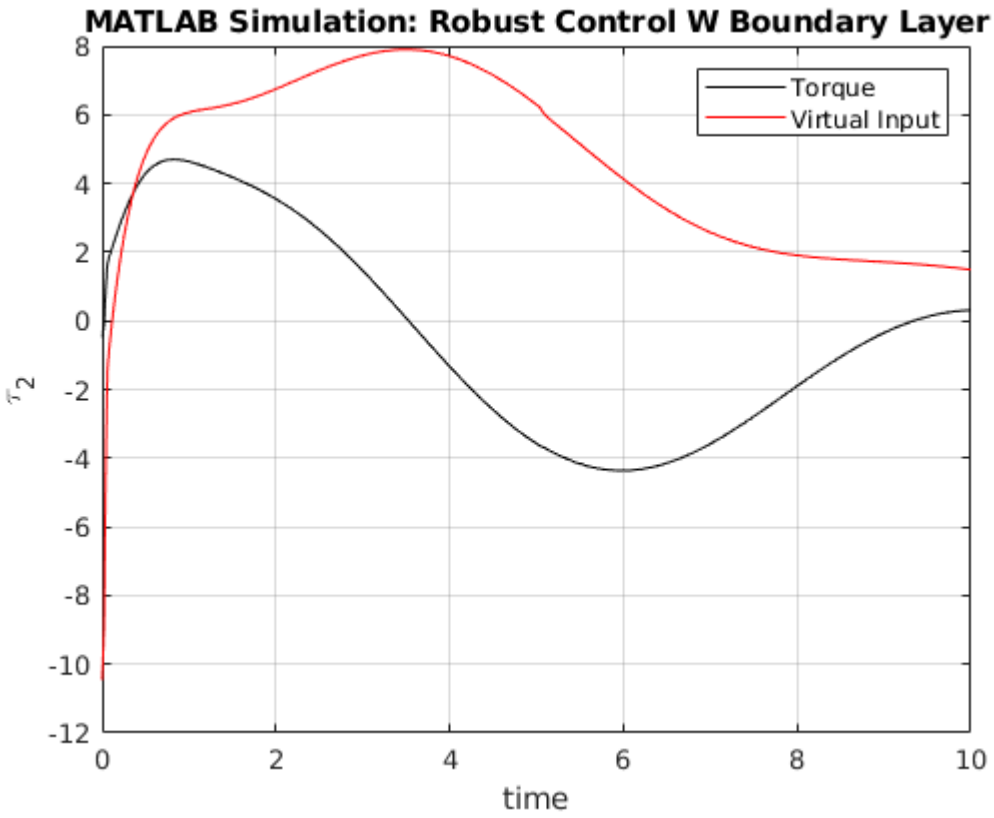
```
system_type =
"MSWRCWBL"
```

```
graph_title = "MATLAB Simulation: Robust Control W Boundary Layer"
```

```
graph_title =
"MATLAB Simulation: Robust Control W Boundary Layer"
```

```
simulate_system;
```

```
initial_params = 4×1
    3.4907
    2.1817
         0
         0
Simulating System
t_size = 337
Calculating System Parameters for Plotting
Plotting System
```




```
x_rcwbl = state_space_matrix;
v_rcwbl = virtual_input;
tau_rcwbl = system_input;
time_rcwbl = time_points;
tau1_max = max(tau_rcwbl(:, 1))
```

```
tau1_max = 6.2892
```

```
tau1_min = min(tau_rcwbl(:, 1))
```

```
tau1_min = -18.4266
```

```
tau2_max = max(tau_rcwbl(:, 2))
```

```
tau2_max = 4.6972
```

```
tau2_min = min(tau_rcwbl(:, 2))
```

```
tau2_min = -4.3648
```

G): Simulate the System W/O Robust Control

```
phi = 0
```

```
phi = 0
```

```
enable_robust_control = false
```

```
enable_robust_control = logical
   0
```

```
system_type = "MSWNRC"
```

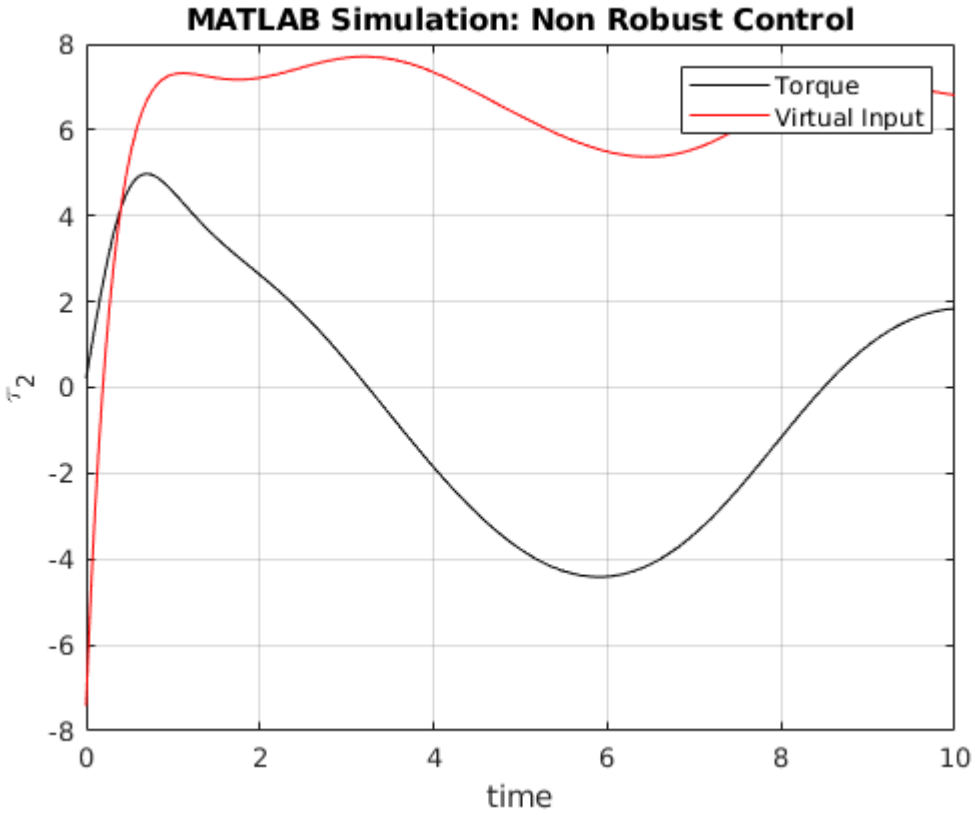
```
system_type =
"MSWNRC"
```

```
graph_title = "MATLAB Simulation: Non Robust Control"
```

```
graph_title =
"MATLAB Simulation: Non Robust Control"
```

```
simulate_system;
```

```
initial_params = 4×1
    3.4907
    2.1817
         0
         0
Simulating System
t_size = 153
Calculating System Parameters for Plotting
Plotting System
```




```
x_nrc = state_space_matrix;
v_nrc = virtual_input;
tau_nrc = system_input;
time_nrc = time_points;
tau1_max = max(tau_nrc(:, 1))
```

```
tau1_max = 5.7893
```

```
tau1_min = min(tau_nrc(:, 1))
```

```
tau1_min = -18.6573
```

```
tau2_max = max(tau_nrc(:, 2))
```

```
tau2_max = 4.9818
```

```
tau2_min = min(tau_nrc(:, 2))
```

```
tau2_min = -4.4127
```

H): Gazebo Simulation

```
system_type = "GSWRCWBL"
```

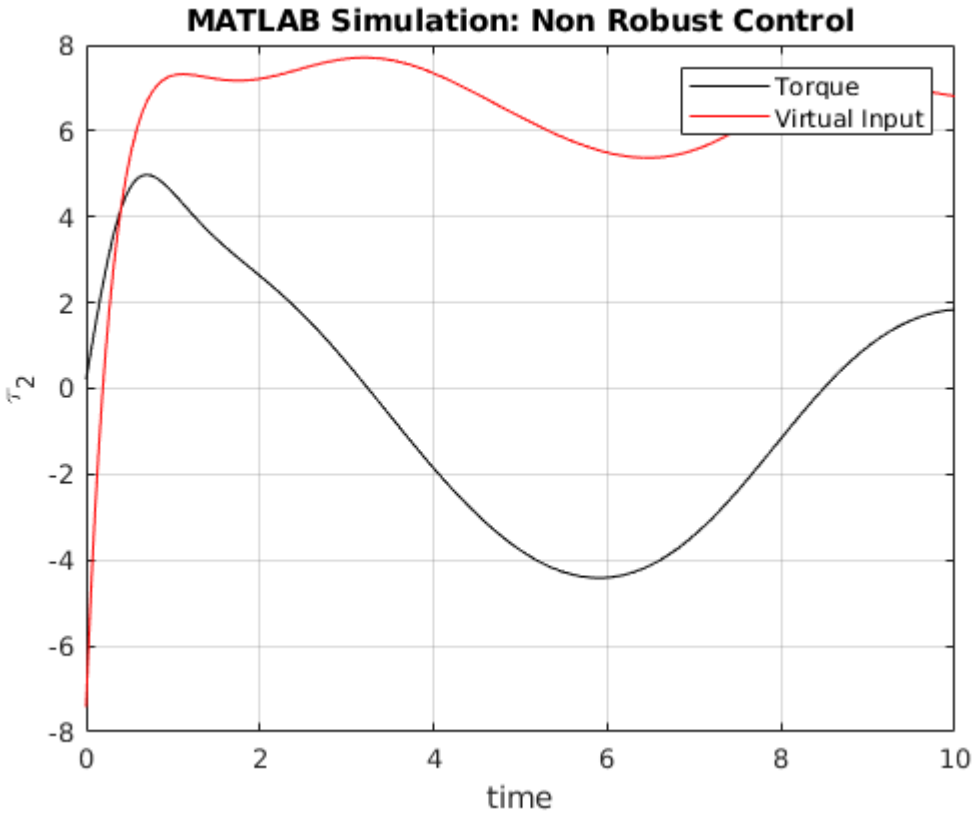
```
system_type =
"GSWRCWBL"
```

```
graph_title = "Gazebo Simulation: Robust Control with Boundary Layer"
```

```
graph_title =
```

"Gazebo Simulation: Robust Control with Boundary Layer"

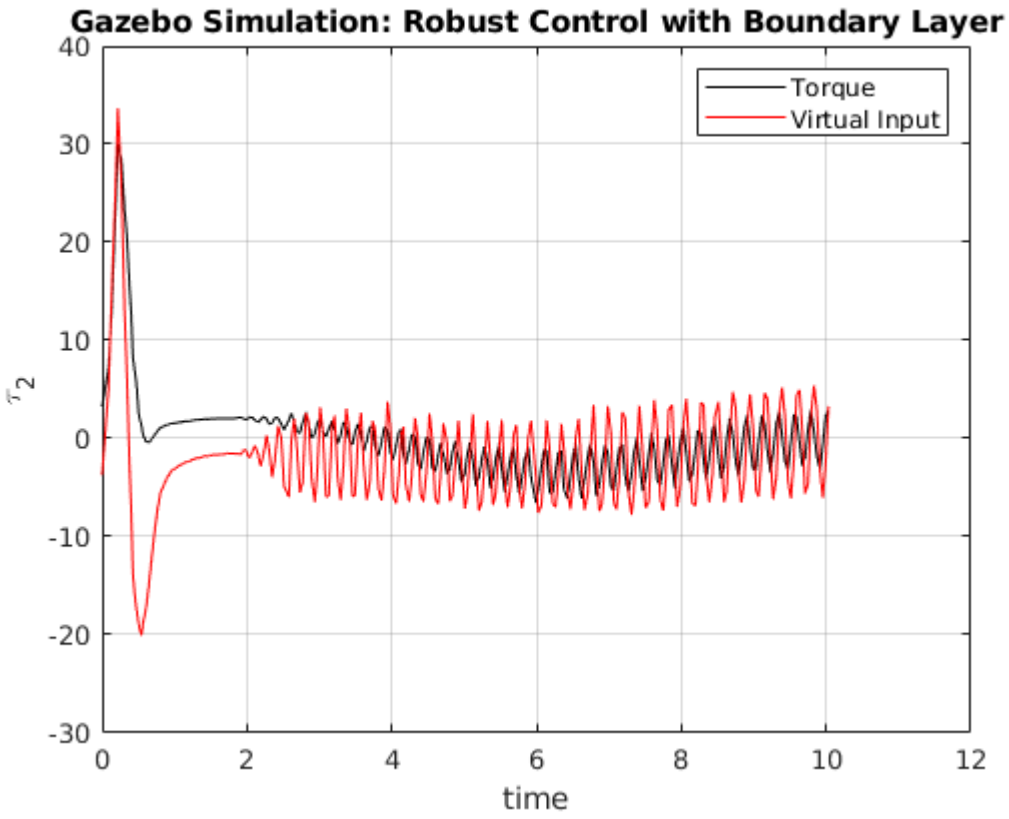
```
enable_robust_control = true;
rrbot_traj_control;
```




```
The value of the ROS_MASTER_URI environment variable, http://localhost:11311, will be used to connect to t
Initializing global node /matlab_global_node_18857 with NodeURI http://Ubuntu-20:37819/
A = 4×4
     0     0     1     0
     0     0     0     1
   -12     0    -7     0
     0   -12     0    -7
Shutting down global node /matlab_global_node_18857 with NodeURI http://Ubuntu-20:37819/
```

```
plot_system;
```

```
t_size = 260
Calculating System Parameters for Plotting
Plotting System
```




```
x_gsrcwobl = state_space_matrix;
v_gsrcwobl = virtual_input;
tau_gsrcwobl = system_input;
time_gsrcwobl =time_points;
tau1_max = max(tau_gsrcwobl(:, 1))
```

```
tau1_max = 68.0895
```

```
tau1_min = min(tau_gsrcwobl(:, 1))
```

```
tau1_min = -23.9650
```

```
tau2_max = max(tau_gsrcwobl(:, 2))
```

```
tau2_max = 29.9985
```

```
tau2_min = min(tau_gsrcwobl(:, 2))
```

```
tau2_min = -6.5811
```

G): Comparing Different System Controls

```
system_type = "comparision"
```

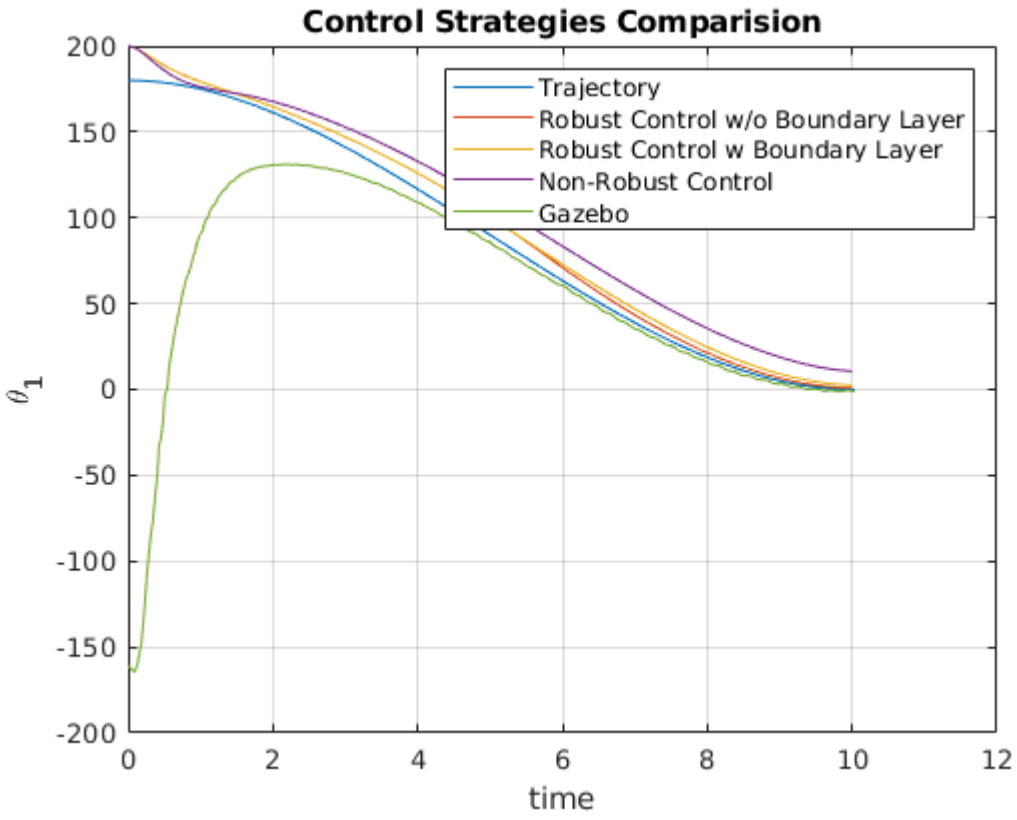
```
system_type =
"comparision"
```

```
graph_title = "Control Strategies Comparision"
```

```
graph_title =
```

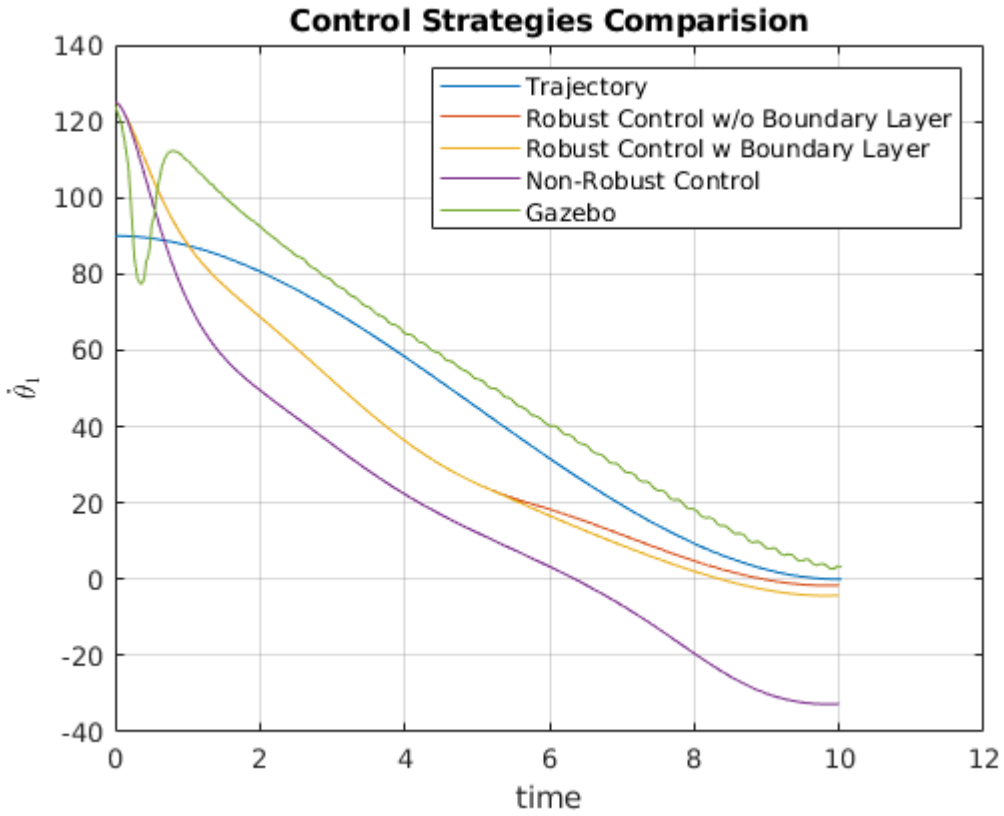
```
"Control Strategies Comparision"
```

```
command = append('mkdir -p ','Photos/',system_type);
system(command);
% Plot Trajectories
plot(time_gsrcwobl, rad2deg(theta_plot_points(:, 1)))
hold on;
plot(time_rcwobl, rad2deg(x_rcwobl(:, 1)))
hold on;
plot(time_rcwbl, rad2deg(x_rcwbl(:, 1)))
hold on;
plot(time_nrc, rad2deg(x_nrc(:, 1)))
hold on;
plot(time_gsrcwobl, rad2deg(x_gsrcwobl(:, 1)))
hold on;
legend('Trajectory', 'Robust Control w/o Boundary Layer', 'Robust Control w Boundary La
hold off;
grid on;
xlabel('time');
ylabel('\theta_{1}')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"theta1.jpg"))
```



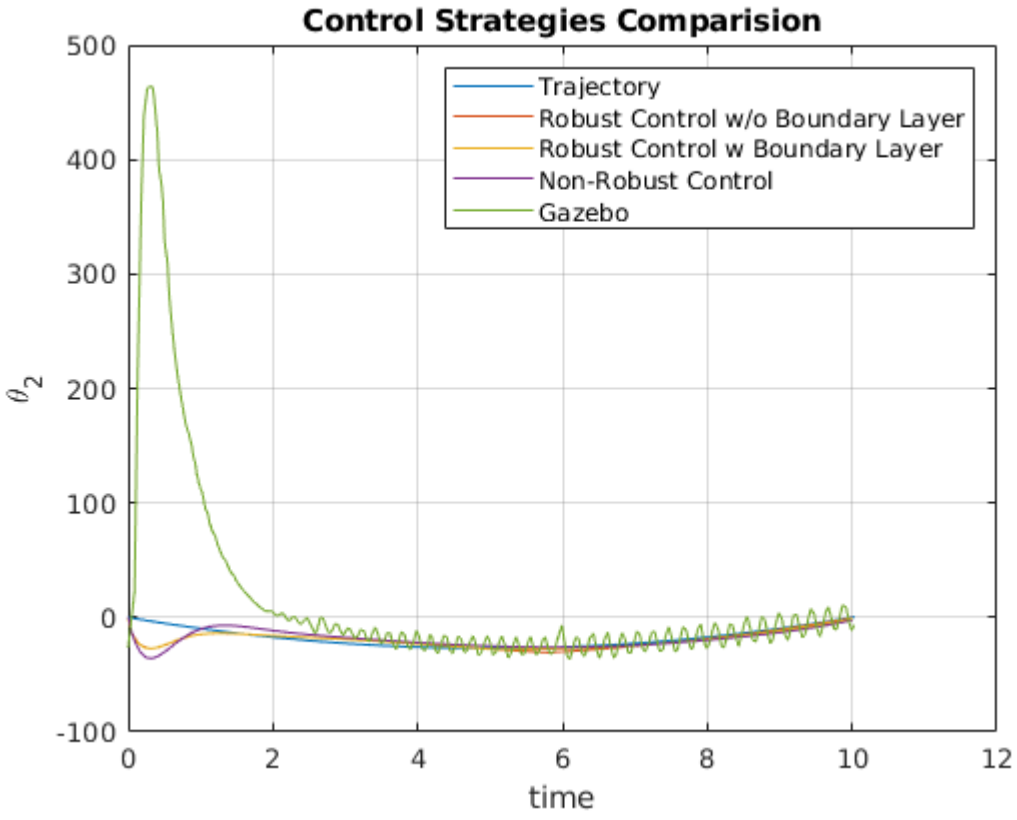

```
plot(time_gsrcwobl, rad2deg(theta_plot_points(:, 2)))
hold on;
plot(time_rcwobl, rad2deg(x_rcwobl(:, 2)))
hold on;
plot(time_rcwbl, rad2deg(x_rcwbl(:, 2)))
```

```
hold on;
plot(time_nrc, rad2deg(x_nrc(:, 2)))
hold on;
plot(time_gsrcwobl, rad2deg(x_gsrcwobl(:, 2)))
hold on;
legend('Trajectory', 'Robust Control w/o Boundary Layer', 'Robust Control w Boundary La
hold off;
grid on;
xlabel('time');
ylabel('$\dot{\theta}_{1}$', 'Interpreter', 'latex')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"dtheta1.jpg"))
```

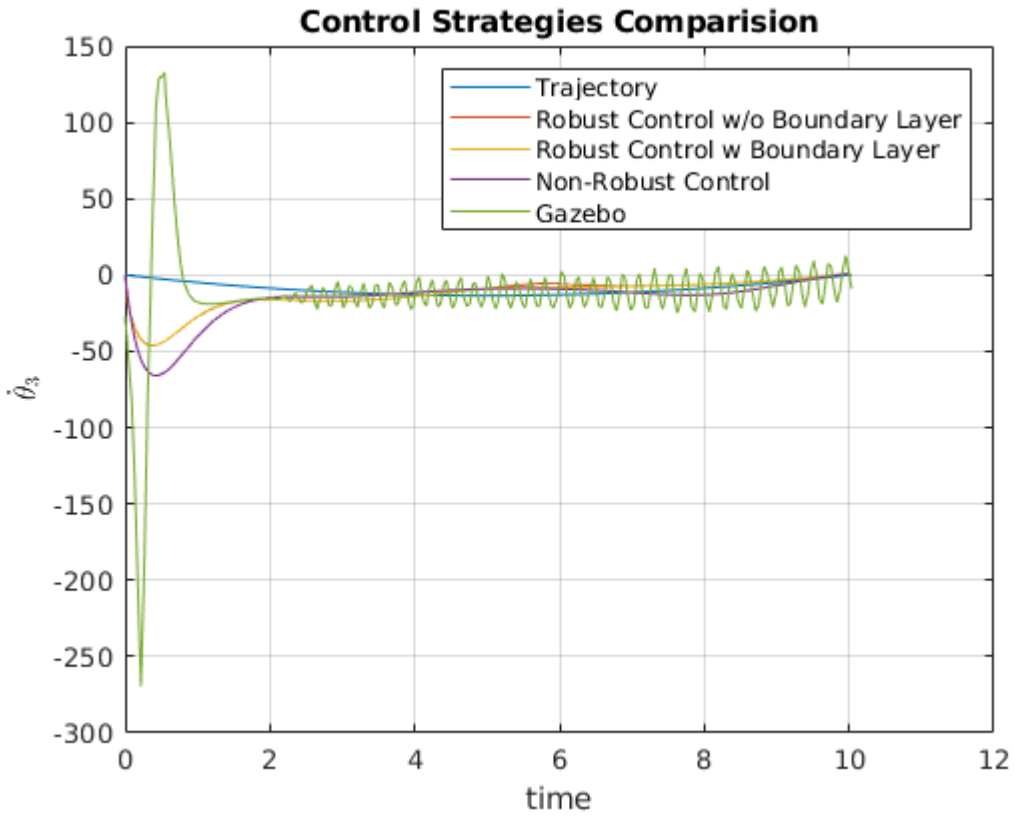



```
plot(time_gsrcwobl, rad2deg(theta_plot_points(:, 3)))
hold on;
plot(time_rcwobl, rad2deg(x_rcwobl(:, 3)))
hold on;
plot(time_rcwbl, rad2deg(x_rcwbl(:, 3)))
hold on;
plot(time_nrc, rad2deg(x_nrc(:, 3)))
hold on;
plot(time_gsrcwobl, rad2deg(x_gsrcwobl(:, 3)))
hold on;
legend('Trajectory', 'Robust Control w/o Boundary Layer', 'Robust Control w Boundary La
hold off;
grid on;
xlabel('time');
```

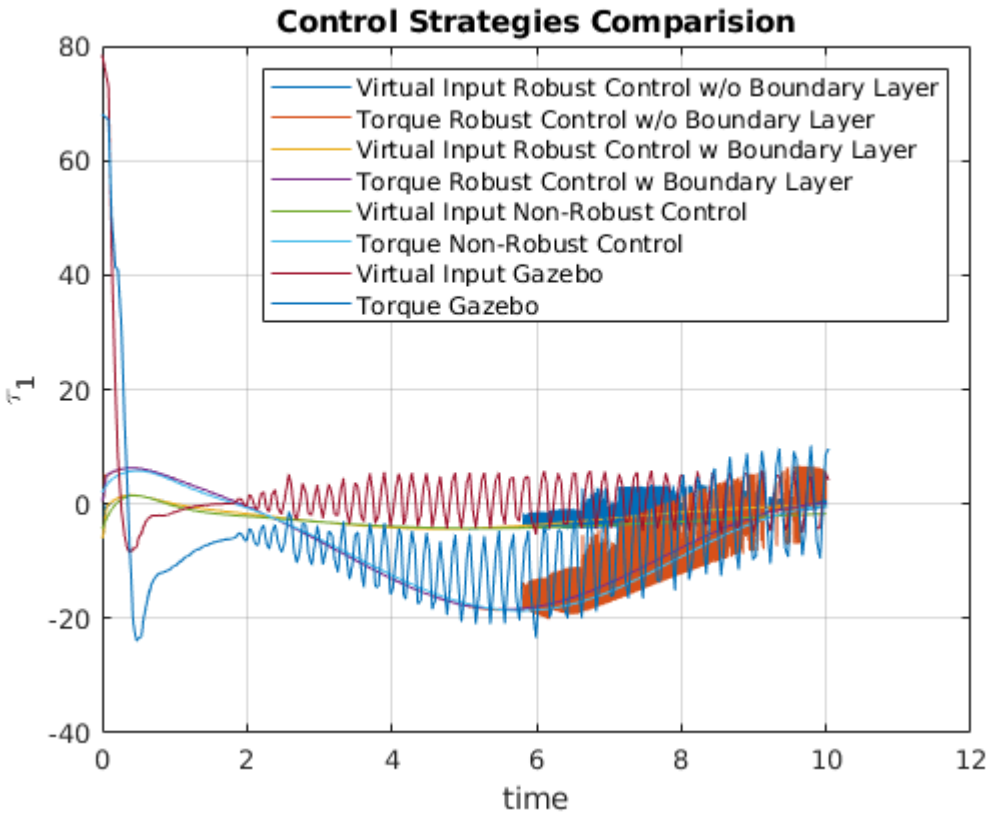
```
ylabel('\theta_{2}')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"theta2.jpg"))
```



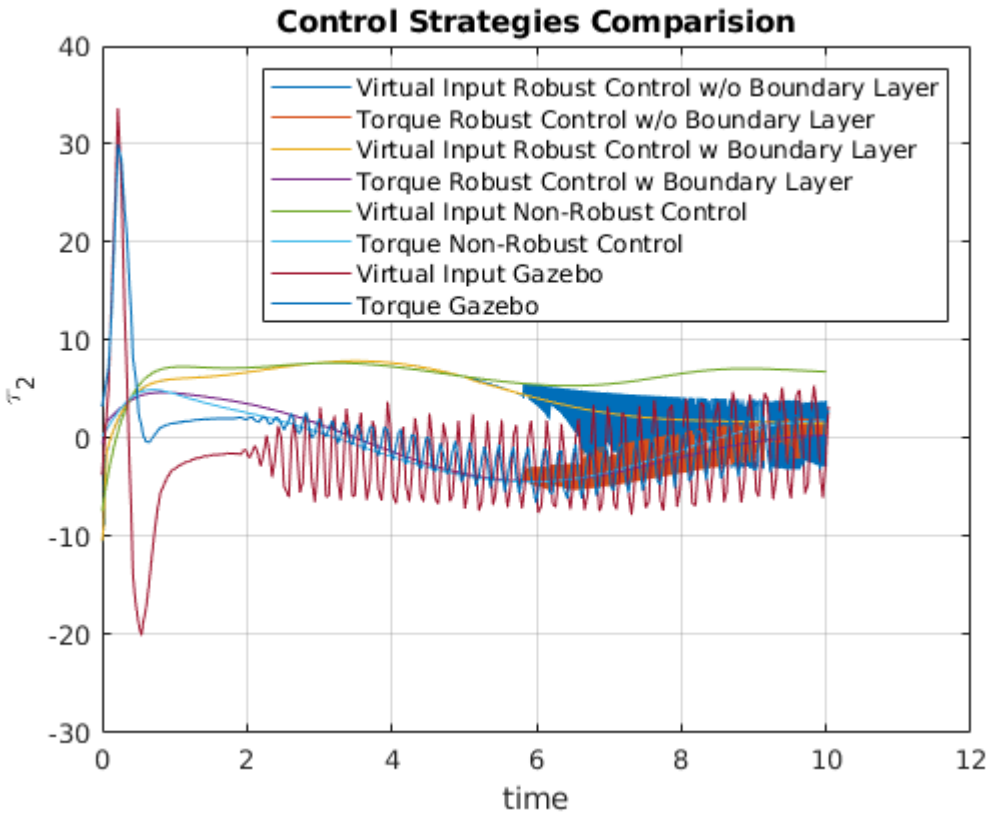

```
plot(time_gsrcwobl, rad2deg(theta_plot_points(:, 4)))
hold on;
plot(time_rcwobl, rad2deg(x_rcwobl(:, 4)))
hold on;
plot(time_rcwbl, rad2deg(x_rcwbl(:, 4)))
hold on;
plot(time_nrc, rad2deg(x_nrc(:, 4)))
hold on;
plot(time_gsrcwobl, rad2deg(x_gsrcwobl(:, 4)))
hold on;
legend('Trajectory', 'Robust Control w/o Boundary Layer', 'Robust Control w Boundary La
hold off;
grid on;
xlabel('time');
ylabel('$\dot{\theta}_{3}$', 'Interpreter', 'latex')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"dtheta3.jpg"))
```




```
plot(time_rcwobl, v_rcwobl(:, 1))
hold on;
plot(time_rcwobl, tau_rcwobl(:, 1))
hold on;
plot(time_rcwbl, v_rcwbl(:, 1))
hold on;
plot(time_rcwbl, tau_rcwbl(:, 1))
hold on;
plot(time_nrc, v_nrc(:, 1))
hold on;
plot(time_nrc, tau_nrc(:, 1))
hold on;
plot(time_gsrcwobl, v_gsrcwobl(:, 1))
hold on;
plot(time_gsrcwobl, tau_gsrcwobl(:, 1))
hold on;
legend('Virtual Input Robust Control w/o Boundary Layer','Torque Robust Control w/o Bou
hold off;
grid on;
xlabel('time');
ylabel('\tau_{1}')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"control1.jpg"))
```




```
plot(time_rcwobl, v_rcwobl(:, 2))
hold on;
plot(time_rcwobl, tau_rcwobl(:, 2))
hold on;
plot(time_rcwbl, v_rcwbl(:, 2))
hold on;
plot(time_rcwbl, tau_rcwbl(:, 2))
hold on;
plot(time_nrc, v_nrc(:, 2))
hold on;
plot(time_nrc, tau_nrc(:, 2))
hold on;
plot(time_gsrcwobl, v_gsrcwobl(:, 2))
hold on;
plot(time_gsrcwobl, tau_gsrcwobl(:, 2))
hold on;
legend('Virtual Input Robust Control w/o Boundary Layer','Torque Robust Control w/o Bou
hold off;
grid on;
xlabel('time');
ylabel('\tau_{2}')
title(graph_title)
saveas(gcf, append('Photos/',system_type,'/',system_type,"control2.jpg"))
```


Control Strategies Comparision
Virtual Input Robust Control w/o Boundary Layer
Torque Robust Control w/o Boundary Layer
Virtual Input Robust Control w Boundary Layer
Torque Robust Control w Boundary Layer
Virtual Input Non-Robust Control
Torque Non-Robust Control
Virtual Input Gazebo
Torque Gazebo
40
30
20
10
0
-10
-20
-30
0
2
4
6
8
10
12
time
τ_2