⊛ Nested Quantifiers

Ex-1 Let x and y be the real numbers and P(x,y) denotes " $x+y=0$. " Find the truth values of

Ⓐ $\forall x \forall y \; P(x,y)$

Ⓑ $\forall x \exists y \; P(x,y)$

Ⓒ $\exists y \forall x \; P(x,y)$

Ⓓ $\exists x \exists y \; P(x,y)$

Sol^n Domain : All real numbers.

Ⓐ $\forall x \forall y \, P(x,y) = \forall x \forall y \, (x+y=0)$

→ " For all real numbers x and y, $x+y=0$ "

↳ Check is it true?

If $x=1$, $y=-1$ $\Rightarrow x+y=0$

Suppose

$\left.\begin{array}{l} x=1 \\ y=2 \end{array}\right\} \Rightarrow 1+2 \neq 0$

$\therefore \forall x \forall y \, P(x,y) \rightarrow$ False

Ⓑ $\forall x \exists y \, P(x,y) = \forall x \exists y \, (x+y=0)$

" For every real number x, there exist a real number y such that $x+y=0$ "

↳ True - Yes

y → Depends on x.

i.e

$x=1$, $y=-1$; $x=-1$, $y=1$; $x=1/2$, $y=-1/2$

1) consider all Diff^n values of x

2) Find just 1 value of y for each x

c) $\exists y \forall x\ P(x,y) = \exists y \forall x\ (x+y=0)$

∴ "There exist some real number y such that for all real number x, $x+y=0$"

↳ is it true ?

⇒ $\boxed{NO}$

i.e.

$\exists y$, for Take $y=1$ ⇒

$\forall x$ For all $x = 1, \frac{1}{2}, \ldots$

∴ $P(x,1) = x+1 = 0$

$P(\frac{1}{2},1) = \frac{1}{2}+1 \neq 0$

$P(1,1) = 1+1 \neq 0$

∴ No matter what y you choose, $P(x,y)$ is always False for all real number x.

D) $\exists x \exists y\ P(x,y) = \exists x \exists y\ (x+y=0)$

∴ "There exists some real numbers x & y such that $x+y=0$"

↳ is it true

→ $\boxed{True}$

Take $x=1$, $y=-1$ (Many such combination you can take)

$x+y=0$

Q: Let x and y be the real numbers and $Q(x,y)$ denotes "$x \cdot y = 0$" Find the truth values of the following.

(I) $\forall x \forall y \; Q(x,y)$ (II) $\forall x \exists y \; Q(x,y)$

(III) $\exists y \forall x \; Q(x,y)$ (IV) $\exists x \exists y \; Q(x,y)$

Sol: (I) $\forall x \forall y \; Q(x,y) \equiv \forall x \forall y \; (x \cdot y = 0)$

"For all real numbers x and y, $x \cdot y = 0$"

· False

i.e., Take $x = 1$ & $y = 2 \Rightarrow 1 \cdot 2 \neq 0$

$x \cdot y = 0$ is not satisfied for every combination

(II) $\forall x \exists y \; Q(x,y) \equiv \forall x \exists y \; Q(x,y)$

"For every real number x $\exists$ a ~~real~~ real number y such that $x \cdot y = 0$"

$\Rightarrow$ True

i.e.,

$x = 1$	$x = -1$	$x = \frac{1}{2}$
$y = 0$	$y = 0$	$y = 0$
$1 \cdot 0 = 0$	$(-1) \cdot 0 = 0$	$\frac{1}{2}(0) = 0$

(III) $\exists y \forall x \; Q(x,y) \equiv \exists y \forall x \; (x \cdot y = 0)$

"There exists some real number y such that for every real number x, $x \cdot y = 0$"

$\Rightarrow$ True

It is plugging into all real values of x

suppose Take $y = 0$.

$\therefore Q(x,y) = Q(x,0) \Rightarrow x \cdot 0 = 0$

(some value of y choose in real no.)

$\therefore \exists y \forall x \; Q(x,y)$ is true,

1) $\exists x \exists y\, Q(x,y) = \exists x \exists y\, (x \cdot y = 0)$

" There exists some real numbers x and y such that $x \cdot y = 0$ "

$\Rightarrow$ True

Take $x = 1,\ y = 0$	$y = 1,\ x = 0$
$x \cdot y = 0$	$x \cdot y = 0$
$1 \cdot 0 = 0$	$0 \cdot 1 = 0$

Ex=3 Let $Q(x, y, z)$ be the statement " $x + y = z$ ". What are the truth values of the statements $\forall x \forall y \exists z\, Q(x, y, z)$ and $\exists z \forall x \forall y\, Q(x, y, z)$ where the domain of all variables consists of all real numbers?

Solu^n (I) $\forall x \forall y \exists z\, Q(x, y, z) = \forall x \forall y \exists z\, (x + y = z)$

" For all real numbers x and y, there is a real number z such that $x + y = z$

Let $x = 1,\ y = -1$
$z = 0$

(Addition of any real number of x & y is some of real no. z)

$\therefore \forall x \forall y \exists z\, (x + y = z)$ is true.

(II) $\exists z \forall x \forall y\, Q(x, y, z) = \exists z \forall x \forall y\, (x + y = z)$

$\therefore$ There exists some real no. z such that for all real numbers x and y.

$\Rightarrow$ False $x + y = z$ is true

$\therefore$ Take $z = 2$

$x = 1$ and $y = 1$ is true

But $x = 2$ & $y = 3$

$x + y = z$ is not true