6. Hidden Layer Models

Models with Hidden Layer

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For the following set of problems, let's consider a simple 2-dimensional classification task. The training set is made up of 4 points listed below:

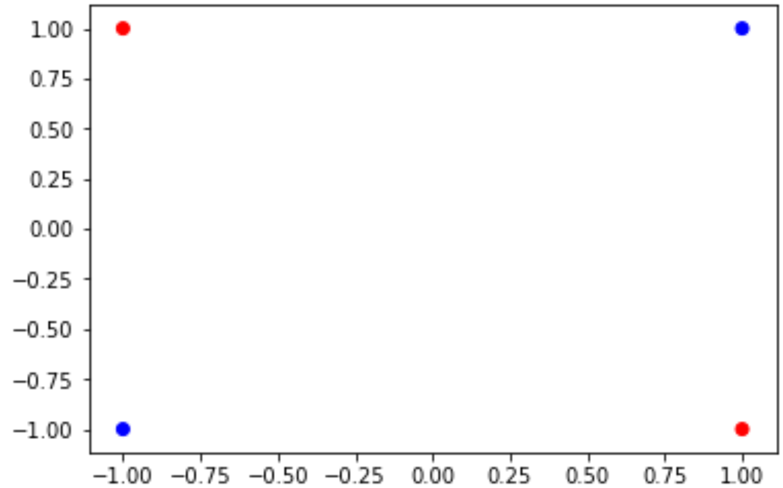
$$x^{(1)} = (-1,-1) \quad , \qquad y^{(1)} = 1$$

$$x^{(2)} = (1,-1) \quad , \qquad y^{(2)} = -1$$

$$x^{(3)} = (-1,1) \quad , \qquad y^{(3)} = -1$$

$$x^{(4)} = (1,1) \quad , \qquad y^{(4)} = 1$$

The dataset is illustrated below (blue - positive, red - negative)

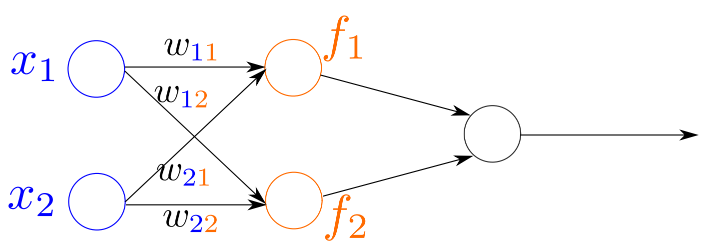


For simplicity, assume that we are only interested in binary classification problems for now. That is, $y^{(i)}$ can be either 1 or -1.

Linear Separability After First Layer

1/1 point (graded)

For this problem, let us focus on a network with one hidden layer and two units in that layer:



Let $f_1^{(i)}, f_2^{(i)}$ denote the output of the two units in the hidden layer corresponding to the input $x^{(i)}$ respectively, i.e.

$$\begin{aligned} f_1^{(i)} &= f\left(w_{01} + \left(w_{11}x_1^{(i)} + w_{21}x_2^{(i)}\right)\right) \\ f_2^{(i)} &= f\left(w_{02} + \left(w_{12}x_1^{(i)} + w_{22}x_2^{(i)}\right)\right) \end{aligned}$$

Consider the set $D' = \left\{ \left(\left[f_1^{(i)}, f_2^{(i)} \right], y^{(i)} \right), \quad i = 1, 2, 3, 4 \right\}$.

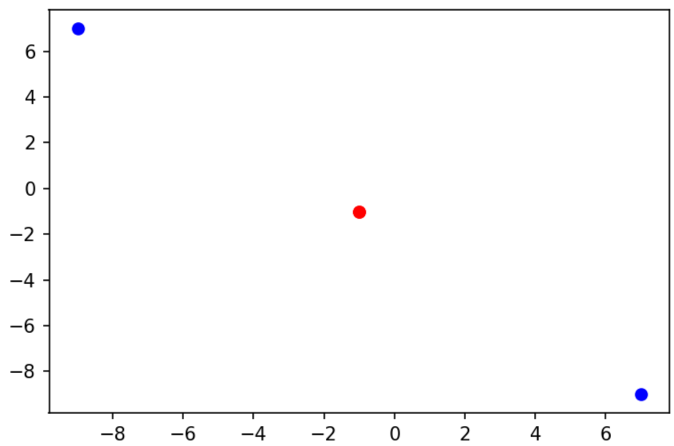
Assume that f is the linear activation function given by $f(z) = 2z - 3$.

For which of the following values of weights would the set D' be linearly separable? (Select all that apply.)

- [ ] $w_{11} = w_{21} = 0, w_{12} = w_{22} = 0, w_{01} = w_{02} = 0$
- [ ] $w_{11} = w_{21} = 2, w_{12} = w_{22} = -2, w_{01} = w_{02} = 1$
- [ ] $w_{11} = w_{21} = -2, w_{12} = w_{22} = 2, w_{01} = w_{02} = 1$
- [x] None of the above

Solution:

First of all note that from the figure in the text above that D is clearly not linearly separable.
Also, $f(z) = 2z - 3$ is a linear activation function.
Any linear transformation of the feature space of a linearly in-separable classification problem would still continue to remain linearly inseparable. For this question, one can compute the feature representations of all the data points and verify visually.
For example, the result of the second answer is plotted here:



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You have used 1 of 2 attempts

Answers are displayed within the problem

Non-linear Activation Functions

1/1 point (graded)

Again, let's focus on a network with one hidden layer with two units and use the same training set as above. The weights of the network are given as follows:

$$w_{11} = 1, w_{21} = -1, w_{01} = 1$$
$$w_{12} = -1, w_{22} = 1, w_{02} = 1$$

Let f_1, f_2 be the outputs of the first and second unit respectively.

Consider the set $D' = \{([f_1^{(i)}, f_2^{(i)}], y^{(i)}), \quad i = 1, 2, 3, 4\}$

For which of the following functions f, would the set D' be linearly separable? (Select one or more that apply.)

- [ ] $f(z) = 5z - 2$
- [x] $f(z) = \text{ReLU}(z)$
- [x] $f(z) = \tanh(z)$
- [ ] $f(z) = z$

Solution:

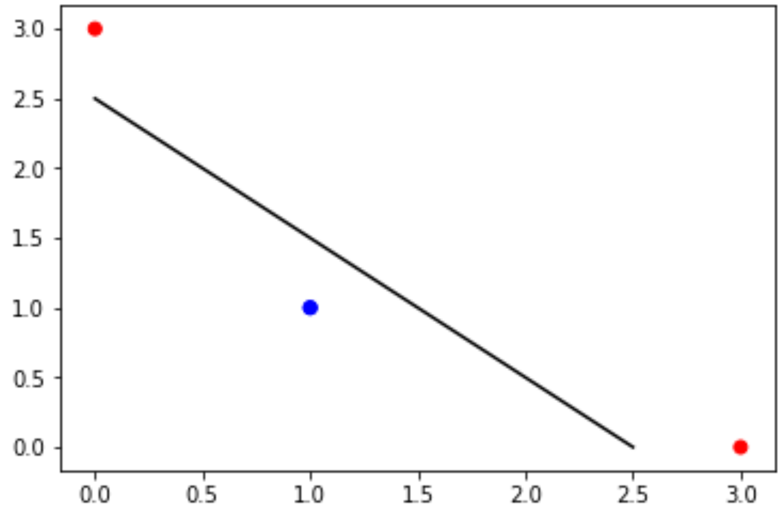
From the above problem, we note that any linear transformation of the feature space of a linearly in-separable classification problem would still continue to remain linearly inseparable. Hence we rule out the two linear functions.
For all of the parts below, note that

$$\begin{aligned} f_1^{(i)} &= f\left(w_{01} + \left(w_{11}x_1^{(i)} + w_{21}x_2^{(i)}\right)\right) \\ f_2^{(i)} &= f\left(w_{02} + \left(w_{12}x_1^{(i)} + w_{22}x_2^{(i)}\right)\right) \end{aligned}$$

- $f(z) = \text{ReLU}(z)$: Substituting for ReLU into f in the above equation gives the following results:

  $$(1,1), (3,0), (0,3), (1,1)$$

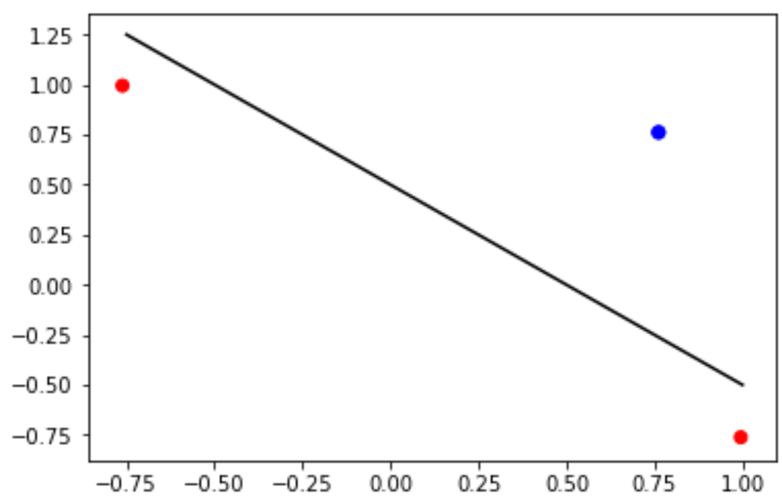
  The following figure plots these points and a potential linear classifier:



- $f\left(x\right) = \tanh\left(x\right)$: Substituting for $tanh$ into f in the above equation gives the following results:

$$\left(0.76, 0.76\right), \left(0.99, -0.76\right), \left(-0.76, 0.99\right), \left(0.76, 0.76\right)$$

The following figure plots these points and a potential linear classifier:



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Answers are displayed within the problem

Neural Network Learned parameters

1/1 point (graded)

Given a neural network with one hidden layer for classification, we can view the hidden layer as a feature representation, and the output layer as a classifier using the learned feature representation.

There're also other parameters that will affect the learning process and the performance of the model, such as the learning rate and parameters that control the network architecture (e.g. number of hidden units/layers) etc. These are often called hyper-paramters.

Which of the following is/are optimized during a single training pass? (Note that cross-validation is tuned before this point.) Check all that apply.

- [ ] The dimension of the feature representation
- [x] The weights that control the feature representation
- [ ] The hyper-parameters
- [x] The weights for the classifier

Solution:

Similar to the linear classifiers that we covered in previous lectures, we need to learn the parameters for the classifier. However, in this case we also learn the parameters that generate a representation for the data.
The dimensions and the hyper-parameters are decided with the structure of the model and are not optimized directly during the learning process but can be chosen

by performing a grid search with the evaluation data or by more advanced techniques (such as meta-learning).

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The linear separability problems were tough
Not conceptually, but as effort / time. Any way to solve them without iterating over all the po... 1

Thank you
This lecture really helped to wrap my head around what activation functions actually do. Kud... 1

[Staff] Typo
*"These are often called hyper-**paramters**."* 2