

Unit 4 Unsupervised Learning (2

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> Lecture 14. Clustering 2 >

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2. Limitations of the K Means Algorithm

Limitations of the K Means Algorithm



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Limitations of the K-Means Algorithm I

1/1 point (graded)

Remember that the K-Means Algorithm is given as below:

- 1. Randomly select z_1,\ldots,z_K
- 2. Iterate
 - 1. Given z_1,\dots,z_K , assign each data point $x^{(i)}$ to the closest z_j , so that

$$\operatorname{Cost}\left(z_{1}, \ldots z_{K}
ight) = \sum_{i=1}^{n} \min_{j=1,...,k} \left\|x^{(i)} - z_{j}
ight\|^{2}$$

2. Given C_1,\ldots,C_K find the best representatives z_1,\ldots,z_K , i.e. find z_1,\ldots,z_K such that

$$z_j = \operatorname{argmin}_z \sum_{i \in C_j} \left\| x^{(i)} - z
ight\|^2 = rac{\sum_{i \in C_j} x^{(i)}}{|C_j|}$$

where $\left|C_{j}\right|$ is the number of points in C_{j} .

Which of the following are **false** about K-Means Algorithm? Please choose all those apply.

- $luellet C_1,\ldots,C_K$ found by the algorithm is always a partition of $ig\{x_1,\ldots,x_nig\}$
- lacksquare It is always guaranteed that the K representatives $z_1,\dots,z_K\inig\{x_1,\dots,x_nig\}$
- The algorithm may output different C_1,\ldots,C_K and z_1,\ldots,z_K depending on the initialization of line 1
- Line 2.2 of the algorithm(Given C_1,\ldots,C_K find the best representatives z_1,\ldots,z_K ...) finds the cost-minimizing representatives $z_1,\ldots z_K$.



Solution:

It is not guaranteed that $z_1,\dots,z_K\in\{x_1,\dots,x_n\}$, because as in line 2.2 of the algorithm above, z_1,\dots,z_K are given by

$$z_j = rac{\sum_{i \in C_j} x^{(i)}}{|C_j|}$$

There is no guarantee that the centroid of all $x^{(i)}$ in a cluster will itself belong to $\left\{x_1,\ldots,x_n\right\}$. Depending on the application context, such as when clustering Google News articles, it can be problematic that a representative of a clustering is not an actual datapoint.

The other 3 choices are true:

- ullet Clustering always outputs C_1,\ldots,C_K that is a partition of $ig\{x_1,\ldots,x_nig\}$
- ullet The result of clustering depends on the initialization of z_1,\ldots,z_K .
- As we saw in the last lecture, line 2.2 of the algorithm

$$z_j = rac{\sum_{i \in C_j x^{(i)}}}{|C_j|}$$

minimizes the cost

$$\operatorname{Cost}\left(C_{1}, \dots C_{K}
ight) = \min_{j=z_{1}, ..., z_{K}} \sum_{j=1}^{k} \sum_{i \in C_{j}} \operatorname{dist}\left(x^{(i)}, z_{j}
ight)$$

where the distance function ${
m dist}\,(x^{(i)},z_j)$ is the squared euclidean distance function $\|x^{(i)}-z_j\|^2.$

Submit

You have used 1 of 3 attempts

1 Answers are displayed within the problem

Limitations of the K-Means Algorithm II

2/2 points (graded)

Suppose we have a 1D dataset drawn from 2 different Gaussian distribution $\mathcal{N}\left(\mu_1,\sigma_1^2\right)$, $\mathcal{N}\left(\mu_2,\sigma_2^2\right)$ where $\mu_1\neq\mu_2$. The dataset contains n data points from each of the two distributions for some large number n.

Define **optimal clustering** to be the assignment of each point to the more likely Gaussian distribution given the knowledge of the generating distribution.

Consider the case where $\sigma_1^2=\sigma_2^2$, would you expect a 2-means algorithm to approximate the optimal clustering?

Yes			
_			



Now if $\sigma_1^2 >> \sigma_2^2$, would you expect a 2-means algorithm to approximate the optimal clustering?

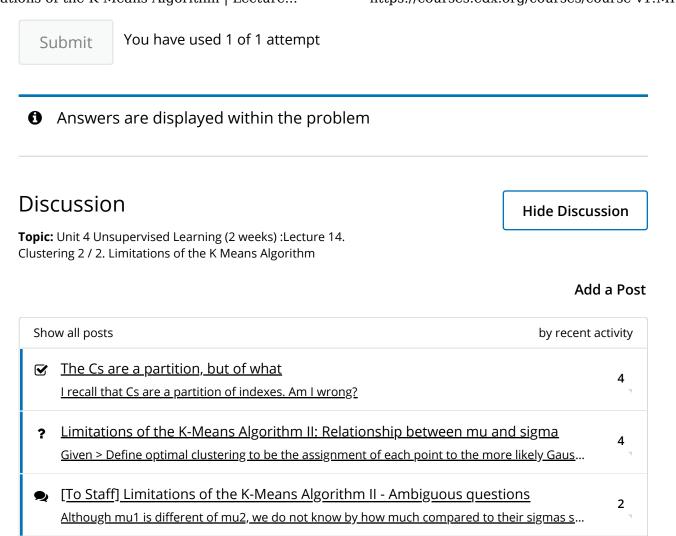
Yes			

● No



Solution:

When $\sigma_1^2=\sigma_2^2$, the boundary between the 2 optimal clusters is the midpoint between μ_1 and μ_2 . The 2 centroids found by the 2-means algorithm will also be approximately equidistant from this boundary (midpoint between μ_1 and μ_2), and therefore the assignment to clusters will be a similar split around the midpoint. When $\sigma_1^2>>\sigma_2^2$, the boundary between the 2 optimal clusters is closer to one centroid then the other. Since the 2-means algorithm will always have an equidistant split between the two centroids, this behavior cannot be reproduced and thus k-means clustering will erroneoously assign more points to the cluster with a smaller variance.



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