# 3. Kernels

In this question, we will practice kernel methods in classification.

## 3. (a)

1.0/1 point (graded)

Let $x, q \in \mathbb{R}^2$ be two feature vectors, and let $K(x, q) = (x^T q + 1)^2$. This is often known as a polynomial kernel. It's simple to compute: you just take the dot product between two feature vectors, add one, and then square the result. But what kind of feature mapping does this kernel implicitly use?

Assuming we can write $K(x, q) = \phi(x)^T \phi(q)$, derive an expression for $\phi(x)$.

Enter the solution as a vector $\phi(x) = [f_1(x_1, x_2), \cdots, f_N(x_1, x_2)]$.

$\phi(x) =$ [1, sqrt(2)*x_1, sqrt(2)*x_2, sqrt(2)*x_1*x_2, x_1^2, x_2^2] ✔

**Answer:** [x_1^2, x_2^2, sqrt(2)*x_1*x_2, sqrt(2)*x_1, sqrt(2)*x_2, 1]

**Solution:**

- We can rewrite the kernel as
$K(x,q) = (x^T q + 1)^2 = \left(1 + \sum_{i=1}^{2} x_i q_i\right)^2 = (x_1 q_1 + x_2 q_2 + 1)^2.$

- Expanding and combining terms gives
$x_1^2 q_1^2 + x_2^2 q_2^2 + 2x_1 x_2 q_1 q_2 + 2x_1 q_1 + 2x_2 q_2 + 1.$

- We can then rewrite this expression as $\phi(x)^T \phi(q)$ where
$\phi(x) = [x_1^2, x_2^2, \sqrt{2}x_1 x_2, \sqrt{2}x_1, \sqrt{2}x_2, 1].$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

## 3. (b)

1/1 point (graded)

As a simple example that uses this kernel, imagine that our feature vectors were bag of words vectors. In this example, give an intuitive interpretation of what the $\sqrt{2}x_1 x_2$ term in the expression for $\phi(x)$ you just wrote down means.

- consecutive co-appearance (bigram)
- co-appeareance in document ✔

**Solution:**

- Each token in the bag-of-word model only represents appearance in the document.
- Hence, $x_1 x_2$ represents co-appearance in a document.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

## Discussion

Hide Discussion

**Topic:** Unit 2 Nonlinear Classification, Linear regression, Collaborative Filtering (2 weeks):Homework 3 / 3. Kernels

Add a Post

Show all posts by recent activity

| | | |
|---|---|---|
| ? | 3.(a)<br>I don't get why it gives me 4/6 here, I checked and phi(x)*phi(q) gives me the same result as K... | 1 |
| 💬 | [Staff] Grader error in 3. (a) | 3 |
| ☑ | [Staff] : I'm getting the below error for 3(a)<br>Staff, please help me out to solve the error Invalid Input: 1 parenthesis was opened without b... | 4 |
| 💬 | This is how I feel<br>I read a comic once and this is how it went: Someone shoots some rays which makes everyo... | 1 |
| ? | Thinking out loud 3(b) [staff]<br>Am I on the right path? Let's say we have a bag of words defined by: [good, not] And we have:... | 4 |
| ☑ | which variables are allowed in the answer?<br>x, x1, x2 are not accepted. | 4 |
| ? | Can't figure out why I am only getting 1/2 point in 3a<br>I feel confident in the expression, plus I am getting half credit. So can only think I am making ... | 7 |
| 💬 | Tip: for expand cuadratic or cubic ecuation | 7 |
| ☑ | What does co-appearance actually mean?<br>Can someone be so kind to define co-appearance in 3(b) by giving an example (with words in ... | 3 |

| | | |
|---|---|---|
| ☑ | Invalid Input<br>I'm fairly certain that the answer to 3a involves a sqrt 2 term (it is even implied in 3b) but I get... | 3 |
| 💬 | I took a lot to do 3.a...<br>...and I will die in the Capstone (provided I survive this one), at this pace. | 7 |
| ? | Question 3a<br>In the lecture we learned that theta(x)= Sum of : alpha * y * phay. I read question 3a and I do... | 3 |

Learn About Verified Certificates