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# 14. Interlude: Polynomials and Geometric

## Quadratic Polynomials

1/1 point (graded)

Recall a **degree** $n$ polynomial in $x_1, x_2, \ldots, x_k$ are all linear combinations of monomials in $x_1, x_2, \ldots, x_k$, where **monimials** in $x_1, x_2, \ldots, x_k$ are **unordered words** using $x_1, x_2, \ldots, x_k$ as the letters.

**Examples:**

1. A degree 2, also known as quadratic, polynomial in the 1 variable $x$ is of the form

$$ax^2 + bx + c$$

   for some numbers $a, b, c$. The polynomial is determined by the 3 coefficients $a, b, c$, and different choices of $(a, b, c)$ result in different polynomials.
   In linear algebraic terms, the space of degree 2 polynomials in 1 variable is of dimension 3 since it consists of all linear combinations of 3 linearly independent vectors $x^2,\ x,$ and $1$.

2. A degree 2 polynomial in 2 variables $x_1, x_2$ is of the form

$$ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f$$

for some numbers $a, b, c, d, e, f$. Different choices of $(a, b, c, d, e, f)$ result in different polynomials.
In linear algebraic terms, the space of degree 2 polynomials in 2 variables is of dimension 6 since it consists of all linear combinations of 6 linearly independent vectors $x_1^2,\ x_2^2,\ x_1x_2,\ x_1,\ x_2,$ and $1$.

Consider degree 2 polynomials in 3 variables $x_1, x_2, x_3$. How many coefficients are needed to completely determine such a polynomial? Equivalently, what is the dimension of the space of polynomials in 3 variables such polynomials?

Number of coefficients needed/ Dimension: 10 ✔

**Answer:** 10

What is dimension of the polynomials of degree $N$ in $K$ variables? (This part of the question is optional and there is no answer box for it.)

**Solution:**

We count the number of monomials of length $2, 1, 0$:

- The monomials of length $2$ are unordered pairs of $x_1, x_2,\ x_3$, hence there are $\binom{3}{2}$ This list consists of $x_1^2, x_2^2, x_3^2, x_1x_2, x_1x_3, x_2x_3$.
- The monomials of length $1$ are $x_1,\ x_2,\ x_3$.
- The monomial of length $0$ is the constant term, i.e. $1$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

## Discussion

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- Fantastic exercise — 4
  Tied me into knots to find the general case solution n, k. But it worth it : ). I will share the solu...
- Degree definition needs something more to be correct — 2
  Maybe 'unordered words of length n or less'? Currently 'n' isn't used in the definition. (I think ...
- Understanding the gist — 2
- Nice exercise at the end — 1
  An actual use of combination with repetition, other than the common example of choosing ic...

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