

#### Unit 0. Course Overview, Homework

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14. Interlude: Polynomials and

Geometric

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# 14. Interlude: Polynomials and Geometric

## **Quadratic Polynomials**

1/1 point (graded)

Recall a **degree** n polynomial in  $x_1, x_2, \ldots, x_k$  are all linear combinations of monomials in  $x_1, x_2, \ldots, x_k$ , where **monimials** in  $x_1, x_2, \ldots, x_k$  are **unordered words** using  $x_1, x_2, \ldots, x_k$  as the letters.

### **Examples:**

1. A degree 2, also known as quadratic, polynomial in the 1 variable  $\boldsymbol{x}$  is of the form

$$ax^2 + bx + c$$

for some numbers a,b,c. The polynomial is determined by the 3 coefficients a,b,c, and different choices of (a,b,c) result in different polynomials. In linear algebraic terms, the space of degree 2 polynomials in 1 variable is of dimension 3 since it consists of all linear combinations of 3 linearly independent vectors  $x^2, x$ , and 1.

2. A degree 2 polynomial in 2 variables  $x_1, x_2$  is of the form

$$ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f$$

for some numbers a,b,c,d,e,f. Different choices of (a,b,c,d,e,f) result in different polynomials.

In linear algebraic terms, the space of degree 2 polynomials in 2 variables is of dimension 6 since it consists of all linear combinations of 6 linearly independent vectors  $x_1^2$ ,  $x_2^2$ ,  $x_1x_2$ ,  $x_1$ ,  $x_2$ , and 1.

Consider degree 2 polynomials in 3 variables  $x_1, x_2, x_3$ . How many coefficients are needed to completely determine such a polynomial? Equivalently, what is the dimension of the space of polynomials in 3 variables such polynomials?

Number of coefficients needed/ Dimension:

10



Answer: 10

What is dimension of the polynomials of degree N in K variables? (This part of the question is optional and there is no answer box for it.)

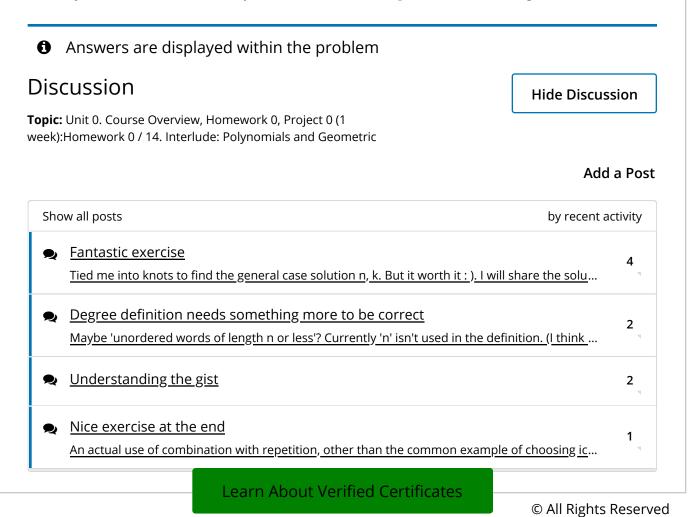
### **Solution:**

We count the number of monomials of length 2, 1, 0:

- ullet The monomials of length 2 are unordered pairs of  $x_1,x_2,\ x_3$ , hence there are  $inom{3}{2}$  This list consists of  $x_1^2,x_2^2,x_3^2,x_1x_2,x_1x_3,x_2x_3$ .
- The monomials of length 1 are  $x_1, x_2, x_3$ .
- ullet The monomial of length 0 is the constant term, i.e. 1.

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You have used 1 of 3 attempts



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