7. Univariate Gaussians

A univariate **Gaussian** or **normal distributions** can be completely determined by its mean and variance.

Gaussian distributions can be applied to a large numbers of problems because of the central limit theorem (CLT). The CLT posits that when a large number of **independent and identically distributed ((i.i.d.)** random variables are added, the cumulative distribution function (cdf) of their sum is approximated by the cdf of a normal distribution.

Recall the probability density function of the univariate Gaussian with mean μ and variance σ^2, $\mathcal{N}\left(\mu, \sigma^2\right)$:

$$f_X\left(x\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}.$$

Probability review: PDF of Gaussian distribution

2/2 points (graded)

In practice, it is not often that you will need to work directly with the probability density function (pdf) of Gaussian variables. Nonetheless, we will make sure we know how to manipulate the (pdf) in the next two problems.

The pdf of a Gaussian random variable X is given by

$$f_X\left(x\right) = \frac{n}{3\sqrt{2\pi}} \exp\left(-\frac{n^2\left(x-2\right)^2}{18}\right),$$

then what is the mean μ and variance σ^2 of X?
(Enter your answer in terms of n.)

$\mu =$ 2 ✓ **Answer:** 2

2

$\sigma^2 =$ 9/n^2 ✓ **Answer:** 9/n^2

$\frac{9}{n^2}$

STANDARD NOTATION

Solution:

Comparing

$$f_X(x) = \frac{n}{3\sqrt{2\pi}}\exp\left(-\frac{n^2(x-2)^2}{18}\right) = \frac{1}{(3/n)\sqrt{2\pi}}\exp\left(-\frac{(x-2)^2}{2(3/n)^2}\right)$$

with

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

yields $\mu = 2$ and $\sigma^2 = \frac{9}{n^2}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Probability review: PDF of Gaussian distribution

1/1 point (graded)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$, i.e. the pdf of X is

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

Let $Y = 2X$. Write down the pdf of the random variable Y. (Your answer should be in terms of y, σ and μ. Type **mu** for μ, **sigma** for σ.)

$f_Y(y) =$

1/(2*sigma*sqrt(2*pi))*exp(-(y-2*mu)^2/(8*sigma^2))

✓ **Answer:** 1/(2*sigma*sqrt(2*pi))* exp(-(y-2*mu)^2/(8*sigma^2))

$$\frac{1}{2\cdot\sigma\cdot\sqrt{2\cdot\pi}}\cdot\exp\left(-\frac{(y-2\cdot\mu)^2}{8\cdot\sigma^2}\right)$$

STANDARD NOTATION

Solution:

If $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$, then $Y = 2X \sim \mathcal{N}\left(2\mu, 4\sigma^2\right)$ by the following general properties of expectations and variance:

$$\begin{aligned} \mathbf{E}\left[2X\right] &= 2\mathbf{E}\left[X\right] \\ \backslash\text{var}\left[2X\right] &= 2^2\backslash\text{var}\left[X\right] = 4\backslash\text{var}\left[X\right]. \end{aligned}$$

Therefore,

$$f_Y\left(y\right) = \frac{1}{2\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(y-2\mu\right)^2}{2\left(4\sigma^2\right)}\right).$$

Alternate solution: In general, for any continuous random variables X and any continuous monotonous (i.e. always increasing or always decreasing) function g, such that $Y = g\left(X\right)$, the pdf of Y is given by:

$$f_Y\left(y\right) \quad = \frac{f_X\left(x\right)}{\left|g'\left(x\right)\right|} \qquad \text{where } x = g^{-1}\left(y\right).$$

In this problem, $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$, $Y = g\left(X\right) = 2X$, and $g'\left(x\right) = 2$. Therefore:

$$\begin{aligned} f_Y\left(y\right) &= \frac{f_X\left(\frac{y}{2}\right)}{\left|g'\left(\frac{y}{2}\right)\right|} \\ &= \frac{1}{g'\left(y/2\right)\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(y/2-\mu\right)^2}{2\sigma^2}\right) \\ &= \frac{1}{2\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(\left(y-2\mu\right)/2\right)^2}{2\sigma^2}\right) \\ &= \frac{1}{2\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(\left(y-2\mu\right)\right)^2}{2\left(4\right)\sigma^2}\right) \end{aligned}$$

and we recover the same answer as above.

Submit

You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

Argmax

1/1 point (graded)
Let $f_X\left(x;\mu,\sigma^2\right)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2. What value of x maximizes this function?

(Enter **mu** for the mean μ, and **sigma^2** for the variance σ^2.)

mu ✓ **Answer:** mu

μ

STANDARD NOTATION

Solution:

The answer is μ, the mean of the distribution. If you look at the graph of the standardized normal distribution, you see that the maximum is at 0, its mean. Any normal distribution with different mean or variance is simply a shifted (different mean) or stretched (different variance) version of this distribution, so our result holds for any normally distributed variable. Alternatively, you can differentiate the PDF and determine the maximum, which gives you the same result.

Submit You have used 1 of 3 attempts

ℹ Answers are displayed within the problem

Maximum of pdf

1/1 point (graded)
As above, let $f_X\left(x;\mu,\sigma^2\right)$ denote the probability density function of a normally distributed variable X with mean μ and variance σ^2.

What is the maximum value of $f_X\left(x;\mu,\sigma^2\right)$?
(Enter **mu** for the mean μ, and **sigma^2** for the variance σ^2.)

1/(sigma*sqrt(2*pi)) ✓

Answer: 1/sqrt(2*pi*sigma^2)

$\frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}$

STANDARD NOTATION

Solution:

From the question above, we know that the maximum value occurs when $x = \mu$. Observe the PDF of a normal variable: setting $x = \mu$ forces the exponent of e to 0, leaving us with the answer above.

Submit You have used 1 of 3 attempts

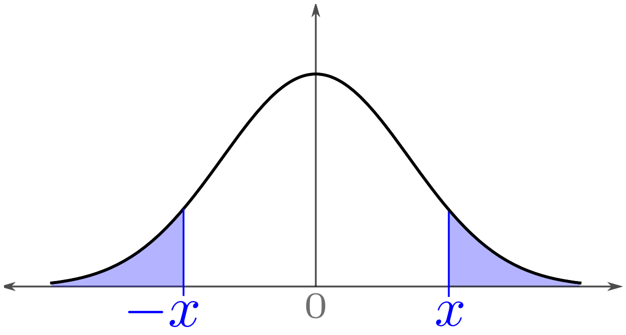
Answers are displayed within the problem

Quantiles

1/1 point (graded)

The **quantile** of order $1 - \alpha$ of a variable X, denoted by q_α (specific to a particular X), is the number such that $\mathbf{P}\left(X \leq q_\alpha\right) = 1 - \alpha$.

Graphed below is the pdf of the normal distribution with generic/unknown (but fixed) variance σ^2. If the total area of the two shaded regions is 0.03, then what is x?
(Choose all that apply.)



The total area of the two shaded regions is 0.03.

☐ $\mathbf{P}\left(|X| \leq 0.03\right)$

☐ $\mathbf{P}\left(|X| \leq 0.015\right)$

☐ 0.97

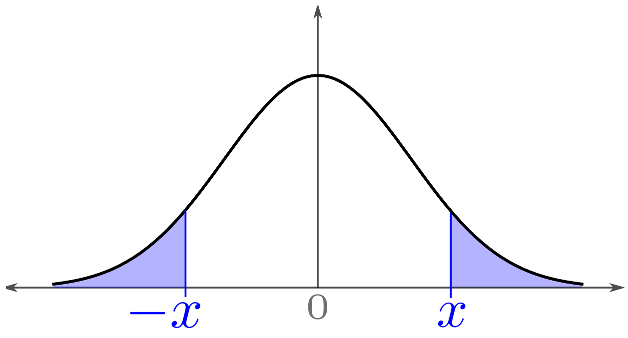
☐ 0.985

☐ $q_{0.03}$

☑ $q_{0.015}$

✓

Solution:



The total area of the two shaded regions equals $\mathbf{P}\left(|X| \geq x\right) = 0.03$. By symmetry, the probability in the positive tail is $\mathbf{P}\left(X \geq x\right) = 0.015$; hence $x = q_\alpha$ with $\alpha = 0.015$.

For the wrong choices:

- The first pair of choices mixed up the values of probability with the value of the variable.
- The third and fourth choices "0.97" and "0.985" are meant to play the role resembling $1 - \alpha$ in this example, but these are wrong for the same reasons as the first pair of choices. In any case, to give a particular numerical value of x, the answer must depend on σ.
- The fifth choice would have been correct again if the area of one of the tails is 0.03.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

Probability

1/1 point (graded)

Let $X \sim \mathcal{N}\left(1, 2\right)$, i.e., the random variable X is normally distributed with mean 1 and variance 2. What is the probability that $X \in [0.5, 2]$?

(Enter your answer accurate to at least 4 decimal places.)

$P\left(X \in [0.5, 2]\right) =$ 0.3984 ✓ **Answer:** 0.3984

STANDARD NOTATION

Solution:

One way to solve this problem is to integrate the PDF, which will give you the answer. Another way is to standardize the normal, giving us the variable $Z = \frac{X-1}{\sqrt{2}}$. We apply Z to the bounds $[0.5, 2]$ and then use a standard normal table to compute the answer.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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? My answer is correct, but it was flagged wrong
Dear TA, Could you please double check my answer for question "Probability review: PDF of Gaussian distribution". I am sure it is th... 2

💬 [Staff] Please check my answer to last question
I have used table to find the answer to last question. Its marked wrong. I can't find a mistake in my calculations. I am not yet familia... 3

? [Staff] Last question
Could you please check my answer to the last question? I use table, not coding. My answer was marked wrong but I couldn't find re... 8

💬 [STAFF] Last question
I couldn't find the reason why my last question was incorrect. Could I try again? Thank you. 2

☑ Unable to understand why my question 4 is wrong
I tried putting the values from R code using the difference of pnorm but I got it wrong all the time. Any idea what's wrong? 2

? Y=2X question
Failed to answer the Y=2X question correctly. Can someone explain what was i missing there... 16

💬 Maximum of PDF [STAFF]
Q4, what is the maximum value of the PDF? The answer seems very obvious and the question is very straight forward, yet my answ... 4

☑ Quantile
TA, please provide me with any reference that can guide me in answering the quantile question. Thank you! 2

? Any Hint on question Argmax and max ?
I was trying to have argMax evaluating fx(x) as per the gaussian pdf function but, it was not finding a globar max for it, now by defin... 3

💬 The last one..
i was so excited just to take the integral... through variable substitution... while guessed finally to google. Em.. such integral is not e... 1

💬 Just a friendly tip: use R if you are ok with the probability theory [this only applied to non-theoretical, numerical-answer questions]
While Probability has a lot of background (most of us already passed the course if we are right here, right now)... just use R for the c... 5

? last question
I confused in the last question I changed it to Standard normal distribution and I get P(-0.25<Z<0.5) then by using The Standard Nor... 2

? (staff) Probability review: PDF of Gaussian distribution

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