3. Introduction to Mixture Models

Gaussian Mixture Model: Definitions

0:00 / 0:00 1.25x

Video

Download video file

Transcripts

Download SubRip (.srt) file
Download Text (.txt) file

Gaussian Mixture Model: Definitions

1/1 point (graded)

Assume a Gaussian mixture model with K Gaussians such that we know all the means and variances. Assume that we also know the mixture weights $p_1, \ldots, p_K$. Let $\mathbf{x}$ be an observation obtained from the Gaussian mixture model. Let all of the parameters of the Gaussian mixture model be collectively represented as θ.

Which of the following are true?

- [x] We should be able to compute the probability density function (likelihood) $p(\mathbf{x}|\theta)$ given the information that we know.
- [x] We should be able to compute the probability that $\mathbf{x}$ belongs to each Gaussian component $j = 1, \ldots, K$ given the information that we know.

✔

Solution:

Both the statements are true. The generative Gaussian mixture model means that if we know all of the parameters of the K Gaussians and the mixture weights, the probability density function $p(\mathbf{x}|\theta)$ can be computed using the law of total probability as

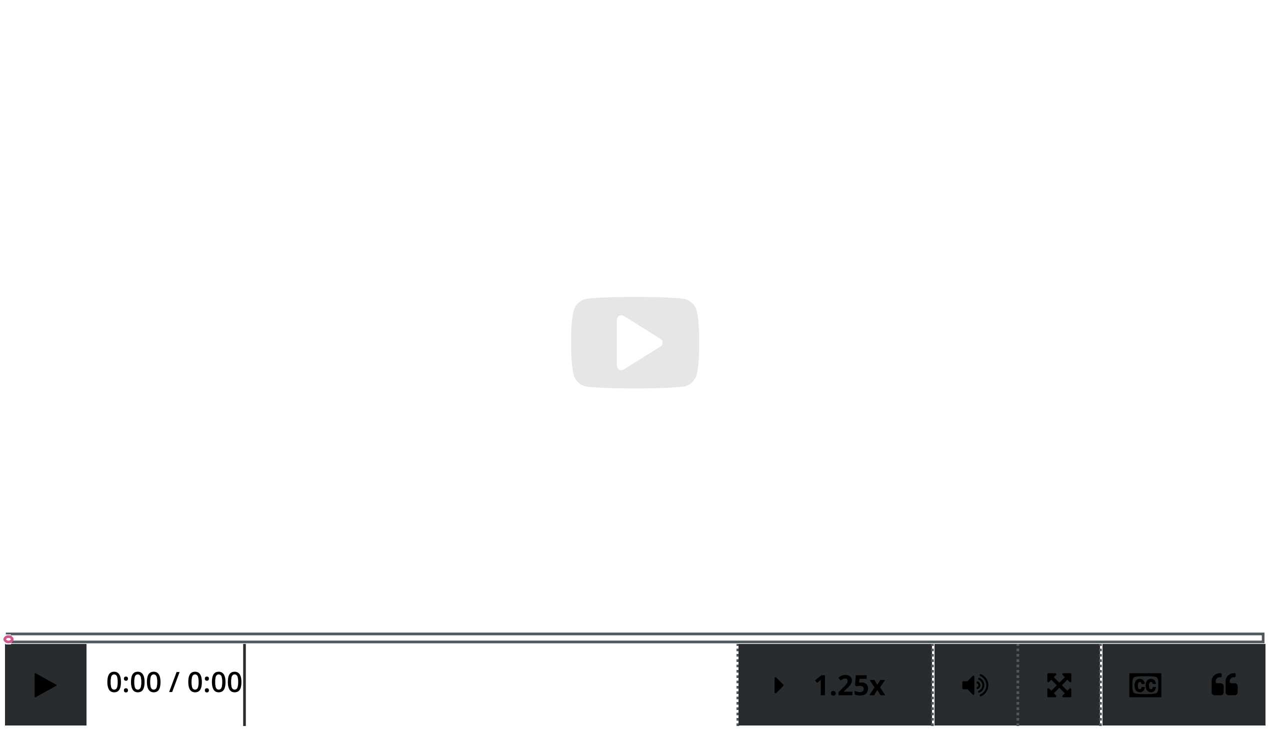
$$p(\mathbf{x}|\theta) = \sum_{j=1}^{K} p_j \mathcal{N}\left(\mathbf{x}; \mu^{(j)}, \sigma_j^2\right).$$

The posterior probability that $\mathbf{x}$ belongs to a Gaussian component j can then be computed using Bayes rule.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Likelihood of Gaussian Mixture Model



Video

Download video file

Transcripts

Download SubRip (.srt) file

Download Text (.txt) file

Gaussian Mixture Model: Definitions

A **Gaussian Mixture Model (GMM)** , which is a generative model for data $\mathbf{x} \in \mathbb{R}^d$, is defined by the following set of parameters:

1. K: Number of mixture components

2. A d-dimensional Gaussian $\mathcal{N}\left(\mu^{(j)}, \sigma_j^2\right)$ for every $j = 1, \ldots, K$

3. $p_1, \ldots, p_K$: Mixture weights

The parameters of a K-component GMM can be collectively represented as $\theta = \left\{p_1, \ldots, p_K, \mu^{(1)}, \ldots, \mu^{(K)}, \sigma_1^2, \ldots, \sigma_K^2\right\}$. Note that we have assumed the same variance σ_j^2 across all components of the j^{th} Gaussian mixture component for $j = 1, \ldots, K$. Further, every Gaussian component is assumed to have a diagonal covariance matrix. These are two assumptions that are made only for simplicity and the methodology presented can be extended to the setting of a general covariance matrix. Also, note that $\mu^{(j)}$ is a d-dimensional vector for every $j = 1, \ldots, K$.

The **likelihood** of a point $\mathbf{x}$ in a GMM is given as

$$p(\mathbf{x} \mid \theta) = \sum_{j=1}^{K} p_j \mathcal{N}\left(\mathbf{x}, \mu^{(j)}, \sigma_j^2\right).$$

The generative model can be thought of first selecting the component $j \in \{1, \ldots, K\}$, which is modeled using the multinomial distribution with parameters $p_1, \ldots, p_K$, and then selecting a point $\mathbf{x}$ from the Gaussian component $\mathcal{N}\left(\mu^{(j)}, \sigma_j^2\right)$.

Discussion

Hide Discussion

Topic: Unit 4 Unsupervised Learning (2 weeks) :Lecture 16. Mixture Models; EM algorithm / 3. Introduction to Mixture Models

Add a Post

Show all posts | by recent activity

? Meaning of "belonging to a Gaussian component". 1
I don't understand the 2nd option in Q1 (Gaussian Mixture Model: Definitions): > We should ...

? Difference between some terms 3
What is the difference between "mixture components" and Clusters ? what is the difference b...