# Controlling Epidemics and Economics Activity in Interacting Communities

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We consider an SAIR model between two communities. Define the following variables for community $i$.

- $S_i(t)$ is the number of susceptible at time $t$.
- $A_i(t)$ is the number of infected at time $t$ but asymptomatic total population in community.
- $I_i(t)$ is the number of infected but symptomatic.
- $R_i(t)$ is the number recovered from infection.
- $D_i(t)$ is the number of deaths from the infection.
- Let $W_i(t) := S_i(t) + A_i(t) + R_i(t)$ be the number of active members available for economic activity in community $i$.

At time $t$, there are $L_{ij}(t)$ meetings between members of community $i$ and community $j$. Each meeting is an economic activity and generates an economic good. Furthermore in each meeting, we assume there are $M_i$ members of community $i$ and $M_j$ members of community $j$. One motivation is a 'service community' in the neighbourhood of a more affluent community. The service could correspond to

Each $i - j$ meeting results in an infection with rate $p_{ij}$. $L_{ij}(t)$ is a control variable. The infected population does not participate in the economic activity. The following o.d.e.s can be written about this interaction. For the discrete time versions of the following, replace $\dot{S}(t) = S(t+1) - S(t)$, etc.

$$\dot{S}_i(t) = -\sum_{j=1}^{2} p_{ij} L_{ij}(t) \frac{A_j(t)}{W_j(t)} \; = \; -\sum_{j=1}^{2} \lambda_{ij}(t) \frac{A_j(t)}{W_j(t)} \tag{1}$$

$$\dot{A}(t) = \sum_{j=1}^{2} \lambda_{ij}(t) \frac{A_j(t)}{W_j(t)} - (\nu_i(t) + \mu_i(t) A_i(t) \tag{2}$$

$$\dot{I}_i(t) = \nu_i(t) A_i(t) - \xi_i(t) I_i(t) \tag{3}$$

$$\dot{R}_i(t) = \mu_i(t) A_i(t) + \xi_i(t) I_i(t) \tag{4}$$

$$\dot{D}_i(t) = \delta_i(t) I_i(t) \tag{5}$$

If $\delta$ is significantly small, we can assume it to be zero to obtain a population conserving model like i. We will assume $\lambda_{ij}(t)$ and $\nu_i(t)$ are controllable and $\mu_i(t)$ and $\xi(t)$ are exogenous.

The following are the kinds of optimisations that the different communities may seek over the period $(0, T)$.

- Community 1, denoted by $C_1$. is sparse, $p_{11}(t)$ is low, just needs to maintain low levels of activity involving $1-1$ meetings. It needs $1-2$ meetings for 'convenience'. It wants to keep its infections low. It can control $L_{11}(t), L_{12}(t)$ and possibly the testing parameter,$\nu_1(t)$

  $$\min \quad \max \quad I_1(t) \tag{6}$$

  subject to

  $$F_1(L_{11}(t)) > a_{11}$$
  $$F_{12}(L_{12}(t) > a_{12}$$

  Alternatively, it could have the objective of maximising convenience resulting form $1-2$ meetings while keeping infections below a threshold.

  Here $F_1(\cdot), F_{12}(\cdot)$ are increasing functions and $a_{11}$ and $a_{12}$ are given constants.

- Community 2, denoted by $C_2$, is dense, $p_{ij}(t)$ is high and it needs high levels of $L_{22}$ and $L_{21} = L_{12}$ for economic sustenance. It wants to keep infection/hospitalisation levels manageable. It can control only $L_{22}(t)$

  $$\max \min G_{11}(L_{11}(t)) + G_{21}(L_{12}(t)) \tag{7}$$

  subject to

  $$I_2(t) < b_{22}$$

  Here $G_{11}$ and $G_{21}$ are increasing functions and $b_{22}$ is a constant.

- The Government has the objective of reducing hospitalisations in $C_2$ and overall deaths. It can control by putting upper limits on $L_{ij}(t)$.

$$\min \max H_1(I_1(t)) + H_2(I_2(t)) \tag{8}$$

  Here $H_1$ and $H_2$ are costs of hospitalisation and the cost of possible deaths. Government may also increase testing in $C_2$ which can result in leaky isolation. This needs a bit more work.

The constants could also be known functions of time.