5. Planes

A hyperplane in n dimensions is a $n-1$ dimensional subspace. For instance, a hyperplane in 2-dimensional space can be any line in that space and a hyperplane in 3-dimensional space can be any plane in that space. A hyperplane separates a space into two sides.

In general, a hyperplane in n-dimensional space can be written as $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n = 0$. For example, a hyperplane in two dimensions, which is a line, can be expressed as $Ax_1 + Bx_2 + C = 0$.

Using this representation of a plane, we can define a plane given an n-dimensional vector $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$ and offset θ_0. This vector and offset combination would define the plane $\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n = 0$. One feature of this representation is that the vector θ is normal to the plane.

Number of Representations

1/1 point (graded)

Given a d-dimensional vector θ and a scalar offset θ_0 which describe a hyperplane $\mathcal{P} : \theta \cdot x + \theta_0 = 0$. How many alternative descriptions θ' and θ'_0 are there for this plane $\mathcal{P}$?

- 0
- 1
- ∞ (selected, correct)

STANDARD NOTATION

Solution:

Given a normal vector θ and an offset θ_0 that uniquely determine the plane $\theta \cdot x + \theta_0 = 0$, we can scale θ and θ_0 by $\alpha > 0$, $\alpha \in \mathbb{R}$ without changing the orientation of the plane. Notice that if we only scale the normal $\theta' = \alpha\theta$ without affecting the offset $\theta'_0 = \theta_0$, then for $\alpha > 1$ the value of the θ'_0 must decrease for $\theta' \cdot x + \theta'_0 = 0$. Thus, there is an infinite number of possible parameter vectors that can describe the plane.

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Answers are displayed within the problem

Orthogonality Check

0/1 point (graded)

To check if a vector x is orthogonal to a plane $\mathcal{P}$ characterized by θ and θ_0, we check whether

- $x = \alpha\theta$ for some $\alpha \in \mathbb{R}$ ✔
- (selected) $x \cdot \theta = 0$
- $x \cdot \theta + \theta_0 = 0$

✖

STANDARD NOTATION

Solution:

A vector x is orthogonal to the plane if and only if it is collinear with the normal vector θ of the plane.

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Answers are displayed within the problem

Perpendicular Distance to Plane

0/1 point (graded)

Given a point x in n-dimensional space and a hyperplane described by θ and θ_0, find the **signed distance between the hyperplane and** x. This is equal to the perpendicular distance between the hyperplane and x, and is positive when x is on the same side of the plane as θ points and negative when x is on the opposite side.

(Enter **theta_0** for the offset θ_0.
Enter **norm(theta)** for the norm $||\theta||$ of a vector θ.
Use ***** to denote the dot product of two vectors, e.g. enter **v*w** for the dot product $v \cdot w$ of the vectors v and w.)

abs(x*theta)/norm(theta) ✖

Answer: (trans(theta)*x+theta_0)/norm(theta)

STANDARD NOTATION

Solution:

The distance from a point x_1 to a plane $\theta \cdot x + \theta_0$ is equal to $|\theta \cdot x_1 + \theta_0|/\|\theta\|$. If $\theta \cdot x_1 + \theta_0 > 0$, then x_1 belongs to a half-space in the direction of θ. Therefore, we can define the signed distance as:

$$d_{x_1} = \frac{\theta \cdot x_1 + \theta_0}{\|\theta\|}$$

Submit You have used 5 of 5 attempts

Answers are displayed within the problem

Orthogonal Projection onto Plane

1.0/1 point (graded)

Find an expression for the **orthogonal projection** of a point v onto a plane $\mathcal{P}$ that is characterized by θ and θ_0. Write your answer in terms of v, θ and θ_0.

(Enter **theta_0** for the offset θ_0.
Enter **norm(theta)** for the norm $\|\theta\|$ of a vector θ.
Use ***** to denote the dot product of two vectors, e.g. enter **v*w** for the dot product $v \cdot w$ of the vectors v and w.)

v-((v*theta+theta_0)/norm(theta)^2)*theta

Answer: v-(((trans(v)*theta)+theta_0)/(norm(theta))^2)*theta

STANDARD NOTATION

Solution:

Since $v - x$ is collinear with the normal, $v - x = \lambda\theta$ for some λ. Also, x lies in the plane, so $\theta \cdot x + \theta_0 = 0$. Solve this to get the value of λ and plug it back to find the orthogonal projection:

$$\begin{aligned} (v - \lambda\theta) \cdot \theta + \theta_0 &= 0 \\ \lambda &= \frac{v \cdot \theta + \theta_0}{\|\theta\|^2} \\ x &= v - \frac{v \cdot \theta + \theta_0}{\|\theta\|}\hat{\theta} \end{aligned}$$

Submit You have used 3 of 5 attempts

Answers are displayed within the problem

Perpendicular Distance to Plane

4/4 points (graded)

Let $\mathcal{P}_1$ be the hyperplane consisting of the set of points $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for which $3x_1 + x_2 - 1 = 0$. (Note that this hyperplane is in fact a line, since it is 1-dimensional.)

What is the signed perpendicular distance of point $a = [-1, -1]$ from $\mathcal{P}_1$?

-5/sqrt(10) **Answer:** -5/sqrt(10)

What is the signed perpendicular distance of the origin from $\mathcal{P}_1$?

-1/sqrt(10) **Answer:** -1/sqrt(10)

What is the orthogonal projection of point $a = [-1, -1]$ onto p_1?

First coordinate:

1/2 **Answer:** 1/2

Second coordinate:

-1/2

Answer: -1/2

STANDARD NOTATION

Solution:

1. For $a = [-1, -1]^T$ the signed distance is:

$$\frac{\theta \cdot a + \theta_0}{\|\theta\|} = \frac{(3)(-1) + (1)(-1) - 1}{\sqrt{(3)^2 + (1)^2}} = -\frac{5}{\sqrt{10}}$$

2. For $a = [0, 0]^T$ the signed distance is:

$$\frac{\theta \cdot 0 + \theta_0}{\|\theta\|} = \frac{-1}{\sqrt{(3)^2 + (1)^2}} = -\frac{1}{\sqrt{10}}$$

3. For $a = [-1, -1]^T$ the orthogonal projection is:

$$\begin{aligned} x &= v - \frac{v \cdot \theta + \theta_0}{\|\theta\|}\hat{\theta} \\ &= [-1, -1]^T - \frac{[-1, -1]^T \cdot [3, 1]^T + (-1)}{\sqrt{(3)^2 + (1)^2}}[3/\sqrt{10}, 1/\sqrt{10}]^T \\ &= [1/2, -1/2]^T \end{aligned}$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

2. (f)

1/1 point (graded)

Consider a hyperplane in a d-dimensional space. If we project a point onto the plane, can we recover the original point from this projection?

no

Answer: no

STANDARD NOTATION

Solution:

Given a projection on a plane, there are infinitely many points that project to that point. They all lie along the normal to the plane which passes through that point.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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Distance to the plane notation 2
I spent quite some time imputing x instead of x_1 and then much more time thinking I was wrong...

? trans symbol 2
what does trans symbol mean in answer??

[Staff] Answer to signed perpendicular distance from origin 7
Dear Staffs, My answer -0.316 is correct upto 3 decimal places. Kindly check and update the grad...

Link for "Projection of a Vector onto a Plane" 1

- Question about grading 2
  Hi Staff! I have a question regarding the grading system, if I did not finish the questions that wasn...
- [Staff] Please extend the deadline by 1 more day 3
  Please give us some more time to complete the prerequisite knowledge. Thank You
- About the solution of "Orthogonal Projection onto Plane" 5
- Differentiate b/w dot product and multiplication 2
  Hi all course mates. I have received lots of problem regarding differentiation between dot produc...
- [STAFF] Scalar Product and Scalar Multiplication 1
  Community TA
- Hyperplane is NOT always a sub-space 5
- Orthogonal Projection onto Plane - can not input the answer 6
  Hello everyone, how are you? I'm trying to compute the expression of the Orthogonal Projection ...
- Perpendicular Distance to Plane 4
  staff : would you please let me know whats the problem in the first 2 questions . I am applying th...
- Orthogonal Projection onto Plane 3

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