9. Gaussian Generative models

Gaussian Generative Models

0:00 / 0:00 1.25x

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Multivariate Gaussian Random Vector

A random vector $\mathbf{X} = \left(X^{(1)}, \ldots, X^{(d)}\right)^T$ is a **Gaussian vector** , or **multivariate Gaussian or normal variable** , if any linear combination of its components is a (univariate) Gaussian variable or a constant (a “Gaussian" variable with zero variance), i.e., if $\alpha^T \mathbf{X}$ is (univariate) Gaussian or constant for any constant non-zero vector $\alpha \in \mathbb{R}^d$.

The distribution of $\mathbf{X}$, the d**-dimensional Gaussian or normal distribution** , is completely specified by the vector mean $\mu = \mathbf{E}[\mathbf{X}] = \left(\mathbf{E}\left[X^{(1)}\right], \ldots, \mathbf{E}\left[X^{(d)}\right]\right)^T$ and the $d \times d$ covariance matrix Σ. If Σ is invertible, then the pdf of $\mathbf{X}$ is

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^d \det(\Sigma)}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1} (\mathbf{x}-\mu)}, \quad \mathbf{x} \in \mathbb{R}^d$$

where $\det(\Sigma)$ is the determinant of the Σ, which is positive when Σ is invertible.

If $\mu = \mathbf{0}$ and Σ is the identity matrix, then $\mathbf{X}$ is called a **standard normal random vector** .

Note that when the covariant matrix Σ is diagonal, the pdf factors into pdfs of univariate Gaussians, and hence the components are independent.

Gaussian Distribution

1/1 point (graded)

Recall that the likelihood of x being generated from a multi-dimensional Gaussian with mean μ and all the components being uncorrelated and having the same standard deviation σ is:

$$P(x|\mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{1}{2\sigma^2} \|x - \mu\|^2\right)$$

Let $x = \begin{bmatrix} \frac{1}{\sqrt{\pi}} \\ 2 \end{bmatrix}$ be a vector in the two-dimensional space.

Let G be a two-dimensional Gaussian distribution with mean μ and standard deviation σ taking values as follows

$$\mu = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \sigma = \sqrt{\frac{1}{2\pi}}$$

Calculate the likelihood $p\left(x|\mu,\sigma^2\right)$ of x being sampled from the Gaussian distribution G with mean μ and variance σ^2 taking values as given above.

Enter the value of $\log p\left(x|\mu,\sigma^2\right)$ below. (Note that we use log for the natural logarithm, i.e. $\log_e()$.)

-1

Answer: -1

Solution:

Note that the likelihood of vector x being sampled from a Gaussian distribution G with mean μ and variance σ^2 is given as follows

$$p\left(x|\mu,\sigma^2\right) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{1}{2\sigma^2}\|x-\mu\|^2\right)$$

Substituing the value of $\sigma = \sqrt{\frac{1}{2\pi}}$ from above, we have

$$p\left(x|\mu,\sigma^2\right) = \frac{1}{2\pi\frac{1}{2\pi}}\exp\left(-\frac{1}{2\frac{1}{2\pi}}\|x-\mu\|^2\right)$$

$$p\left(x|\mu,\sigma^2\right) = \exp\left(-\pi\|x-\mu\|^2\right)$$

Substituing the value of $x = \begin{bmatrix} \frac{1}{\sqrt{\pi}} \\ 2 \end{bmatrix}$ and $\mu = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, we have

$$p\left(x|\mu,\sigma^2\right) = \exp\left(-\pi\left[\left(\frac{1}{\sqrt{\pi}}-0\right)^2 + (2-2)^2\right]\right)$$

$$p\left(x|\mu,\sigma^2\right) = \exp\left(-\pi\frac{1}{\pi}\right)$$

$$p\left(x|\mu,\sigma^2\right) = \exp\left(-1\right)$$

$$\log\left(p\left(x|\mu,\sigma^2\right)\right) = \log\left(\exp\left(-1\right)\right) = -1$$

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Answers are displayed within the problem

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? Isn't $| x - \mu \| ^2$ = to SQRT of differences squared? 3

? P(X|mu, sigma) 1

Is not P(X|mu, sigma) always equal to 0? NVM I can see below it refers to p(X|mu,sigma) - but...