

Unit 4 Unsupervised Learning (2

Course > weeks)

9. Gaussian Generative models

> Lecture 15. Generative Models >

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9. Gaussian Generative models Gaussian Generative Models





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Multivariate Gaussian Random Vector

A random vector $\mathbf{X} = (X^{(1)}, \dots, X^{(d)})^T$ is a **Gaussian vector**, or **multivariate Gaussian or normal variable**, if any linear combination of its components is a (univariate) Gaussian variable or a constant (a "Gaussian" variable with zero variance), i.e., if $\boldsymbol{\alpha}^T\mathbf{X}$ is (univariate) Gaussian or constant for any constant non-zero vector $\boldsymbol{\alpha} \in \mathbb{R}^d$.

The distribution of ${\bf X}$, the d-dimensional Gaussian or normal distribution , is completely specified by the vector mean

 $\mu = \mathbf{E}\left[\mathbf{X}\right] = \left(\mathbf{E}\left[X^{(1)}\right], \dots, \mathbf{E}\left[X^{(d)}\right]\right)^T$ and the $d \times d$ covariance matrix Σ . If Σ is invertible, then the pdf of \mathbf{X} is

$$f_{\mathbf{X}}\left(\mathbf{x}
ight) = rac{1}{\sqrt{\left(2\pi
ight)^{d}\mathrm{det}\left(\Sigma
ight)}}e^{-rac{1}{2}\left(\mathbf{x}-\mu
ight)^{T}\Sigma^{-1}\left(\mathbf{x}-\mu
ight)}, \;\;\; \mathbf{x} \in \mathbb{R}^{d}$$

where $\det\left(\Sigma\right)$ is the determinant of the Σ , which is positive when Σ is invertible.

If $\mu=\mathbf{0}$ and Σ is the identity matrix, then \mathbf{X} is called a **standard normal random vector** .

Note that when the covariant matrix $\,\Sigma\,$ is diagonal, the pdf factors into pdfs of univariate Gaussians, and hence the components are independent.

Gaussian Distribution

1/1 point (graded)

Recall that the likelihood of x being generated from a multi-dimensional Gaussian with mean μ and all the components being uncorrelated and having the same standard deviation σ is:

$$P\left(x|\mu,\sigma^{2}
ight)=rac{1}{\left(2\pi\sigma^{2}
ight)^{d/2}}\mathrm{exp}\left(-rac{1}{2\sigma^{2}}\left\Vert x-\mu
ight\Vert ^{2}
ight)$$

Let
$$x=\left[egin{array}{c} rac{1}{\sqrt{\pi}} \\ 2 \end{array}
ight]$$
 be a vector in the two-dimensional space.

Let G be a two-dimensional Gaussian distribution with mean μ and standard deviation σ taking values as follows

$$\mu = \left[egin{array}{c} 0 \ 2 \end{array}
ight], \sigma = \sqrt{rac{1}{2\pi}}$$

Calculate the likelihood $p\left(x|\mu,\sigma^2\right)$ of x being sampled from the Gaussian distribution G with mean μ and variance σ^2 taking values as given above.

Enter the value of $\log p\left(x|\mu,\sigma^2\right)$ below. (Note that we use \log for the natural logarithm, i.e. $\log_e\left(\right)$.)

Solution:

Note that the likelihood of vector x being sampled from a Gaussian distribution G with mean μ and variance σ^2 is given as follows

$$p\left(x|\mu,\sigma^{2}
ight)=rac{1}{2\pi\sigma^{2}}\mathrm{exp}\left(-rac{1}{2\sigma^{2}}\left\Vert x-\mu
ight\Vert ^{2}
ight)$$

Substituing the value of $\sigma=\sqrt{rac{1}{2\pi}}$ from above, we have

$$p\left(x|\mu,\sigma^{2}
ight)=rac{1}{2\pirac{1}{2\pi}}\mathrm{exp}\left(-rac{1}{2rac{1}{2\pi}}\left\Vert x-\mu
ight\Vert ^{2}
ight)$$

$$p\left(x|\mu,\sigma^{2}
ight)=\exp\left(-\pi\left\|x-\mu
ight\|^{2}
ight)$$

Substituing the value of $x=\left[egin{array}{c} rac{1}{\sqrt{\pi}} \\ 2 \end{array}
ight]$ and $\mu=\left[egin{array}{c} 0 \\ 2 \end{array}
ight]$, we have

$$p\left(x|\mu,\sigma^2
ight) = \exp\left(-\pi\left[\left(rac{1}{\sqrt{\pi}}-0
ight)^2 + (2-2)^2
ight]
ight)$$

$$p\left(x|\mu,\sigma^2
ight)=\exp\left(-\pirac{1}{\pi}
ight)$$

$$p\left(x|\mu,\sigma^2\right) = \exp\left(-1\right)$$

$$\log(p(x|\mu, \sigma^2)) = \log(\exp(-1)) = -1$$

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You have used 2 of 2 attempts

1 Answers are displayed within the problem

Discussion

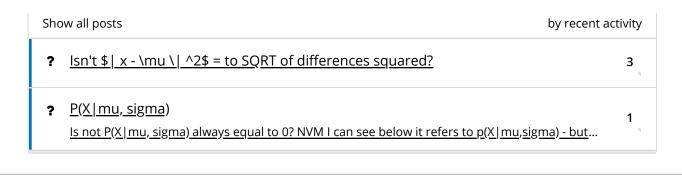
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