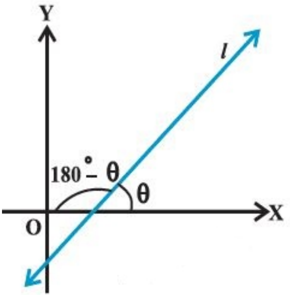
SECTION 2

MODULE 3 : Analytical Geometry

Slope of a Line

A line in a coordinate plane forms two angles with the x-axis, which are supplementary (i.e. sum of angles is 180 degree.)



The angle θ made by the line l with positive direction of x-axis and measured anti clockwise is called the inclination of the line.

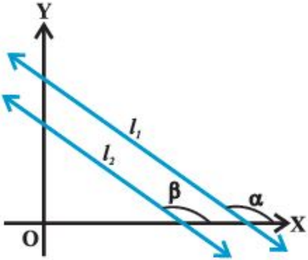
If θ is the inclination of a line l, then $\tan\theta$ is called the slope of the line l. The slope of a line whose inclination is 90^o is not defined. The slope of a line is denoted by m.

$$m = \tan\theta.$$

• Slope of a line when $P(x_1, y_1)$ and $Q(x_2, y_2)$ are coordinates of any two points on the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

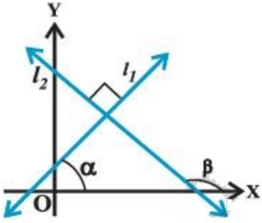
• Conditions for parallelism and perpendicularity of lines in terms of their slopes



If the line l_1 is parallel to l_2 then their slopes are equal

$$m_1 = m_2$$

where $m_1 = \tan\alpha$ and $m_2 = \tan\beta$



If the line l_1 is perpendicular to l_2 then

$$m_1 \cdot m_2 = -1$$

where $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

Example

(1) Find the slope of the line passing through the points $(3,-2)$ and $(-1,4)$.

(2) Find the slope of the line passing Making inclination of 60^o with the positive direction of x-axis.

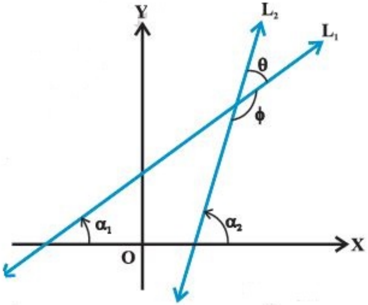
Solution : (1) Here $P(x_1,y_1) = P(3,-2)$ and $Q(x_2,y_2) = Q(-1,4)$.
Slope of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-1-3} = -\frac{3}{2}.$$

(2) Here $\theta = 60^o$.
Slope of a line

$$m = \tan\theta = \tan 60^o = \sqrt{3}.$$

- Angle between two lines



Let L_1 and L_2 be two non-vertical lines with slopes

$$m_1 = \tan\alpha_1 \quad \text{and} \quad m_2 = \tan\alpha_2,$$

respectively.

The acute angle θ between lines L_1 and L_2 with slopes m_1 and m_2, respectively, is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|, \quad 1 + m_2 \cdot m_1 \neq 0$$

The obtuse angle ϕ can be found by using

$$\phi = 180^o - \theta.$$

Example

(1) If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution : Here

$$m_1 = \frac{1}{2}, \;\; m_2 = m, \;\; \theta = \frac{\pi}{4}.$$

The acute angle is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|$$

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + m \cdot \frac{1}{2}}\right|$$

$$1 = \left|\frac{2m-1}{2+m}\right|$$

Therefore

$$\frac{2m-1}{2+m} = \pm 1$$

$$\frac{2m-1}{2+m} = 1 \text{ implies } m = 3$$

and

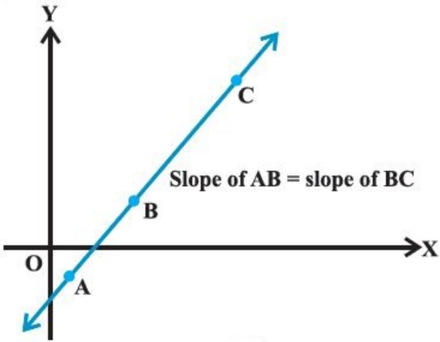
$$\frac{2m-1}{2+m} = -1 \text{ implies } m = -\frac{1}{3}$$

Hence the slope of other line is 3 or $-\frac{1}{3}$.

(2) Find the value of x if the line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(8,12)$ and $(x,24)$.

Solution : $m_1 = \frac{1}{3}$, $m_2 = \frac{12}{x-8}$, $x = 4$

- Collinearity of three points



If A, B and C are three points in the XY-plane, then they will lie on a line, i.e., three points are collinear if and only if

$$\text{slope of } AB = \text{slope of } BC.$$

Example

Three points P (h, k), Q (x_1, y_1) and R (x_2, y_2) lie on a line. Show that

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1).$$

Since points P, Q and R are collinear, we have

Slope of PQ = Slope of QR, i.e., $\dfrac{y_1 - k}{x_1 - h} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{k - y_1}{h - x_1} = \frac{y_2 - y_1}{x_2 - x_1},$$

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1).$$

(2) Find the value of x for which the points $(x, 1), (2,-1)$ and $(4, 5)$ are collinear.

Solution : Let $A = (x,-1), B = (2, 1)$ and $C = (4, 5)$. Since A, B and C are collinear, we have

$$\text{slope of } AB = \text{slope of } BC$$

so

$$\frac{1-(-1)}{2-x} = \frac{5-1}{4-2}.$$
$$\implies \frac{2}{2-x} = \frac{4}{2}$$
$$\implies 2-x = 1$$
$$\implies x = 1.$$

(3) The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$ find the slopes of the lines.

Solution : Let m_1 and m_2 be the slope. It is given that

$$m_2 = 2m_1 \quad \text{and} \quad \tan\theta = \frac{1}{3}$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right|$$

$$\frac{1}{3} = \left|\frac{2m_1 - m_1}{1 + 2m_1^2}\right|$$

Therefore

$$\frac{m_1}{1 + 2m_1^2} = \pm\frac{1}{3}$$

Let

$$\frac{m_1}{1+2m_1^2} = \frac{1}{3}. \quad \text{--------------------} \quad (1)$$

Then

$$\begin{aligned} 3m_1 &= 1 + 2m_1^2 \\ 2m_1^2 - 3m_1 + 1 &= 0 \\ (2m_1 - 1)(m_1 - 1) &= 0 \end{aligned}$$

$$\boxed{m_1 = 1 \quad \text{or} \quad m_1 = \frac{1}{2}}$$

which gives $m_2 = 2$ or $m_2 = 1$. Next let

$$\frac{m_1}{1+2m_1^2} = -\frac{1}{3} \quad \text{--------------------} \quad (2)$$

On solving

$$\boxed{m_1 = -1 \quad \text{or} \quad m_1 = -\frac{1}{2}}$$

which gives $m_2 = -2$ or $m_2 = -1$.

- Any equation of the form

$$Ax + By + C = 0,$$

where A and B are not zero simultaneously is called general linear equation or general equation of a line. The slope of the line $Ax + By + C = 0$ is

$$m = -\frac{A}{B}$$

$$y \;\; \text{intercept} = -\frac{C}{B}$$

$$x \;\; \text{intercept} = -\frac{C}{A}$$

Example

(1) Equation of a line is $3x - 4y + 10 = 0$. Find its (i) slope, (ii) x - and y-intercepts.

Solution : Here $3x - 4y + 10 = 0$ so

$$A = 3, \;\; B = -4, \;\; C = 10$$

The slope of the line $3x - 4y + 10 = 0$ is

$$m = -\frac{A}{B} - \frac{3}{-4} = \frac{3}{4}$$

$$y \;\; \text{intercept} = -\frac{C}{B} = -\frac{10}{-4} = \frac{10}{4}$$

$$x \;\; \text{intercept} = -\frac{C}{A} = -\frac{10}{3}$$

(2) Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

Solution : Let

$$L_1 : y - \sqrt{3}x - 5 = 0$$

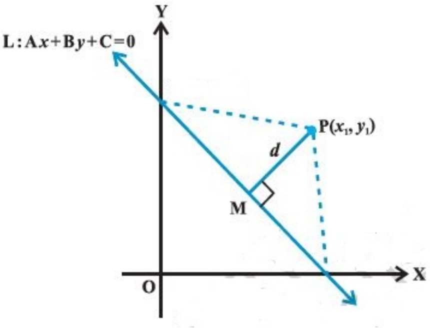
and

$$L_2 : \sqrt{3}y - x + 6 = 0$$

and m_1 and m_2 be the slopes respectively. Then

$$m_1 = -\frac{-\sqrt{3}}{1} = \sqrt{3}, \;\; m_2 = -\frac{-1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\begin{aligned}
\tan\theta &= \left|\frac{m_2 - m_1}{1 + m_2 \cdot m_1}\right| \\
&= \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}\right| \\
&= \frac{1}{\sqrt{3}} \\
\theta &= 30^o \;\text{ or }\; 180^o - 30^o = 150^o.
\end{aligned}$$



- The perpendicular distance of a Point $P(x_1, y_1)$ from a Line

$$L : Ax + By + C = 0$$

is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

• The distance d between two parallel lines

$$Ax + By + C_1 = 0 \quad \text{and} \quad Ax + By + C_2 = 0,$$

is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Example

(1) Find the distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$.

(2) Find the distance between the parallel lines $3x-4y+7 = 0$ and $3x-4y+5 = 0$

(3) Find the distance of the point $(-1, 1)$ from the line $12(x+6) = 5(y-2)$.
ANS : 5

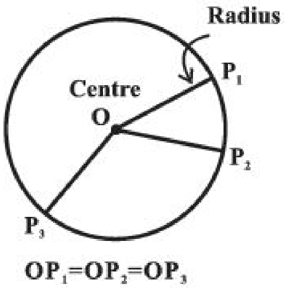
(4) Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.
ANS : $(-2, 0)$ and $(8, 0)$

(5) Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.
ANS : $3x - 4y + 18 = 0$

(6) Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.
ANS : $y + 7x = 21$

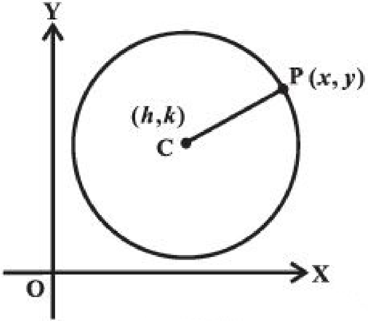
(7) Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.
ANS : 30^o

Circle



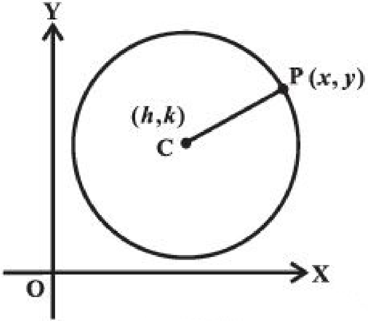
A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

The fixed point is called the centre of the circle and the distance from the centre to a point on the circle is called the radius of the circle



(1) The equation of the circle with centre at (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2.$$



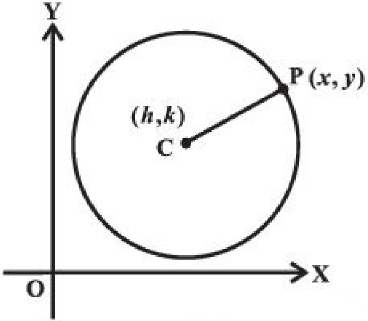
(1) The equation of the circle with centre at (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2.$$

(2) The general equation of the circle is

$$x^2+y^2+2gx+2fy+c=0$$

having centre



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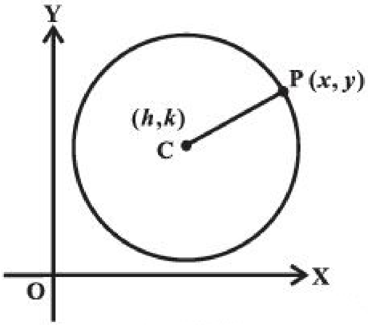
$$(-g,-f)$$

and radius



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(2) The general equation of the circle is

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having centre

$$(-g,-f)$$

and radius

$$r=\sqrt{g^2+f^2-c}$$

1) Find the equation of the circle whose centre is (2,-1) and radius 3 units.

Solution:

Equation of the circle with centre (h,k) and radius 'r' is

$(x-h)^2+(y-k)^2=r^2$ (h,k) =(2,-1) r=3

$\therefore (x-2)^2+(y+1)^2=3^2$ $x^2+y^2-4x+2y-4=0$

2) Find the centre and radius of the circle $x^2+y^2-6x+4y+2=0$

Solution:

Here 2g = -6 and 2f = 4

g = -3 f = 2

Centre is (-g,-f)

Centre (3,-2)

$\therefore$ r = $\sqrt{(-3)^2+2^2-2}$

$\therefore$ r = $\sqrt{11}$ units

3) Find the centre and the radius of the circle

$$x^2 + y^2 + 8x + 10y - 8 = 0.$$

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4) Find the equation of the circle which passes through the points $(2, -2)$, and $(3, 4)$ and whose centre lies on the line $x + y = 2$.

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$$x^2 + y^2 + 8x + 10y - 8 = 0.$$

4) Find the equation of the circle which passes through the points $(2, -2)$, and $(3, 4)$ and whose centre lies on the line $x + y = 2$.

Solution : Let the required equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

3) Find the centre and the radius of the circle

$$x^2 + y^2 + 8x + 10y - 8 = 0.$$

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Solution : Let the required equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Given that circle passes through points $(2, -2)$, and $(3, 4)$ we have

$$2^2 + (-2)^2 + 2g(2) + 2f(-2) + c = 0.$$

$$4g - 4f + c = -8 \tag{1}$$

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$$x^2 + y^2 + 8x + 10y - 8 = 0.$$

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$$2^2 + (-2)^2 + 2g(2) + 2f(-2) + c = 0.$$
$$4g - 4f + c = -8 \tag{1}$$

and

$$3^2 + 4^2 + 2g(3) + 2f(4) + c = 0.$$
$$6g + 8f + c = -25 \tag{2}$$

Also it is given that centre $(-g,-f)$ lies on the line $x+y=2$.

$$-g-f=2 \implies g+f=-2 \tag{3}$$

Also it is given that centre $(-g, -f)$ lies on the line $x + y = 2$.

$$-g - f = 2 \implies g + f = -2 \tag{3}$$

From (1) and (2), we have

$$2g + 12f = -17 \tag{4}$$

Now solving (3) and (4), we get

$$f = -\frac{13}{10}, \quad g = -\frac{7}{10}.$$

Also it is given that centre $(-g, -f)$ lies on the line $x + y = 2$.

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From (1) and (2), we have

$$2g + 12f = -17 \tag{4}$$

Now solving (3) and (4), we get

$$f = -\frac{13}{10}, \quad g = -\frac{7}{10}.$$

Substituting value of f and g in (1), we get $c = -\frac{104}{10}$.

Hence centre is

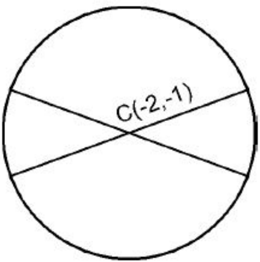
$$(-g, -f) = (0.7, 1.3)$$

and radius

$$r = \sqrt{12.58}.$$

5) If $3x - y + 5 = 0$ and $4x + 7y + 15 = 0$ are the equations of two diameters of a circle of radius 4 units write down the equation of the circle.

Solution :



5) If $3x - y + 5 = 0$ and $4x + 7y + 15 = 0$ are the equations of two diameters of a circle of radius 4 units write down the equation of the circle.

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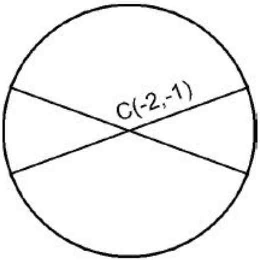


Here we are given that

$$3x - y + 5 = 0$$

and

$$4x + 7y + 15 = 0$$

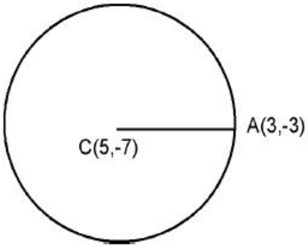
are the equations of two diameters of a circle. Solving them we get centre as $(-2, -1)$. Hence the equation of circle is

$$(x - (-2))^2 + (y - (-1)) = 4^2$$

$$(x + 2)^2 + (y + 1)^2 = 16$$

6) Find the equation of the circle whose centre is $(5, -7)$ and passing through the point $(3, -3)$.

Solution :



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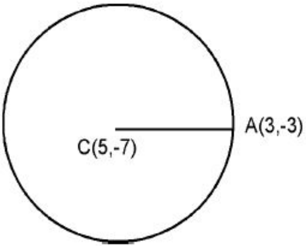
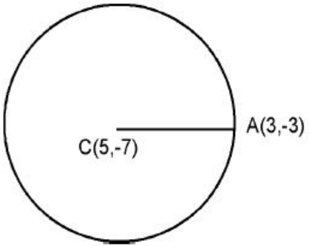


Let the centre and a point on the circle be $C(5, -7)$ and $A(3, -3)$

Radius $= CA = \sqrt{(5-3)^2 + (-7+3)^2}$
$r = 4 + 16 = \sqrt{20}$

6) Find the equation of the circle whose centre is $(5,-7)$ and passing through the point $(3,-3)$.

Solution :



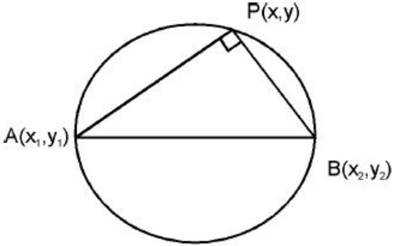
Let the centre and a point on the circle be $C(5,-7)$ and $A(3,-3)$

Radius $= CA = \sqrt{(5-3)^2+(-7+3)^2}$
$r = 4+16 = \sqrt{20}$
Hence the equation of circle is

$$(x-(5))^2+(y-(-7))^2 = 20$$

$$(x-5)^2+(y+7)^2 = 20$$

- Equation of circle with end points of a diameter



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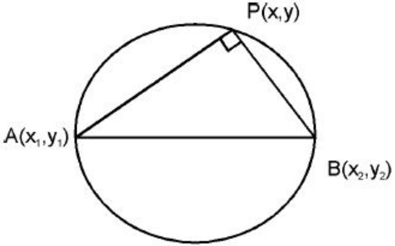
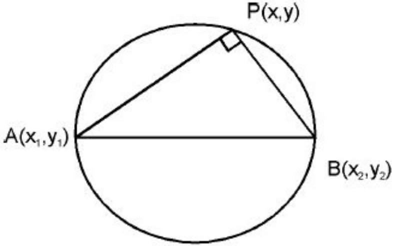


Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be given two end points of a diameter. Let $P(x, y)$ be any point on the circle.

- Equation of circle with end points of a diameter

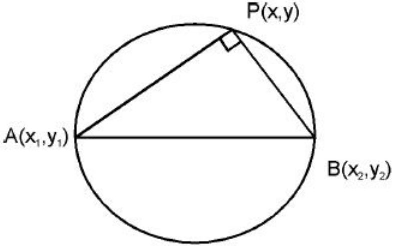


Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be given two end points of a diameter. Let $P(x, y)$ be any point on the circle.

The equation of circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

- Equation of circle with end points of a diameter



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be given two end points of a diameter. Let $P(x, y)$ be any point on the circle.

The equation of circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Example

Find the equation of the circle joining the points $(1, -1)$ and $(-2, 3)$ as diameter.

Solution : Here end points of diameter are given as

$$(x_1, y_1) = (1, -1), \quad (x_2, y_2) = (-2, 3).$$

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The equation of circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$(x - 1)(x - (-2)) + (y - (-1))(y - 3) = 0$$

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$$(x_1, y_1) = (1, -1), \quad (x_2, y_2) = (-2, 3).$$

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$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$(x - 1)(x - (-2)) + (y - (-1))(y - 3) = 0$$

$$(x - 1)(x + 2) + (y + 1)(y - 3) = 0$$

Solution : Here end points of diameter are given as

$$(x_1, y_1) = (1, -1), \quad (x_2, y_2) = (-2, 3).$$

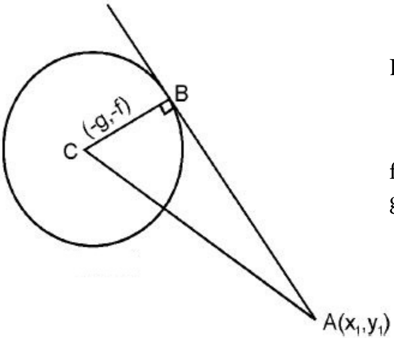
The equation of circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$(x - 1)(x - (-2)) + (y - (-1))(y - 3) = 0$$

$$(x - 1)(x + 2) + (y + 1)(y - 3) = 0$$

$$x^2 + y^2 + x - 2y - 5 = 0$$

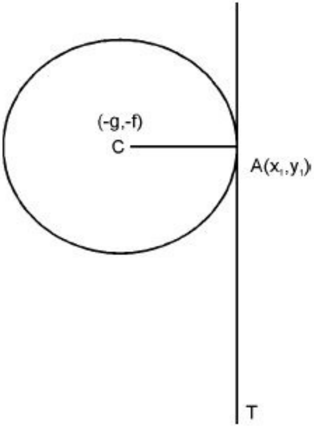


Length of the Tangent to a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

from a point $A(x_1, y_1)$ lies outside the circle is given by

$$AB = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$



Equation of the Tangent to a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

at the point $A(x_1, y_1)$ on the circle is given by

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Example

(1) Find the length of the tangent from $(2,3)$ to circle $x^2+y^2-2x+4y+1=0$.

Solution : Here $(x_1, y_1) = (2,3)$ and equation of circle is $x^2+y^2-2x+4y+1=0$.
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(3) Find the equation of the tangent at $(4, 1)$ to the circle $x^2+y^2-8x-6y+21 = 0$
ANS : $y - 1 = 0$

Solution : Here $(x_1, y_1) = (4, 1)$ and the given circle is $x^2 + y^2 - 8x - 6y + 21 = 0$. So

$$g = -4, \quad f = -3, \quad c = 21.$$

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