Q4. Formal Proof :

Page No.
Date

* Push()

Let's assume (capacity = C_0) : (C_0 = some).

The edge case is the case, when the size = capacity.

; now, we generate a new array double the capacity of original array. and copy the elements from that to it.

∴ if the start size of array (before generating new)

= n;

then it takes = n (to generate new array)

= N (to copy elements from original array to new array...)

∴ A total of O(2n) time.

∴ For rest of normal Push operation, it takes place is O(1) time.

∴ for a given n Push operations =

$$\frac{1 + 1 + 1 + \cdots + 3.2^0 + 1 + \cdots + 3.2^1 + \cdots + (3.2^{\log_2 n})}{n}$$

$$\approx \frac{3(2n-1)}{n} + n \approx O(1).$$

* POP();

while popping out the elements;

Reverse the threshold to 25% of original size.

The edge case arises when;

$$\left(\text{size} = \frac{\text{capacity}}{4} + 1\right);$$

when this happens; a new memory is allocated of capacity = $\left(\frac{\text{capacity}}{4}\right)$;

& the elements are copied;

If $\frac{\text{capacity}}{4} = N$

This takes; $\left(\frac{N}{4}\right) * \left(\frac{N}{4}\right) = \left(\left(\frac{N}{4}\right)^2\right)$ time.

Otherwise fixed;

POP() operation takes O(1) time.

1.

(H) $H = 1 + 1 + 1 + \ldots + 2^{\log_2 n} + \ldots + 2^2 + 2^1 + 2^0 + \ldots + 1 + 1 + 1 + \ldots$

$\log_2 n$

$= \frac{n(n+1)}{n} + n = O(1)$ (Theory amortized cost)

Comment:

Page No.
Date

Here the growing factor was 2 ;

while the shrinking factor was 1/4.

This was done to ensure hysteresis

(which will Avert simultaneous growing and shrinking ...

This Making the average cost

still $O(1)$.