

<u>Unit 0. Course Overview, Homework</u>

Course > 0, Project 0 (1 week)

> Homework 0 > 5. Planes

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5. Planes

A hyperplane in n dimensions is a n-1 dimensional subspace. For instance, a hyperplane in 2-dimensional space can be any line in that space and a hyperplane in 3-dimensional space can be any plane in that space. A hyperplane separates a space into two sides.

In general, a hyperplane in n-dimensional space can be written as $heta_0+ heta_1x_1+ heta_2x_2+\cdots+ heta_nx_n=0.$ For example, a hyperplane in two dimensions, which is a line, can be expressed as $Ax_1+Bx_2+C=0.$

Using this representation of a plane, we can define a plane given an n-dimensional

vector
$$heta=egin{bmatrix} heta_1 \\ heta_2 \\ \vdots \\ heta_n \end{bmatrix}$$
 and offset $heta_0$. This vector and offset combination would define the

plane $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n = 0$. One feature of this representation is that the vector θ is normal to the plane.

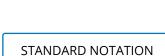
Number of Representations

1/1 point (graded)

Given a d-dimensional vector θ and a scalar offset θ_0 which describe a hyperplane $\mathcal{P}: \theta \cdot x + \theta_0 = 0$. How many alternative descriptions θ' and θ'_0 are there for this plane \mathcal{P} ?







Solution:

Given a normal vector θ and an offset θ_0 that uniquely determine the plane $\theta \cdot x + \theta_0 = 0$, we can scale θ and θ_0 by $\alpha > 0$, $\alpha \in \mathbb{R}$ without changing the orientation of the plane. Notice that if we only scale the normal $\theta' = \alpha \theta$ without affecting the offset $\theta'_0 = \theta_0$, then for $\alpha > 1$ the value of the θ'_0 must decrease for $\theta' \cdot x + \theta'_0 = 0$. Thus, there is an infinite number of possible parameter vectors that can describe the plane.

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You have used 1 of 1 attempt

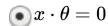
1 Answers are displayed within the problem

Orthogonality Check

0/1 point (graded)

To check if a vector x is orthogonal to a plane ${\mathcal P}$ characterized by θ and θ_0 , we check whether

 $\bigcap x = \alpha \theta \text{ for some } \alpha \in \mathbb{R} \checkmark$



$$\bigcirc x \cdot \theta + \theta_0 = 0$$



STANDARD NOTATION

Solution:

A vector x is orthogonal to the plane if and only if it is collinear with the normal vector θ of the plane.

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You have used 1 of 1 attempt

1 Answers are displayed within the problem

Perpendicular Distance to Plane

0/1 point (graded)

Given a point x in n-dimensional space and a hyperplane described by θ and θ_0 , find the **signed distance between the hyperplane and** x. This is equal to the perpendicular distance between the hyperplane and x, and is positive when x is on the same side of the plane as θ points and negative when x is on the opposite side.

(Enter **theta_0** for the offset θ_0 .

Enter **norm(theta)** for the norm $\|\theta\|$ of a vector θ .

Use * to denote the dot product of two vectors, e.g. enter ${\bf v}^*{\bf w}$ for the dot product $v\cdot w$ of the vectors v and w.)

abs(x*theta)/norm(theta)



Answer: (trans(theta)*x+theta_0)/norm(theta)

STANDARD NOTATION

Solution:

The distance from a point x_1 to a plane $\theta \cdot x + \theta_0$ is equal to $|\theta \cdot x_1 + \theta_0|/\|\theta\|$. If $\theta \cdot x_1 + \theta_0 > 0$, then x_1 belongs to a half-space in the direction of θ . Therefore, we can define the signed distance as:

$$d_{x_1} = rac{ heta \cdot x_1 + heta_0}{\| heta\|}$$

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You have used 5 of 5 attempts

1 Answers are displayed within the problem

Orthogonal Projection onto Plane

1.0/1 point (graded)

Find an expression for the **orthogonal projection** of a point v onto a plane $\mathcal P$ that is characterized by θ and θ_0 . Write your answer in terms of v, θ and θ_0 .

(Enter **theta_0** for the offset θ_0 .

Enter **norm(theta)** for the norm $\|\theta\|$ of a vector θ .

Use * to denote the dot product of two vectors, e.g. enter $\mathbf{v^*w}$ for the dot product $v\cdot w$ of the vectors v and w.)

v-((v*theta+theta_0)/norm(theta)^2)*theta

✓ Answer: v-(((trans(v)*theta)+theta_0)/(norm(theta))^2)*theta

STANDARD NOTATION

Solution:

Since v-x is collinear with the normal, $v-x=\lambda \theta$ for some λ . Also, x lies in the

plane, so $heta \cdot x + heta_0 = 0$. Solve this to get the value of λ and plug it back to find the orthogonal projection:

$$egin{array}{ll} (v-\lambda heta)\cdot heta+ heta_0&=&0 \ \lambda&=&rac{v\cdot heta+ heta_0}{\| heta\|^2} \ x&=&v-rac{v\cdot heta+ heta_0}{\| heta\|}\hat{ heta} \end{array}$$

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You have used 3 of 5 attempts

1 Answers are displayed within the problem

Perpendicular Distance to Plane

4/4 points (graded)

Let \mathcal{P}_1 be the hyperplane consisting of the set of points $x=\left[egin{array}{c} x_1 \ x_2 \end{array}
ight]$ for which

 $3x_1+x_2-1=0$. (Note that this hyperplane is in fact a line, since it is 1-dimensional.)

What is the signed perpendicular distance of point a=[-1,-1] from \mathcal{P}_1 ?

✓ Answer: -5/sqrt(10)

What is the signed perpendicular distance of the origin from \mathcal{P}_1 ?

✓ Answer: -1/sqrt(10)

What is the orthogonal projection of point a=[-1,-1] onto p_1 ?

First coordinate:

1/2

✓ Answer: 1/2

Second coordinate:

-1/2

✓ Answer: -1/2

STANDARD NOTATION

Solution:

1. For $a=\left[-1,-1\right]^T$ the signed distance is:

$$\frac{\theta \cdot a + \theta_0}{\|\theta\|} = \frac{(3)(-1) + (1)(-1) - 1}{\sqrt{(3)^2 + (1)^2}} = -\frac{5}{\sqrt{10}}$$

2. For $a=\left[0,0\right]^T$ the signed distance is:

$$rac{ heta \cdot 0 + heta_0}{\| heta\|} = rac{-1}{\sqrt{{(3)}^2 + {(1)}^2}} = -rac{1}{\sqrt{10}}$$

3. For $a = \left[-1, -1\right]^T$ the orthogonal projection is:

$$egin{aligned} x &= v - rac{v \cdot heta + heta_0}{\| heta\|} \hat{ heta} \ &= \left[-1, -1
ight]^T - rac{\left[-1, -1
ight]^T \cdot \left[3, 1
ight]^T + \left(-1
ight)}{\sqrt{\left(3
ight)^2 + \left(1
ight)^2}} \left[3/\sqrt{10}, 1/\sqrt{10}
ight]^T \ &= \left[1/2, -1/2
ight]^T \end{aligned}$$

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You have used 2 of 3 attempts

1 Answers are displayed within the problem

2. (f)

1/1 point (graded)

Consider a hyperplane in a d-dimensional space. If we project a point onto the plane, can we recover the original point from this projection?

no

Answer: no

STANDARD NOTATION

Solution:

Given a projection on a plane, there are infinitely many points that project to that point. They all lie along the normal to the plane which passes through that point.

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You have used 1 of 1 attempt

1 Answers are displayed within the problem

Discussion

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Distance to the plane notation I spent quite some time imputing x instead of x 1 and then much more time thinking I was wrong trans symbol what does trans symbol mean in answer?? [Staff] Answer to signed perpendicular distance from origin Dear Staffs, My answer -0.316 is correct upto 3 decimal places. Kindly check and update the grad Link for "Projection of a Vector onto a Plane" 1	Show all posts	by recent activity
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Link for "Projection of a Vector onto a Plane" 1	<u> </u>	7 ne grad
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Question about grading Hi Staff! I have a question regarding the grading system, if I did not finish the questions that wasn	2
[Staff] Please extend the deadline by 1 more day Please give us some more time to complete the prerequisite knowledge. Thank You	3
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